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SHOCK WAVE DEVELOPMENT IN THE COLLAPSE OF A CLOUD OF BUBBLES

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ABSTRACT

A numerical simulation of the collapse of a cloud of bubbles has been used to demonstrate the development of an inwardly propagating shock wave which grows rapidly in magnitude. The fully non-linear non-barotropic homogeneous flow equations are coupled with single bubble dynamics and solved by a stable numerical scheme. The computational results demonstrate the structure of the shock wave as well as its strengthening effect due to the coupling of the single bubble dynamics with the global dynamics of the flow through the pressure and velocity fields. This appears to confirm the speculation of Mørch and his co-workers that such shock formation is an important part of cloud collapse.

Nomenclature

A	Dimensionless radius of the bubble cloud
A_0	Dimensionless radius of the bubble cloud at the undisturbed reference condition
C_P	Pressure coefficient, $(p-p_0)/\frac{1}{2}\rho U^2$
$C_{P\infty}$	Pressure coefficient at infinity, $(p_\infty - p_0)/\frac{1}{2}\rho U^2$
$C_{P_{MIN}}$	Minimum pressure coefficient at infinity
D	Reference body size
R	Dimensionless bubble radius
R_0	Initial radius of bubble at undisturbed reference condition
Re	Reynolds number, UR_0/ν
S	Surface tension of the liquid
U	Reference velocity of the flow
We	Weber number, $\rho U^2 R_0/S$
k	Polytropic index of behavior of the gas inside the bubbles
p	Fluid pressure
p_0	Fluid pressure at undisturbed reference condition
p_v	Vapor pressure inside the bubble
p_∞	Pressure at infinity
r	Dimensionless Eulerian radial coordinate measured from the center of cloud
r_0	Dimensionless Lagrangian radial coordinate measured from the center of cloud and equal to r at undisturbed reference condition
t	Dimensionless time
t_G	Dimensionless duration of the low pressure perturbation
u	Dimensionless radial velocity of fluid
α	Void fraction of the bubbly mixture
α_0	Void fraction of the bubble mixture at the undisturbed reference condition
ρ	Density of liquid
σ	Cavitation number, $(p_0 - p_v)/\frac{1}{2}\rho U^2$
η	Dimensionless bubble population per unit liquid volume
ν	Kinematic viscosity of the liquid

1 Introduction

Much recent interest has focused on the dynamics and acoustics of finite clouds of cavitation bubbles because of the very destructive effects which are observed to occur when such clouds form and collapse in a flow (see, for example, Bark and Berlekom 1978, Soyama *et al.* 1992). This paper addresses the issue of the modelling of the dynamics of cavitation clouds, a subject whose origins can be traced to the work of van Wijngaarden (1964) who first attempted to model the behavior of a collapsing layer of bubbly fluid next to a solid wall. Later investigators explored numerical methods which incorporate the individual bubbles (Chahine 1982) and continuum models which, for example, analyze the behavior of shock waves in bubbly liquid (Noordzij and van Wijngaarden 1974) and identify the natural frequencies of spherical cloud of bubbles (d'Agostino and Brennen 1983). Indeed the literature on the linearized dynamics of clouds of bubbles is growing rapidly (see, for example, Omta 1987, d'Agostino *et al.* 1988 & 1989,

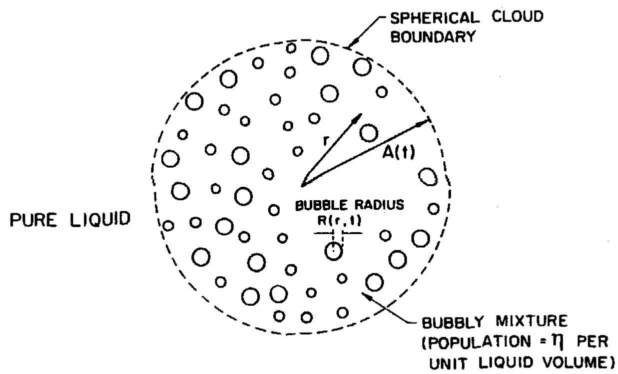


Figure 1: Schematic of a spherical cloud of bubbles

Prosperetti 1988). However, apart from some weakly non-linear analyses (Kumar and Brennen 1991, 1992, 1993) only a few papers have addressed the highly non-linear processes involved during the collapse of a cloud of bubbles. Chahine and Duraiswami (1992) have recently conducted numerical simulations using a number of discrete bubbles and demonstrated how the bubbles on the periphery of the cloud develop inwardly directed re-entrant jets. However, most clouds contain many thousands of bubbles and it therefore is advantageous to examine the non-linear behavior of continuum models, which is the subject of this paper.

Another perspective on the subject of collapsing clouds was that introduced by Mørch and his co-workers (Mørch 1980 & 1981, Hanson *et al.* 1981). They speculated that the collapse of a cloud of bubbles involves the formation and inward propagation of a shock wave and that the geometric focusing of this shock at the center of cloud creates the enhancement of the noise and damage potential associated with cloud collapse. One of the purposes of the present work is to examine whether or not continuum models of the cloud manifest such phenomena.

2 Basic Equations

The paper addresses the problem of the dynamics of a spherical cloud of bubbles in an unbounded liquid at rest at infinity, as shown in figure 1. To incorporate the interactive effects that the cavitating bubbles have on themselves and on the pressure and velocity of the liquid flow, the bubble dynamics must be included in the non-barotropic, homogeneous flow model. The basic equations used are same as those of d'Agostino and Brennen (d'Agostino & Brennen 1983; d'Agostino et al. 1988, 1989) except that all the nonlinear convective terms are retained since these are important in the context of the highly non-linear growth and collapse of the cloud.

The dimensionless forms of continuity and momentum equations for the one-dimensional spherical bubbly mixture can be written as

$$\frac{1}{r^2}\frac{\partial(r^2u)}{\partial r} = \frac{12\pi\eta R^2}{3+4\pi\eta R^3}\frac{DR}{Dt} \quad ,r \le A(t) \qquad (1)$$

$$\frac{Du}{Dt} = -\frac{1}{6}(3+4\pi\eta R^3)\frac{\partial C_P}{\partial r} \quad ,r \le A(t) \qquad (2)$$

where D/Dt indicates the Lagrangian derivative, $R(r,t)$ is the individual bubble radius, $C_P(r,t)$ is the pressure coefficient in the mixture, and the bubble population per unit liquid volume, η, is related to the void fraction, α, by $(\frac{4}{3}\pi R^3)\eta = \alpha/(1-\alpha)$. The variables and equations are non-dimensionalized using the initial bubble radius, R_0, a reference flow velocity, U, and the time scale, R_0/U. The pressure coefficient, C_P, and the bubble radius, R, are related by the Rayleigh-Plesset equation:

$$\begin{aligned} R\frac{D^2R}{Dt^2} &= -\frac{1}{2}C_P + \frac{2}{We}[R^{-3k} - R^{-1}] - \frac{3}{2}(\frac{DR}{Dt})^2 \\ &\quad +\frac{\sigma}{2}[R^{-3k} - 1] - \frac{4}{Re}\frac{1}{R}\frac{DR}{Dt}, r \le A(t) \quad (3) \end{aligned}$$

where σ is the cavitation number, $We = \rho U^2 R_0/S$ is the Weber number, S is the surface tension of liquid, $Re = U R_0/\nu$ is the Reynolds number, ν is the kinematic viscosity of liquid, and k is the polytropic index of the gas inside the bubble.

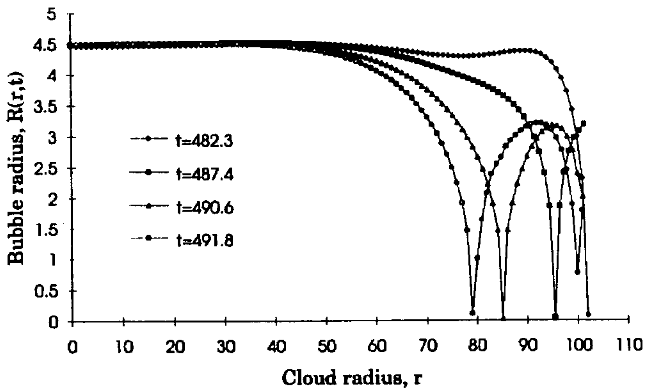
The above three equations (1), (2) and (3) with the appropriate boundary conditions can, in theory, be solved to find the unknown $C_P(r,t)$, $u(r,t)$ and $R(r,t)$ for any spherical bubbly cavitating flow. However the non-linearities in the Rayleigh-Plesset equation as well as those in the Lagrangian derivative, $D/Dt = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$, mean that only linearized and weakly non-linear solutions have been obtained previously.

The boundary conditions for the present analysis are as follows. The incompressible liquid flow outside the cloud, $r \ge A(t)$, has the standard solution of the form:

$$\begin{aligned} C_P(A(t),t) &= C_{P\infty}(t) + \frac{2}{A(t)}\frac{d[A^2(t)u(A(t),t)]}{dt} \\ &\quad -u^2(A(t),t) \qquad (4) \end{aligned}$$

where $C_{P\infty}(t)$ corresponds to the known driving pressure at infinity. At the center of cloud the symmetry of the problem requires

$$u(0,t) = 0. \qquad (5)$$



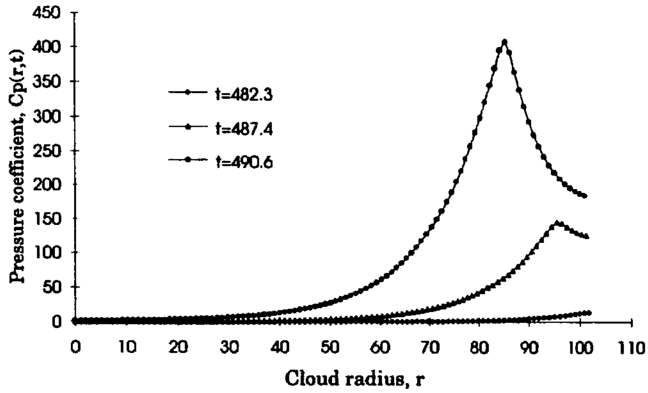


Figure 2: The dimensionless bubble size distribution in the cloud as a function of the dimensionless cloud radius at various times for σ=0.4, $C_{P_{MIN}}$=-0.5, α_0=0.08%, A_0= 1 cm, R_0=100 μm and the dimensionless time, t_G= 500.

Figure 3: The pressure coefficient distribution in the cloud as a function of the dimensionless cloud radius at various times. Parameters as in figure 2.

At time t=0, it is assumed that the whole flow field is in equilibrium. It is also assumed, for simplicity, that all the bubbles have the same initial radius R_0. Thus we have the following simple initial conditions: $R(r,0) = R_0$, $\frac{DR}{Dt}(r,0) = 0$, $u(r,0) = 0$, $C_P(r,0) = 0$. In the present solutions we have used a simple sinusoidal driving pressure as follows:

$$C_{P\infty}(t) = \begin{cases} C_{P_{MIN}}\{\cos(\frac{2\pi}{t_G}t) - 1\} & t < t_G \\ 0 & t > t_G \end{cases} \qquad (6)$$

where $C_{P_{MIN}}$ is the minimum pressure coefficient at infinity and t_G is the dimensionless duration of the low pressure perturbation. Consequently, for a cloud flowing with velocity U past a body of size D, t_G will be of the order of magnitude of D/U, $C_{P_{MIN}}$ will be the minimum pressure coefficient of the flow and σ is the conventional cavitation number.

3 Numerical Method

A stable numerical scheme was derived which is second order accurate in time and third order accurate in space. The scheme involves iteration of the governing equations at each time step. The program automatically adjusts the interval of each time step to ensure that the fractional change in bubble radius between any two consecutive times does not exceed some specific value, say 0.1. This is essential for time marching through a violent bubble collapse. Let a tilde represent the quantities which change during an iteration. Superscripts i and $i+1$ represent the quantities at time t_i and t_{i+1} respectively. At the beginning of this iterative procedure, the solution of the last time step is used for the initial tilde quantities. A complete time step proceeds as follows:

1) R^{i+1} and $\widetilde{(\frac{DR}{Dt})}^{i+1}$ are calculated using a Taylor's series and the solution of the previous time step, R^i, $(\frac{DR}{Dt})^i$ and $(\frac{D^2R}{Dt^2})^i$.

2) Equation (3) is used to calculate $\widetilde{(\frac{D^2R}{Dt^2})}^{i+1}$ using R^{i+1}, $\widetilde{(\frac{DR}{Dt})}^{i+1}$ and $\widetilde{C_P}^{i+1}$.

3) Integrated versions of equations (1) and (2) are used with R^{i+1}, $\widetilde{(\frac{DR}{Dt})}^{i+1}$, $\widetilde{C_P}^{i+1}$ and $\tilde{u}^{i+1}$ to obtain new values of $\widetilde{C_P}^{i+1}$ and $\tilde{u}^{i+1}$.

4) Steps 2 and 3 are repeated until $\tilde{u}^{i+1}$ and $\widetilde{C_P}^{i+1}$ converge. Under-relaxation must be used in iterating $\widetilde{C_P}^{i+1}$ to get convergence.

5) The new values of $\widetilde{(\frac{DR}{Dt})}^{i+1}$ are calculated using $(\frac{DR}{Dt})^i$, $(\frac{D^2R}{Dt^2})^i$, and $\widetilde{(\frac{D^2R}{Dt^2})}^{i+1}$ and the iteration is repeated by going back to step 2 until $\widetilde{(\frac{DR}{Dt})}^{i+1}$ converges.

The rate of convergence in the loop 2) - 4) depends strongly on the initial void fraction α_0; the larger α_0, the slower the convergence.

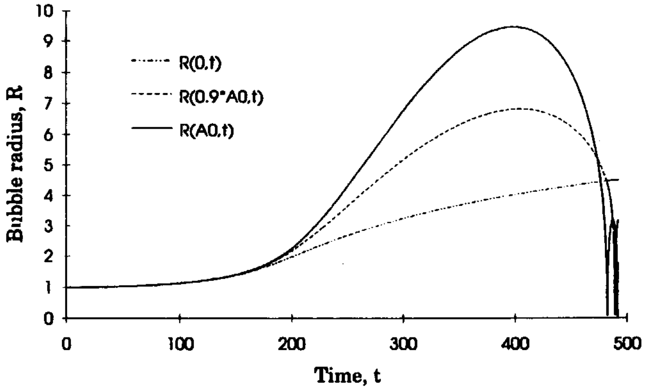


Figure 4: The time history of dimensionless bubble size at three different positions in the cloud, $r_0=0$, $r_0=0.9\ A_0$ and $r_0=A_0$. Parameters as in figure 2.

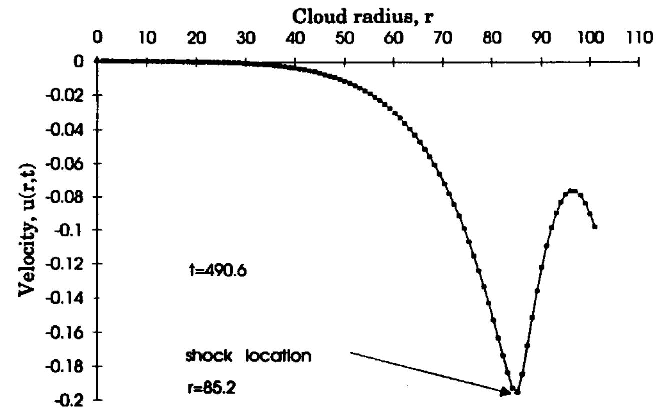


Figure 5: Typical dimensionless velocity distribution in the cloud as a function of the dimensionless cloud radius. The shock is passing the position of r=85.183 at this moment. Parameters as in figure 2.

4 Results and Discussion

For the purpose of illustrative calculations, parameters were chosen as follows. A cloud, composed of air bubbles (k=1.4) of initial radius R_0=100 μm and water at 20^oC (ρ=1000 kg/m^3, μ=0.001 $N sec/m^2$, S=0.0728 N/m), flows with velocity U=10 m/sec and has a void fraction α_0=0.08 %. The cavitation number of the flow is σ=0.4. The pressure perturbation experienced by the cloud has a duration of $t_G=5\times10^{-3}sec$ with $C_{P_{MIN}}=$ -0.5. Under these conditions the lowest resonant frequency of the cloud (d'Agostino and Brennen 1983) is about one-third of the single bubble natural frequency and therefore the calculations should exhibit the effects of non-linear cloud dynamics.

The calculations clearly demonstrated the formation of an inwardly propagating shock wave as can been seen in figures 2 and 3. The location of shock waves can be identified by the minimum bubble radius as well as the peak of the pressure coefficient in the cloud. The shielding of the interior bubbles by the outer shell of bubbles is readily apparent in these results. This shielding is a common feature of the linearized dynamics of clouds (see, for example, Omta 1987, d'Agostino and Brennen 1989, Chahine and Duraiswami 1992). Note also that the structure of the shock wave in figure 2 is qualitatively similar to that of the steady state shock propagation analysis of Noordij and van Wijngaarden (1974). However, quantitative comparison is difficult to make because of the rapid strengthening (figure 3) of the spherical shock wave which leads to its rapid inward acceleration. The present calculation had to be terminated when the strength of the shock became very large for then the adjusted time step became so small that the progress in time had essentially ceased.

Figure 4 shows the time history of the growth and collapse of bubbles at several different positions in the cloud. All the bubbles grow initially in the same way, but are later affected by the cloud dynamics.

A typical velocity distribution in the cloud is shown in figure 5. As anticipated, the magnitude of u as well as the values of $\partial u/\partial r$ mean that the global convective effects are important in the non-linear dynamics of the cloud.

5 Conclusions

This paper presents some preliminary results of the non-linear growth and collapse of a cloud of cavitating bubbles. We have developed an algorithm which allows simultaneous solution of the equations governing the cloud and have utilized this to investigate the bubble dynamics within a collapsing cloud. The objective is to investigate the significant enhancement of the cavitation noise (and damage potential) associated with the collective effects in cloud cavitation. Several years ago, Mørch and his co-workers speculated that one of the reasons for this enhancement was that an inward propagating shock wave forms during cloud collapse. The present calculations confirm the formation of such a shock wave and indicate that it rapidly gains strength. The investigations into the resulting cloud dynamics and noise are continuing.

Acknowledgments

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References

[1] Bark, G. and Berlekom, W.B. (1978). Experimental investigations of cavitation noise. *Proc. 12th ONR Symp. on Naval Hydrodynamics*, 470-493.

[2] Chahine, G.L. (1982). Cloud cavitation: theory. *Proc. 14th ONR Symp. on Naval Hydrodynamics*, 165-194.

[3] Chahine, G.L. and Duraiswami, R. (1992). Dynamical interactions in a multibubble cloud. *ASME J. Fluids Eng.*, **114**, 680-686.

[4] d'Agostino, L. and Brennen, C.E. (1983). On the acoustical dynamics of bubble clouds. *ASME Cavitation and Multiphase Flow Forum*, 72-75.

[5] d'Agostino, L. and Brennen, C.E., Acosta, A.J. (1988). Linearized dynamics of two-dimensional bubbly and cavitating flows over slender surfaces. *J. Fluid Mech.*, **192**, 485-509.

[6] d'Agostino, L. and Brennen, C.E. (1988). Acoustical absorption and scattering cross-sections of spherical bubble clouds. *J. Acoust. Soc. Am.*, **84**, 2126-2134.

[7] d'Agostino, L. and Brennen, C.E. (1989). Linearized dynamics of spherical bubble clouds. *J. Fluid Mech.*, **199**, 155-176.

[8] Hanson, I., Kedrinskii, V.K. and Mørch, K.A. (1981). On the dynamics of cavity clusters. *J. Appl. Phys.*, **15**, 1725-1734.

[9] Kumar, S. and Brennen, C.E. (1991). Non-linear effects in the dynamics of clouds of bubbles. *J. Acoust. Soc. Am.*, **89**, 707-714.

[10] Kumar, S. and Brennen, C.E. (1993). Some nonlinear interactive effects in bubbly cavitating clouds. *J. Fluid Mech.*, **253**, 565-591.

[11] Kumar, S. and Brennen, C.E. (1992). Harmonic cascading in bubble clouds. *Proc. Int. Symp. on Propulsors and Cavitation, Hamburg*, 171-179.

[12] Mørch, K.A. (1980). On the collapse of cavity cluster in flow cavitation. *Proc. First Int. Conf. on Cavitation and Inhomogenieties in Underwater Acoustics, Springer Series in Electrophysics*, **4**, 95-100.

[13] Mørch, K.A. (1981). Cavity cluster dynamics and cavitation erosion. *Proc. ASME Cavitation and Polyphase Flow Forum*, 1-10.

[14] Noordij, L. and van Wijngaarden, L. (1974). Relaxation effects, caused by relative motion, on shock waves in gas-bubble/liquid mixtures. *J. Fluid Mech.*, **66**, 115-143.

[15] Omta, R. (1987). Oscillations of a cloud of bubbles of small and not so small amplitude. *J. Acoust. Soc. Am.*, **82**, 1018-1033.

[16] Prosperetti, A. (1988). Bubble-related ambient noise in the ocean. *J. Acoust. Soc. Am.*, **84**, 1042-1054.

[17] Soyama, H., Kato, H. and Oba, R. (1992). Cavitation observations of severely erosive vortex cavitation arising in a centrifugal pump. *Proc. Third I.Mech.E. Int. Conf. on Cavitation*, 103-110.

[18] van Wijngaarden, L. (1964). On the collective collapse of a large number of gas bubbles in water. *Proc. 11th Int. Conf. Appl. Mech.*, Springer-Verlag, Berlin, 854-861.