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Granular Flow in a Vertically Vibrating Hopper

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Abstract

The behavior of the flow of glass spheres in a vertically vibrating hopper is examined. A two-dimensional hopper is mounted on a shaker that provides sinusoidal, vertical vibrations. Both the frequency and amplitude of the vibrations are adjustable. Hopper discharge rates and flow patterns, are measured as the acceleration amplitude of the vibrations is increased from 0 to $4g$'s. Comparisons are made with unvibrated hopper flows and with a two-dimensional discrete element simulation model.

Introduction

Perhaps the most common devices used for transporting and storing granular material are hoppers. These devices are sometimes subjected to localized vibrations to induce flow that has stopped due to bridging, a condition where particles interlock and form a bridge that supports the material above it. The applied vibration in this situation is either a hammer blow to the side of the hopper or the use of an out-of-balance motor attached to the hopper wall.

Few studies have examined the behavior of a granular material flow from a vibrating hopper. Takahashi *et al.* (1968) studied the trajectories and velocities of glass spheres in a two-dimensional wedge-shaped hopper (with a closed exit), which was subjected to vertical, sinusoidal vibrations. They found that particles moved in a circulatory motion where particles travel up at the inclined side walls and down in the central region. For a vibrating container with vertical walls the circulation is in the opposite direction (Evesque *et al.*, 1986, Knight *et al.*, 1993). Suzuki *et al.* (1968a) measured the discharge rate of glass spheres in the same hopper used by Takahashi *et al.* They found that the discharge rate increases with frequency, f, at a constant dimensionless acceleration amplitude, $\Gamma=4\Pi^2af^2/g$, where a is the amplitude of the

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vibrations and g is gravitational acceleration. For larger values of Γ the discharge rate was found to decrease. Further work by this same group (Suzuki *et al.*, 1968b) examined particle segregation in a vibrating hopper.

Evesque and Meftah (1993) observed a similar discharge effect for a vertically vibrated hourglass filled with sand. They found that the flow rate in the hourglass increased slightly for vibrations levels between 0 and just less than $1g$ but increases rapidly for Γ greater than $1g$. They were able to stop the flow for vibration frequencies between 40 and 60 Hz for large enough acceleration amplitudes.

It is clear that the behavior of a vertically vibrating hopper flow is quite complicated. This paper examines the behavior of glass spheres discharging from a vertically vibrated two-dimensional wedge-shaped hopper. Experimental observations and discharge rate measurements are compared with previous results and with results from two-dimensional discrete particle dynamics simulations.

Experiments

The experimental apparatus consisted of a two-dimensional wedge hopper mounted on a shaker that provided vertical, sinusoidal vibrations with controllable frequency, f, and amplitude, a. The interior walls of the hopper consisted of smooth glass on the front and rear faces and smooth lucite on the side faces. The hopper depth was 1.27 cm, the exit width was 4 mm, and the wall angle measured from the horizontal was 45 degrees. These dimensions were chosen to produce a funnel flow condition where a region of stagnant material formed along the inclined hopper side walls while the core flow continued to discharge.

The granular material was of 1.28 mm diameter glass spheres that were dyed black and fluorescent yellow to aid in flow visualization. The angle of repose of the material was roughly 30 degrees. Enough material was added to the hopper so that the total time to discharge the particles was always more than 20 seconds.

During discharge without vibration the free surface of the material formed a 'v' shaped valley. Particles continuously avalanched down the sloped surfaces. Next to the inclined walls two stagnant flow regions were observed that did not extend down to the exit. As the level of the material decreased, the stagnant flow regions became smaller until all of the material in the hopper flowed.

With vibration and a closed exit, a circulatory pattern was found, which was similar to that observed by both Takahashi *et al.* (1968) and Knight *et al.* (1993). A single layer of particles at the inclined side walls traveled up to the free surface, then avalanched down the surface to the centerline of the hopper and finally circulated to the bottom of the hopper. The region close to the exit of the hopper always remained stagnant.

Using a strobe light, the same vibrating flows were observed to form small amplitude standing waves on the free surface for levels of vibration greater than roughly $2.5g$. These waves were similar to those observed by Fauve *et al.* (1989), Wassgren *et al.* (1994), and Melo *et al.* (1994).

For discharging vibrated flows, the stagnant regions were not present. Instead, particles at the walls flowed slowly down toward the exit. The slope along the free surface of the material decreased as Γ increased until the small amplitude waves formed at which point the average slope of the surface was zero. The waves on the surface of the heap continued as the material discharged until the free surface was a few exit widths from the exit.

Measurements were made of the mean discharge rate by recording the time it took for the hopper to completely discharge for a given mass of material. The results are plotted in figure 1. For vibration levels below 1g the flow rate increased slightly for frequencies greater than 20 Hz while for flowrates between 1 and 4g the flow rates decreased and appeared to be approaching a constant value. These results are similar to those reported by both Takahashi *et al.* (1968) and Evesque *et al.* (1989).

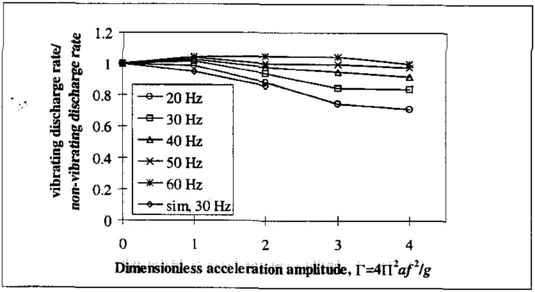

Figure 1. Hopper discharge rate as a function of dimensionless acceleration amplitude, $\Gamma=4\Pi^2af^2/g$, for various frequencies.

Discrete Element Simulations

In addition to the previous experimental measurements, a two-dimensional soft-particle discrete element simulation was performed to gain further insight into the flow dynamics. The only forces acting on the particles were gravity and contact forces. The contact force model consisted of a linear spring and dashpot in parallel acting in the normal and tangential directions with an additional sliding friction limiter in the tangential direction. All the simulations were performed with the parameters given in table 1. Due to the limited number of particles used in the simulations, particles that were discharged were subsequently used to refill the hopper. This provided a steady flow condition for the hopper and did not affect the flow near the hopper exit.

Number of particles	1000	Normal spring constant	1x10^6 N/m
Vibration frequency	30 Hz	Normal dashpot coeff.	6.279 N/(m/s)
Number of oscillation cycles	150	Tangential spring constant	1x10^6 N/m
Hopper exit width	5 mm	Tangential dashpot coeff.	6.279 N/(m/s)
Hopper wall angle from horizontal	45 deg	Tangential sliding friction coeff.	0.3
Particle diameters	0.90 - 1.10 mm	Integration time step	3.0x10^-6 sec

Table 1. Discrete element simulation parameters.

Flow patterns from the simulated hopper showed stagnant regions of the type similar to those observed in the experiments for the non-vibrating case. For the vibrating flows the stagnant regions slowly disappeared over time. Waves were also observed along the free surface of the heap for a simulation that did not have particles refilling the hopper.

Measurements of the simulated hopper discharge rate for a frequency of 30 Hz are displayed in figure 1. The discharge rate decreases with increasing Γ and has values less than the experimentally observed values. The latter effect is most likely due to the two-dimensionality of the simulation. Current simulations examine the discharge rates for other frequencies and acceleration amplitudes, and investigate how the discharge rate varies as a function of phase angle over an oscillation cycle.

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