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Convective heat transfer to rapidly flowing, granular materials

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Abstract—Convective heat transfer to a rapidly flowing, granular material is experimentally investigated over a flat plate in an inclined chute. Two different sizes of glass beads are used as the granular materials. A technique was developed to measure the average density of the flowing material, which allowed the more accurate determination of the average velocity as well as of parameters depending on the velocity. The results are presented in terms of the Nusselt number and a model is proposed relating this Nusselt number to a Péclet number and a Froude number. The predicted results of the model are compared with the experimental data.

1. INTRODUCTION

A SURPRISINGLY large number of materials such as coal, shale, metal ores, dry chemicals and grain are handled in granular form. In many applications, the material has to be heated or cooled. For example, in the preparation of gypsum, calcium sulfate has to be heated for dehydration. Also, charcoal is cooled after processing and before packaging. In the concepts for some future systems, heat exchange with the aid of a stream of granules is visualized for a solar energy converter. As a further example, flowing lithium oxide pellets may be made to act as a coolant for the walls in a rotary fusion energy reactor.

The importance of granular flow has been recognized for some time and a considerable amount of effort has been devoted to this subject [1]. Less attention has been paid to the processes of heat transfer to granular materials. This paper addresses heat transfer to a rapidly flowing, granular material.

2. BACKGROUND OF PRESENT INVESTIGATION

Much of the previous work directly related to the present study has been reviewed by Schlunder [2]. As pointed out by this author, the generally accepted concept of the convective heat transfer to a granular material includes a resistance to heat transfer caused by the interstitial fluid near the wall and the contact time between the moving material and the wall. The experiments conducted by Schlunder and others support this kind of model. Schlunder's work is concerned principally with flows in which there is little change in the average density of the flowing material. For higher flow rates (or shear rates) these density changes become important, but little is known about the heat transfer characteristics in such a case. In the limiting case, however, conditions approach those in a fluidized bed. The effects of the void spaces on heat transfer in a fluidized bed have been analyzed by Kubie and Broughton [3], Martin [4] and others and show characteristics similar to those of the present study as the density decreases over the heating plate.

The early work in our laboratory by Sullivan and Sabersky [5] also used the concept of an interstitial resistance near the wall. Based on this model, they arrived at an expression for the Nusselt number of the form

$$\overline{Nu_{d}^{*}} = \frac{1}{\chi + (\sqrt{\pi}/2)(1/\sqrt{Pe_{L}^{*}})}.$$

Sullivan and Sabersky's experimental data were represented very satisfactorily in terms of this equation. It should be noted that their experiment was performed in a hopper-bin configuration. At the low flow rates of their experiment, the material is not likely to experience the large shear rates which cause significant density changes. Thus, the model proposed by Sullivan and Sabersky which includes a constant thermal resistance between the heating plate and the granular material would seem appropriate. Following ref. [5], Spelt *et al.* [6] obtained experimental data for flow in a chute covering a wider range of flow variables. Spelt *et al.* found that Sullivan and Sabersky's correlation closely fit their data for the slow, high density flows, but beyond a certain flow region, this representation was inadequate. Spelt *et al.* found that by assuming constant density, the data could be presented by a family of curves, each corresponding to a given depth of flow. For each depth, the Nusselt number would at first increase with increasing Péclet number (or convective velocity), but would then reach a maximum and decrease for further increases in the Péclet number. This behavior is most unusual for convective heat transfer. It was speculated that the higher velocities brought about a decrease in the density of the flow and that this decrease in density caused the reduction of the heat transfer even though the velocity continued to increase.

NOMENCLATURE

c_p	specific heat of the bulk material
d	particle diameter
Fr	Froude number, $U^2/gh\cos\theta$
Fr^*	modified Froude number, $(U^2/gh\cos\theta)(\nu_c/\nu)(d/L)(k/k_g)$
g	gravitational acceleration
h	depth of flow
k	conductivity of the bulk material
k_g	conductivity of the interstitial fluid
L	length of heating plate
$\overline{Nu}_d^*$	Nusselt number, $\bar{h}d/k_g$
Pe_L^*	modified Péclet number, $(UL/\alpha)(d/L)^2(k/k_g)^2$
q	heat flux
T_m	temperature of the material
T_1	temperature at position '1' in the flow
T_w	wall temperature
U	average convective velocity
U_m	velocity at position 'm' in the flow
U_1	velocity at position '1' in the flow.

Greek symbols

α	bulk diffusivity
β	parameter, $\left[\frac{2}{\sqrt{\pi}}\left(\frac{U_m-U_1}{U}\right)\frac{\varepsilon}{\varepsilon_h}\frac{1}{\mu}\right]$
χ	experimental constant
ε	turbulent viscosity
ε_h	turbulent diffusivity
μ	friction coefficient
ν	solid fraction
ν_c	critical solid fraction ($\nu_c \simeq 0.56$)
Π	parameter
ρ_p	particle density
τ_w	wall shear
θ	chute angle.

3. EXPERIMENTAL INSTALLATION

The present experiments were conducted in a large chute, 3 m long and 150 mm wide. A mechanical conveyor delivers the material to an upper hopper from which the material flows into the chute. The discharge from the chute is collected in a lower hopper which, in turn, feeds the conveyor so that the chute can be operated continuously (see Fig. 1). The test section consists of a heated plate (see Fig. 2) which forms part of the base of the chute. The plate itself is made of copper and is flush mounted. The plate is heated by a series of flat 50-W ribbon heaters placed under the copper plate and backed by a phenolic plate

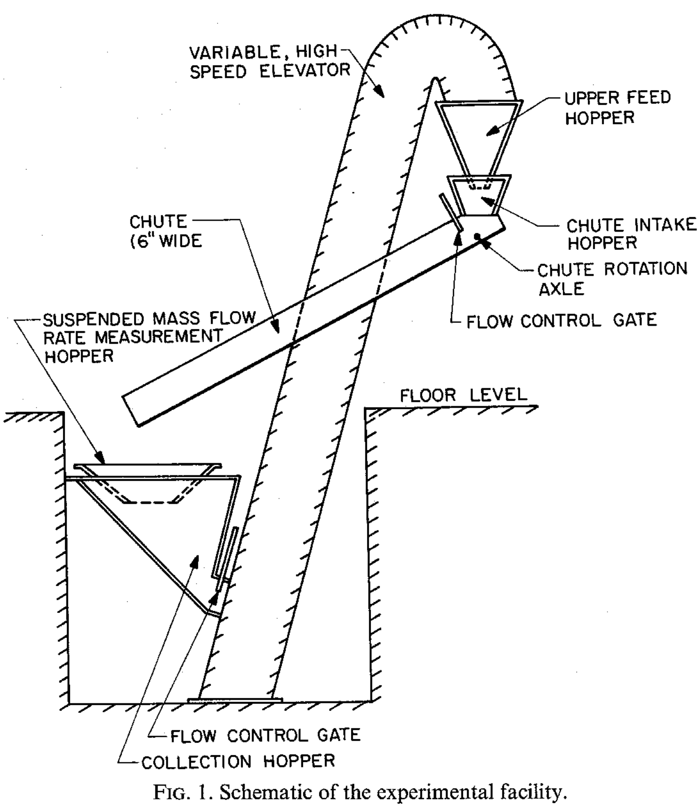

FIG. 1. Schematic of the experimental facility.

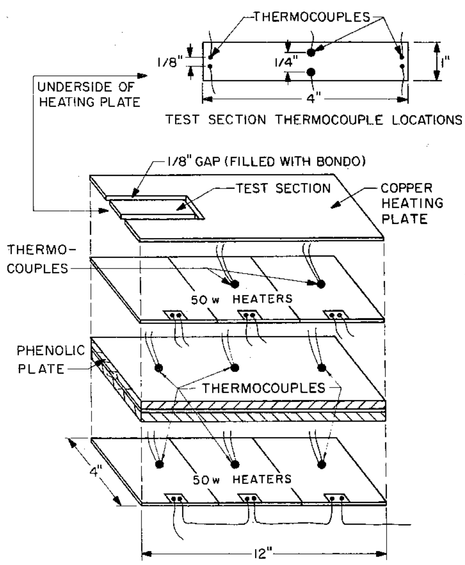

FIG. 2. Schematic of the heating plate.

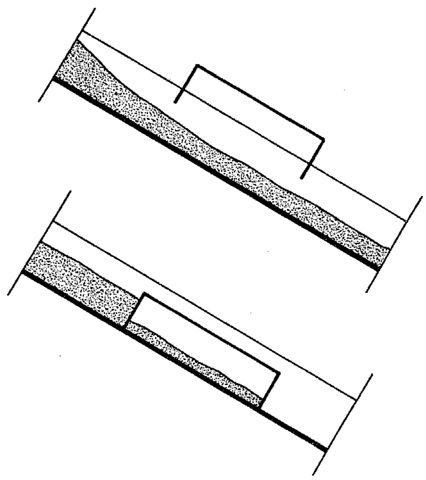

FIG. 3. Schematic of the density gauge.

of the same size. The phenolic plate served as an insulator to these heaters. As an added precaution, the phenolic plate is backed by a second group of 50-W ribbon heaters which act as guard heaters. By adjusting the guard heater, so that there is essentially no temperature differential across the phenolic plate, virtually all of the heat generated by the main heater flows through the copper plate and into the granular stream. To minimize further experimental errors, a section of the copper plate, which acted as the test section, was isolated from the remainder of the plate by a small gap which was filled with insulating material. This gap was designed to reduce any lateral flow of heat in the isolated section. The heat transfer rate was evaluated by measurement of the electric power provided to the heaters. The temperature of the test section was measured using three thermocouples. The temperature of the test section was calculated taking the average of these three thermocouples. The temperature of the granules was obtained from measurements of the material in the chute upstream of the heating plate. The temperature of the granules at the test section was taken to be equal to this measured value.

In addition to the temperatures, the various flow variables were also measured. The mass flow rate was determined by monitoring the rate of depletion of the upper feed hopper through a graduated transparent panel. The depth of the flow was measured in the test section by means of point probes similar to those commonly used in open-channel hydraulics. Furthermore, a simple yet effective method to measure the average density (or solid fraction) of the flowing material was developed (see [7]). This method involved two plates connected by a handle which were suddenly pushed into the flow thereby trapping the material in the space between the two plates (see Fig. 3). By collecting and weighing the trapped material, the average density of the flowing stream could then be computed using the previously measured depth of the flowing stream. Using this density, a meaningful average velocity could be computed. The ability to obtain an average density was an important factor in allowing a more realistic interpretation of the data and it contributes a significant improvement over the earlier work on heat transfer in chute flows.

The overlap in the Péclet number between ref. [5] and ref. [6] was quite small. To allow for a more direct comparison with Sullivan and Sabersky's data, the present series of experiments was designed to include relatively low velocities. These low velocities were conveniently produced by placing an obstruction into the downstream portion of flow and allowing a hydraulic jump to propagate upstream to the inlet gate.

4. PRESENTATION OF DATA

A large number of tests were conducted to determine the heat transfer coefficient to glass beads with diameters of 0.3 mm and 3.0 mm. Some of the physical properties of these beads are listed in Table 1. The heat transfer results, presented in terms of a Nusselt number and a Péclet number, are shown in Fig. 4. These two parameters are defined as

$$\overline{Nu_{\mathrm{d}}^*} = \frac{\bar{h}d}{k_{\mathrm{g}}}$$

$$Pe_{\mathrm{L}}^* = \frac{UL}{\alpha}\left(\frac{d}{L}\right)^2\left(\frac{k}{k_{\mathrm{g}}}\right)^2$$

and are the same as those used in refs. [5, 6]. Also shown in Fig. 4 is the curve corresponding to Sullivan and Sabersky's equation.

The present data shown in Fig. 4 exhibit the following phenomena. Up to a certain Péclet number, the Nusselt number closely follows the Sullivan–Sabersky correlation. The Nusselt number then reaches a maxi-

Table 1. Material properties

	Mean particle size (mm)	Bulk specific gravity	Thermal conductivity ($W\,m^{-1}\,{}^{\circ}C^{-1}$)	Thermal diffusivity ($m^2\,s^{-1}$)	Wall friction angle (aluminum)	Internal friction angle
Small glass beads	0.262	1.50	0.21	0.17	~16.0°	~16.0°
Large glass beads	2.94					

mum and decreases as the Péclet number is further increased. Both sizes of glass beads exhibit this phenomenon but the maximum Nusselt number for each size occurs at different Péclet numbers. Spelt *et al.* observed this same phenomenon. For each size of beads, however, the present results seem to satisfy a single relationship and it is no longer necessary to separate the data into individual curves, corresponding to different depths of flow. It appears that this simplification was brought about in a rather subtle way. The Péclet number computed by Spelt *et al.* was based on a velocity which assumed that the density of the material was constant and equal to the density at rest. The present measurements of the average density of the flowing material allow a more accurate value of the average velocity to be computed. With the aid of a Péclet number based on this improved value of the velocity, the heat transfer data for either material exhibits a single curve independent of the depth of flow. The Péclet number alone seems to reflect properly the effect of both velocity and density. However, the two sizes of beads define two different curves.

It is clear that $\overline{Nu_d^*}$ should depend not only on Pe_L^* but also on other parameters such as the ratio of particle diameter, d, to the depth of flow, h, and a parameter representing the interparticle pressure in the material in contact with the heated place. The latter would affect the density and would obviously include gravity and the depth, h. We shall use a Froude number to represent this effect.

It is, therefore, apparent that the process of heat transfer to a rapidly flowing, granular material is a complicated phenomenon which is intimately affected by the rheology of the material. Nevertheless, we believe this phenomenon can be represented by a simple model which accounts for a variable thermal resistance due to the flow of the material.

5. A MODEL FOR THE HEAT TRANSFER TO FLOWING, GRANULAR MATERIAL

In this section we summarize an attempt to model the process of heat transfer to a rapidly flowing, granular material which was first proposed by Patton [7]. Following an approach often taken for heat transfer in turbulent flows, the flow field is split into two regions (see Fig. 5). The inner region extends from the wall to a position in the flow denoted by '1'. In this region the mechanism controlling the heat transfer is

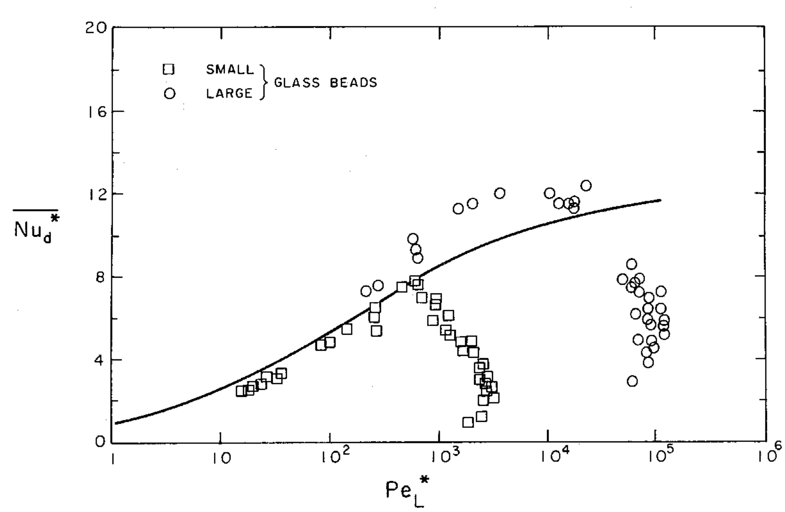

FIG. 4. The variation of the Nusselt number, $\overline{Nu_d^*}$, as a function of Péclet number, Pe_L^*, for both sizes of glass beads.

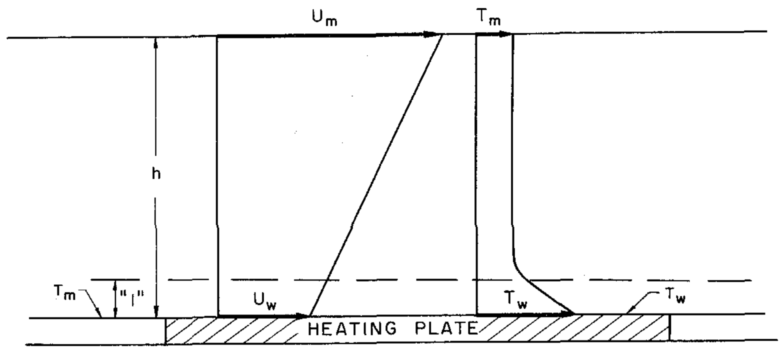

FIG. 5. Schematic of heat transfer process.

assumed to be the same as that proposed by Sullivan and Sabersky. Therefore:

$$\frac{1}{\chi+(\sqrt{\pi}/2)(1/\sqrt{Pe_L^*})}=\frac{q}{T_w-T_l}\frac{d}{k_g}. \quad (1)$$

In the outer region extending from position '1' to a point in the flow where the temperature is equal to the bulk temperature of the material, the transport mechanism is dominated by particle mixing and will be represented by using Reynolds analogy. Thus, using these concepts, the outer region may be represented by:

$$-\frac{q_l}{c_p\varepsilon_h(T_m-T_l)}=\frac{\tau_l}{\varepsilon(U_m-U_l)}. \quad (2)$$

Further assuming that the shear is constant from the wall to position '1' in the flow and that the heat transfer rate is the same through both layers of the flow, these two equations may be combined as follows,

$$\frac{k_g}{d}\frac{(T_w-T_m)}{q}=\chi+\frac{\sqrt{\pi}}{2}\frac{1}{\sqrt{Pe_L^*}}+\frac{\varepsilon}{\varepsilon_h}\frac{k_g}{d}\frac{U_m-U_l}{\tau_w}. \quad (3)$$

A friction coefficient μ is now introduced as

$$\mu=\frac{\tau_w}{\rho_p vgh\cos\theta}$$

where τ_w is the wall shear stress. The data of Patton [7] indicate that μ is a constant for slow flows, but increases as the velocity increases and the density decreases. More generally μ seems to be a function of a Froude number Fr, where

$$Fr=\frac{U^2}{gh\cos\theta}.$$

Substituting for τ_w and rearranging, the expression for the Nusselt number may be written as

$$\overline{Nu_d^*}=\left[\chi+\frac{\sqrt{\pi}}{2}\frac{1}{\sqrt{Pe_L^*}}\times\left(1+\frac{2}{\sqrt{\pi}}\frac{U_m-U_l}{U}\frac{1}{\mu}\frac{\varepsilon}{\varepsilon_h}\frac{v_c}{v}\frac{U^2}{gh\cos\theta}\sqrt{\frac{\alpha}{UL}}\right)\right]^{-1} \quad (4)$$

Thus the Nusselt number depends on two parameters, the Péclet number (Pe_L^*) and the parameter, Π, defined by

$$\Pi=\frac{2}{\sqrt{\pi}}\frac{U_m-U_l}{U}\frac{\varepsilon}{\varepsilon_h}\frac{1}{\mu}\frac{v_c}{v}\frac{U^2}{gh\cos\theta}\sqrt{\frac{\alpha}{UL}}.$$

The constant, χ, which represents the effective interstitial resistance near the wall has the same meaning as that defined by Sullivan [5]. The present data suggest a value of $\chi=0.065$.

To evaluate Π properly, the flow profile would have to be determined. At this point the measurement of this quantity is not possible. In an attempt to approximate further equation (4), several more simplifications are introduced as a first attempt to compare this equation with experimental data.

First, following the concepts of the Reynolds analogy, we will make the usual assumption that $\varepsilon/\varepsilon_h$ is a constant. Next, the quantity $(U_m-U_l)/U$ which depends on the ratio of the velocity gradient to the average velocity is not expected to vary much and will be treated as a constant. The friction coefficient, μ, remains fairly constant for a range of Froude numbers although it does increase at larger values of this parameter. The variations in μ will be neglected for the present. Since the assumptions are rather sweeping the resulting relation should not be regarded as a carefully derived solution. We may, however, be justified in using the form of the equation as a basis for an empirical relation for the Nusselt number. The proposed form is

$$\overline{Nu_d^*}=\left[\chi+\frac{\sqrt{\pi}}{2}\frac{1}{\sqrt{Pe_L^*}}\left(1+\beta\frac{Fr^*}{\sqrt{Pe_L^*}}\right)\right]^{-1} \quad (5)$$

where Π is now written as

$$\Pi=\beta\frac{Fr^*}{\sqrt{Pe_L^*}}.$$

The quantity β is given by

$$\beta=\frac{2}{\sqrt{\pi}}\frac{U_m-U_l}{U}\frac{\varepsilon}{\varepsilon_h}\frac{1}{\mu}$$

and will be taken to be a constant for the reasons described earlier. A value of $\beta = 15$ allowed an acceptable fit of the data.

With this rearrangement, $\overline{Nu_d^*}$ now becomes a function of the Péclet number, Pe_L^*, defined as

$$Pe_L^* = \frac{UL}{\alpha}\left(\frac{d}{L}\right)^2\left(\frac{k}{k_g}\right)^2.$$

and of a Froude number, Fr^*, defined as

$$Fr^* = \frac{U^2}{gh\cos\theta}\frac{v_c}{v}\frac{k}{k_g}\frac{d}{L}.$$

The comparison of the experimental results with those given by equation (5) are shown in Fig. 6. The experimental points agree rather well with those predicted by equation (5) for each Froude number and for both sizes of glass beads. For the lowest range of Froude numbers the points follow the line computed for $Fr^* = 0$ which is identical to the Sullivan–Sabersky correlation. But as the Froude number increases, the $\overline{Nu_d^*}$ number decreases and the experimental points cross the lines of constant Froude number. In addition the data for the two sizes of glass beads cover different ranges of both Pe_L^* and Fr^*. Even if it had been recognized that the Nusselt number depends on a Péclet and a Froude number, it would have hardly been possible to predict the intricate relationship that exists between these parameters represented in equation (5).

The semi-empirical equation together with Fig. 6 will hopefully form the basis for further advances. First, Fig. 6 indicates that the modified Froude number and the modified Péclet number are the principal factors determining the Nusselt numbers. Secondly, in the derivation of the equation (5) the various assumptions have been stated in sufficient detail so as to offer the opportunity to introduce improvements as new information becomes available. Furthermore, a large number of additional experimental data will be needed to cover a sufficient range of the components of the various parameters in order to be able to verify the presently proposed correlation or any that may be offered in the future.

6. SUMMARY AND CONCLUSIONS

Extensive experiments were conducted to determine the heat transfer coefficient for the flow of a granular material in a chute. Glass beads of both 0.3 mm and 3.0 mm were used as the granular material. The range of conditions investigated included rapid flows in which the density changes were significant and an experimental technique was developed to allow the measurement of this density. On the basis of this information more meaningful values of the velocity and thus the Péclet number could be computed. The Nusselt number could then be presented in terms of this Péclet number. The results show the same trends observed by Spelt et al. [6], namely that the Nusselt number would at first increase with the Péclet number, reach a maximum and then decrease with further increase in the Péclet number. For each material a single curve was sufficient to describe the results. This fact constitutes an improvement over Spelt et al.'s work and is the result of properly accounting for the change in density.

In the search for a general representation of the data a model was proposed which extends Sullivan and Sabersky's [5] correlation to the rapid flow regime in which density changes are of importance. This motivated the development of a semi-empirical equation relating the Nusselt number to a Péclet number and a Froude number. The experimental results cor-

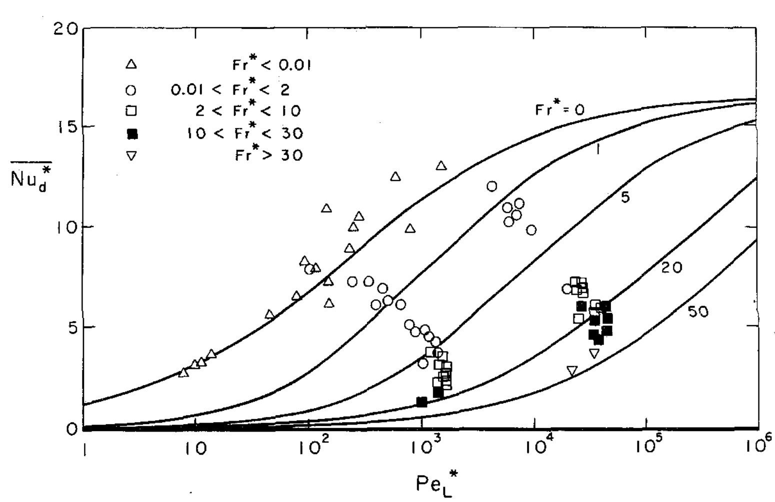

FIG. 6. Comparison of the predicted values of $\overline{Nu_d^*}$, as a function of Pe_L^* and Fr^* from equation (5) with the experimental data from the present investigation.

respond surprisingly well to the predictions of the equation. Many simplifying assumptions were made in developing this model. However, it is hoped that in the future more detailed experimental measurements will allow significant improvements in the model.

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CONVECTION THERMIQUE AUTOUR DE MATERIAUX GRANULAIRES EN MOUVEMENT RAPIDE

Résumé—La convection thermique autour de matériaux granulaires en écoulement rapide est étudiée expérimentalement sur une plaque plane dans une chute inclinée. Deux tailles différentes de granules de verre sont utilisées. Une technique a été utilisée pour mesurer la densité moyenne du matériau en écoulement avec la détermination la plus précise de la vitesse moyenne aussi bien que des paramètres dépendant de la vitesse. Les résultats sont présentés à l'aide du nombre de Nusselt et un modèle est proposé qui relie ce nombre de Nusselt au nombre de Froude et au nombre de Péclet. Les résultats du modèle sont comparés aux données expérimentales.

DER KONVEKTIVE WÄRMEÜBERGANG AN SCHNELL FLIESSENDE GRANULÖSE STOFFE

Zusammenfassung—Es wird der konvektive Wärmeübergang an einen schnell über eine ebene geneigte Gleitbahn fließenden granulösen Stoff experimentell untersucht. Zwei Glasperlenarten unterschiedlicher Größe werden als Versuchsmedien verwendet. Es wurde eine Technik entwickelt, um die durchschnittliche Dichte des fließenden Materials zu messen, die es erlaubt, sowohl die Durchschnittsgeschwindigkeit als auch die geschwindigkeitsabhängigen Parameter genauer zu bestimmen. Die Ergebnisse werden in Form von Nusselt-Zahlen vorgestellt, und es wird ein Modell zur Verknüpfung der Nusselt-Zahl mit der Péclet- und der Froude-Zahl vorgeschlagen. Die berechneten Daten werden mit den experimentellen verglichen.

КОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС В БЫСТРОДВИЖУЩИХСЯ ГРАНУЛИРОВАННЫХ МАТЕРИАЛАХ

Аннотация—Экспериментально исследуется конвективный теплоперенос над плоской пластиной в наклонном желобе в быстродвижущемся гранулированном материале, в качестве которого использовались стеклянные шарики двух размеров. Разработана методика измерения осредненной плотности движущегося материала, позволяющая более точно определить осредненную скорость и параметры, зависящие от нее. Рассчитано число Нуссельта; предложена модель, в которой число Нуссельта зависит от чисел Пекле и Фруда. Расчеты по модели сравниваются с экспериментальными данными.