FUNNEL FLOW IN HOPPERS

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ABSTRACT

Detailed observations of funnel flows of dry granular materials in wedge-shaped hoppers of different geometries are presented. The variations of the flow regime with changes in the height of material in the hopper/vertical bin configuration, the width of the vertical bin, the hopper angle and the hopper opening width were investigated and a number of specific flow regimes identified (mass flow and several forms of funnel flow). In the first part of the paper particular attention is paid to the conditions for transition from one flow regime to another; in particular it is shown that the existence of a funnel depends not only on the hopper angle but is also strongly dependent on the geometry of the hopper/bin system. In the second part of the paper the variations in the shape of the funnel near the exit opening are explored in detail.

NOMENCLATURE

b	Hopper breadth
d	Width of exit opening
H	Height of material above the exit opening
S	Distance along side wall from edge of exit opening to merge point
W	Hopper width
X	Dimensionless horizontal position of funnel boundary
Y	Dimensionless vertical position of funnel boundary
β	Inclination of funnel to vertical at the discharge
θ_w	Hopper angle

1. INTRODUCTION

This paper is concerned with the flow of dry, non-cohesive, granular materials in hoppers. It is well known that for plane (wedge-shaped) or conical hoppers of fairly small included angle all of the granular material flows in a fairly uniform and regular way. Such devices have been referred to as "mass flow hoppers" and have been the subject of considerable study and analyses (for example, references [1] to [6]). The convergence of experimental observations and theoretical predictions suggests that there exists some understanding of the mechanics of granular media flow in these circumstances. However, as the included angle is increased and a vertical bin is added to the top of the inclined sides of the hopper changes occur in the flow pattern which are much less well understood. Most of the motion occurs in a central core, funnel or "rat-hole" and stagnant regions of material tend to occur near the walls of the bin or hopper. This paper presents experimental observations of funnel flows in plane hoppers with vertical bins. Particular attention is paid to the transition between mass flow and funnel flow since it is not only of fundamental interest but is also important to the hopper designer. In this regard comparison is made with some of the existing design criteria such as that proposed by Jenike [1].

The various types of flow pattern which were observed in the present experiments are indicated in Fig. 1 (also shown are the definitions of θ_w, d, W, and H, the hopper angle and opening width, the bin width and the total height to the upper free surface respectively). Figure 2(a) is an example of type A or mass flow. The flows with stagnant regions are subdivided into two basic types,

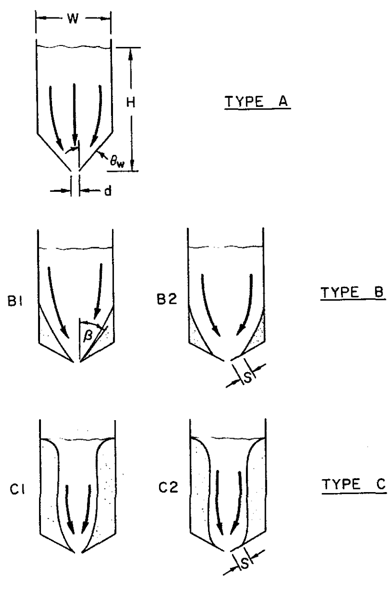
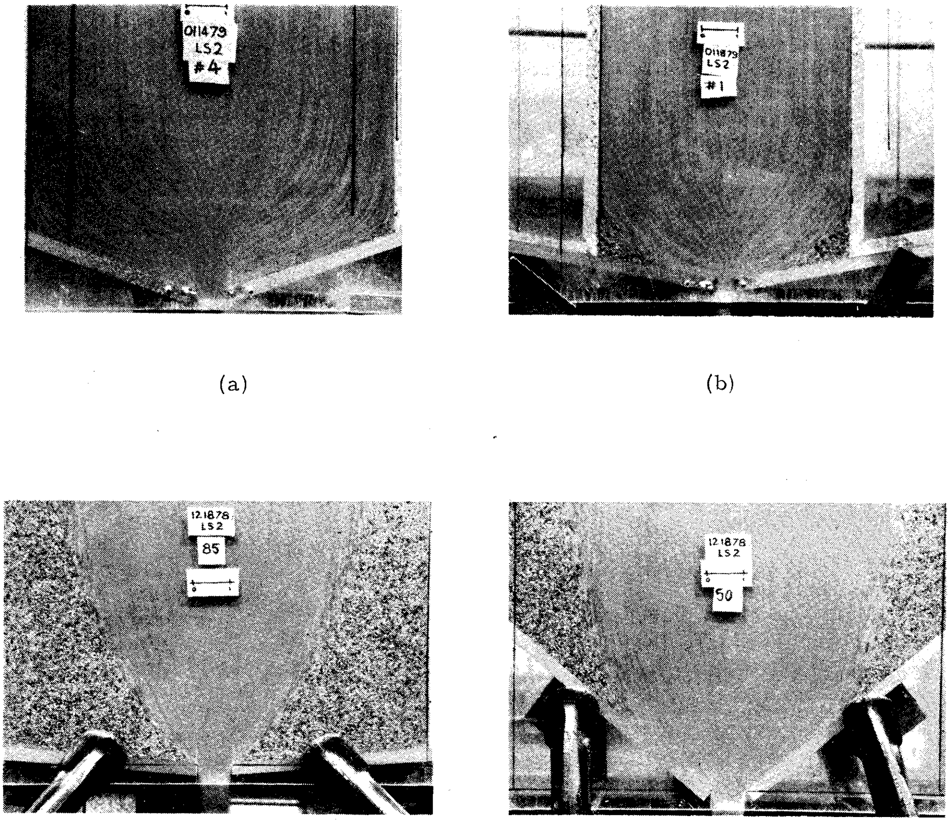


Figure 1. Schematic indicating the different flow patterns observed. Type A is mass flow, Type B has a stagnant corner and Type C has a stagnant side.

B and C. Type B has sliding along the upper part of the bin wall and stagnant regions near the bin/hopper wall corner (for example Fig. 2(b)). Type C has no sliding along the bin walls and larger stagnant regions on either side of the "rat-hole" (for example Fig. 2(c) and (d)). Two subtypes were also noted. In some instances slip occurred along the walls of the hopper (types B2 and C2) whereas in other cases the stagnant regions extended to the opening (types B1 and C1). Figures 2(b) and 2(d) are representative of types B2 and C2 whereas Fig. 2(c) is of type C1.

2. BACKGROUND

A number of observations of flows with stagnant regions have been made previously; for example, Gardner [7] measured the boundary of a flow of type B in a plane hopper and Levinson et al [8] reported the existence of stagnant regions in their experiments in plane flow (though only the results for the case $\theta_w = 90°$ are reported). Brown and Richards [9] studied the angle of



(a) (b)

(c) (d)

Figure 2. Photographic examples of the flow patterns for the sand (angle of repose = 35°, grain size = 0.5 – 1mm). The breadth, b, is 15.2cm in all cases. The following are the values of θ_w, H, W, d (in cm): (a) 70°, 36.8, 22.9, 1.91 (b) 80°, 58.4, 17.8, 1.37 (c) 70°, 35.6, 30.5, 2.54 (d) 50°, 35.6, 30.5, 2.54.

approach β which the funnel makes with the vertical (see Fig. 1). They concluded that the angle of approach in a plane hopper is greater than that in a conical hopper. Weighardt [10] claimed that the angle of approach β must be the same for both the plane and conical flow fields. He distinguishes two cases depending on whether the stress at the bottom of the hopper is independent of the depth or not. In the first case, the angle of approach is equal to $90° - \alpha$; α being the angle of repose of the granular material. In the second case, β is equal to $45° - \alpha/2$. The experimental results of Brown and Richards [9] are observed to lie between these limits. Toyama [11] observed the flow of type B in a plane hopper with $\theta_w = 90°$ when the width of the exit slot is varied. The observations of the boundary of the flow field were similar to those of Gardner [7]. A comment which could be made is that these references do not address the different conditions under which the different flow regimes occur.

3. EXPERIMENTAL APPARATUS

A plane hopper with adjustable geometry as described by the dimensions H, W, d and θ_w (see Fig. 1) was made from lucite. The hopper angle, θ_w, could be varied continuously; exit openings ranging from 5.1mm up to 38.1mm were employed and several breadths, b, of the hopper normal to the sketch were investigated.

The granular materials used were sand (angle of repose = 35°, grain size = .5 - 1mm) and glass beads (internal friction angle = 24.6°, grain size = 0.32mm); most of the observations were made with sand. The hopper/bin system was loaded with material in a loose state; no attempts at dense packing were made.

In the first series of experiments, the critical condition for transition from one type of flow to another were studied. Following this, detailed observations of the flow patterns were made using long time exposure photographs of the flow field.

4. OBSERVATION OF THE TRANSITION BETWEEN FLOW PATTERNS

The transition criteria for the flow of sand in a hopper of breadth, b = 15.2cm will be discussed first. When the hopper angle, θ_w, was less than about 60° mass flow (type A) was observed to occur until the free surface reached a critical height H_{AC}. Below this value of H, stagnant side flow of type C occurred. In Fig.3, the critical ratio H_{AC}/W is plotted versus the ratio W/d for various hopper angles. For θ_w less than about 60°, the critical ratio H_{AC}/W is more or less constant for all angles θ_w. This implies that the transition from mass flow to type C is caused by the presence of the vertical bin on top of the hopper rather than by the inclination of the hopper walls.

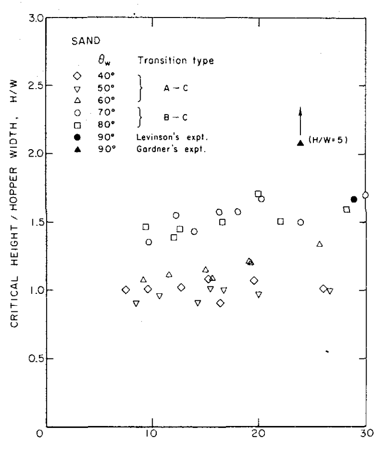


Figure 3. Critical values of H/W plotted against W/d for various hopper angles. The material is sand (angle of repose = 35°, grain size = 0.5 - 1mm).

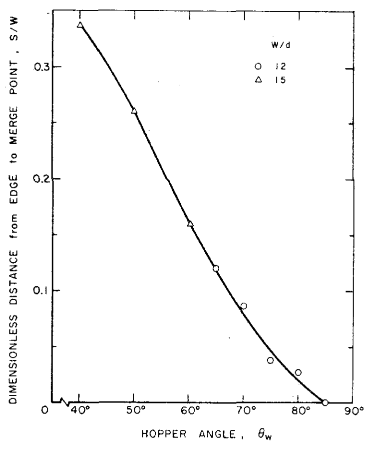


Figure 4. Dimensionless hopper wall slip length, S/W, plotted against the hopper angle, θ_w, for H=35.6cm, d = 2.54cm, b = 15.2cm and W = 30.5 and 38.1cm.

For hopper angles greater than about 60°, flow of type B occurred for large H ; however, the flow underwent a transition to type C when the free surface reached a critical height H_{BC}. The critical ratio H_{BC}/W is plotted in Fig.3. Again, this critical value is more or less independent of the hopper angle. It is worth stressing that at small W/d and large H/W, mass flow can occasionally occur even for hopper angles as large as 70°.

The difference between the flows of type B1 and B2 (or between C1 and C2) is the presence of slip along a part of the hopper walls near the discharge opening. The length of hopper wall over which slip occurs was measured for a variety of flows and will be denoted by S. Flows of type B2 (or C2) were observed to occur when the hopper angle was less than about 85° (see Fig. 3). As the angle is decreased below this the distance S increases monotonically as the flow begins a transition to mass flow. The magnitude of S also depended upon the width W. However the values of S/W for the cases plotted in Fig. 4, indicates that this ratio is primarily a function of θ_w.

The glass beads exhibited similar qualitative flow patterns; however, the values of H_{AC}/W and H_{BC}/W were virtually the same as can be seen in Fig. 5; a single critical value of H/W equal to 1.3 and increasing only slightly with W/d defines the A to C or B to C transition quite well.

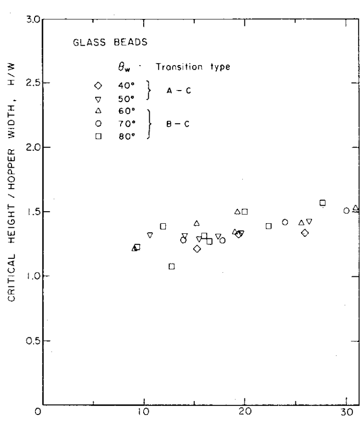


Figure 5. Critical values of H/W plotted against W/d for various hopper angles. The material is glass beads (internal friction angle = 24.6°, grain size = .32mm).

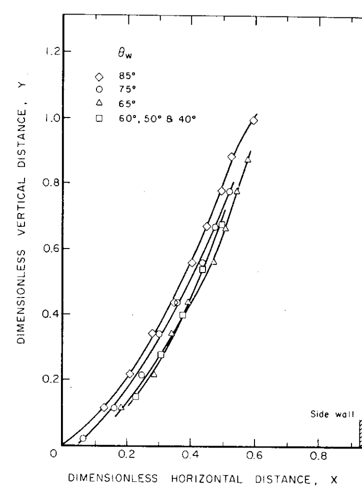


Figure 6. Funnel shapes for various hopper angles and H/W = 1.22, W/d = 12, b/W = 0.5 (d = 2.54cm).

Comparison between the sand and glass beads suggests that the critical value of H/W for the B → C transition is decreased when the internal friction is decreased; the material with less internal friction continues to flow for lower H than the material with greater internal friction. But this argument does not appear to work for A → C transition which actually occurs at somewhat larger values of H/W in the case of the glass beads. Clearly the situation is not a simple one for one could equally well argue that for the same H (and therefore roughly the same normal stress) stagnant material is acted upon by larger shear forces in the sand flows and therefore smaller values of H (and hence normal stress) are required to cause the stagnant corner material to move. Such a trend would be consistent with the observed shift of the critical H/W value for the A → C transition.

As previously stated all of the above observations were made with a breadth, b, equal to 15.2cm. Some limited observations with breadths of

7.6cm and 22.8cm indicated the same qualitative transitional phenomena and only minor quantitative differences. Some data on the variation of funnel shape with b/W is given in Section and these suggest that the results have asymptoted to those of pure plane flow for values of b equal to 15.2 and 22.8cm. Some differences were however observed for $b = 7.6$cm.

It is of interest to examine the regimes of H/W and W/d in which the experiments of Gardner [7] and Levinson [8] were conducted. These are indicated in Fig. 3. The points do not refer to critical values but to regimes of operation. Gardner's photographs provide clear confirmation that his flows were of type B. Levinson's value of H/W is marginal and this is reflected in the fact that the flow with clover seeds tended toward type B whereas the flow with sand, having a larger internal friction angle is closer to type C. Though Brown and Richards [9] did not give the dimensions of their apparatus, the ratio of H/W appears to be about 1.5 and flows of type C were observed for a large number of different granular materials. In Toyama's [11] experiments flows of type B were encountered with large values of H/W of 10 and 6.5.

Jenike [1, 12] has presented several criteria for the evaluation of flow regimes in hopper/bin systems. In reference [12] a condition is given for the determination of whether flow of type A or C will prevail. Implementation of this condition requires the comparison of a quantity q_H for the hopper (which depends only on the hopper angle and material friction angles) with a value q_B for the bin. The appropriate choice of the value of q_B for the bin depends on whether the material in it is in a "passive" or "active" state. Hence it probably depends implicitly though not explicitly on the geometry H/W , W/d , etc. Because of uncertainties in this relation a direct comparison with the present results has not as yet been made. In an earlier work Jenike [1] presented a criterion for the transition between mass flow and funnel flow in a hopper which consisted of a relation between the hopper angle, θ_w, the internal friction angle and the wall friction angle. Specifically for typical wall friction angles of 15° and 20° (the value for glass beads on lucite is about 17° [3]) and internal friction angles less than 45°, Jenike's criterion yields critical hopper angles of 42° and 35° respectively. It seems clear from the present results that the addition of a vertical bin can radically alter this value; then the geometry as given by H/W , W/d , etc strongly influences the flow regime. Indeed as shown in Fig. 2(a) mass flow can occur for angles as large as 70°.

Finally it is also worth mentioning that trends similar to those reported above for plane hoppers also appear to occur in conical hoppers. Van Zanten [13] and Giunta [14] observed flows of type C in conical systems with values of H/W or 2.57 and 1.33 respectively. On the other hand, Novosad and Surapati [15] obtained flows of type B for H/W ranging from 4 to 8. McCabe [16] observed a change in the flow field for values of H/W of about 2. Thus, it would appear that conical hoppers exhibit results qualitatively similar to those reported here for wedge-shaped hoppers and that the critical value of H/W for conical hoppers is between 2 and 3 depending on the properties of the material.

5. VARIATIONS IN THE SHAPE OF THE FUNNEL

Having established the conditions for the $A \rightarrow C$ and $B \rightarrow C$ transitions we now proceed to examine the changes in the shape of the funnel with geometry, primarily for type C flows. In the figures which follow these shapes are plotted non-dimensionally by dividing all lengths by W/2; the origin of the resulting X, Y coordinates in which Y is vertical is taken at the end of the hopper wall at the discharge opening. Hence type C1 profiles pass through the origin. For type C2 in which the funnel merges with the hopper wall at a point $(S \cos\theta_w/W,\ S \sin\theta_w/W)$ these merging points are identified in the figures by the letter S. The objective of this section will be to identify the variations in funnel shape with the parameters θ_w, H/W , W/d , b/W and the material properties. The last effect has not been fully documented as yet and we shall restrict our attention to the variation with the geometric parameters for one particular material, namely the sand.

Figure 6 displays the type C funnel shapes for various hopper angles, θ_w, at fixed values of H, W, d and b as indicated. As one proceeds to smaller angles the type C2 flow slides over longer lengths of the hopper wall; this feature was previously described in Fig. 4. The funnel becomes wider but the shape of the funnel remains much the same; indeed the profiles simply appear to have been shifted outward in the x direction. The inclination of the funnel to the vertical at the merge point, S , appears to decrease somewhat. These trends halt at $\theta_w = 60°$ and the profiles for 60°, 50° and 40° all correspond. Hence in the type A to C transition the resulting funnel seems to be independent of the angle whereas in the type B to C transition ($\theta_w > 60°$) the funnel shape depends on θ_w or more specifically on the position of the merge point as given by S/W.

A typical variation of the funnel shape with H/W for fixed values of θ_w, W , d and b is presented in Fig. 7. All of the shapes shown are for H/W

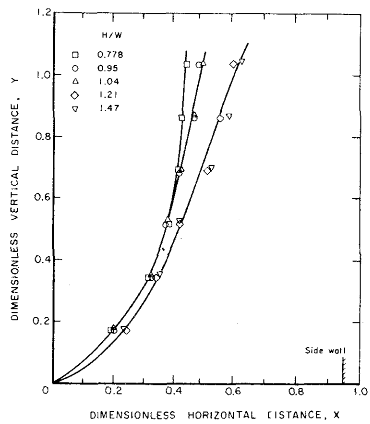


Figure 7. Funnel shape for various values of the head of material above the exit opening, H (or H/W) for fixed $\theta_w = 70°$, W = 30.5cm, W/d = 16, b/W = 0.5.

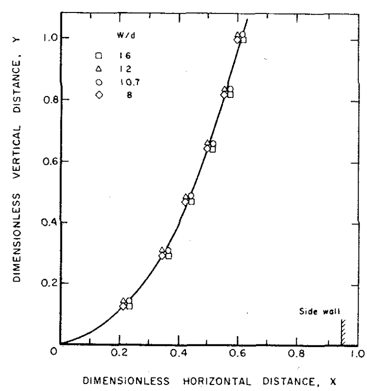


Figure 8. Funnel shapes for various exit width, d, and fixed values of θ_w, H, W and b ($\theta_w = 70°$, H/W = 1.22, b/W = 0.5, W = 30.5cm).

below the critical and hence are type C flows. For the smaller values of H/W the shape of the funnel appears to be relatively independent of this parameter. The only significant departure seems to occur for the two largest values of H/W. These are in fact quite close to the critical value of H_{BC}/W (see Fig. 3) and the widening of the funnel in these cases would seem to represent the beginning of the transition to type B flow in which the boundary is displaced outward and intersects the vertical side wall.

Typical funnel profiles for various values of the opening width, d , are shown in Fig. 8 for fixed θ_w, H , W and b. Clearly these type C flows have funnel shapes which are independent of d and this suggests that they should be independent of W if W rather than d is varied. The results of such a comparison are presented in Fig. 9 where both W and d were varied for the same θ_w, H and b. The results seem to confirm the invariance with W/d in addition to the non-dimensional form in which the funnels have been presented. The greatest discrepancy in Fig. 9 occurs with the funnel for the largest H/W = 1.17; since this is quite close to the critical value of H_{BC}/W ,

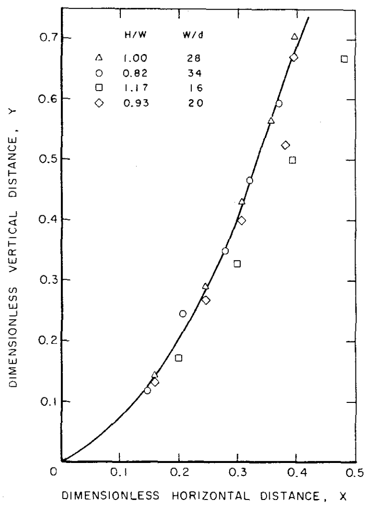


Figure 9. Funnel shapes for various values of the bin width, W for fixed $\theta_w = 70°$, H = 35.6 and b = 15.2cm. Two values of the exit opening, d, of 1.27 and 1.91cm were used.

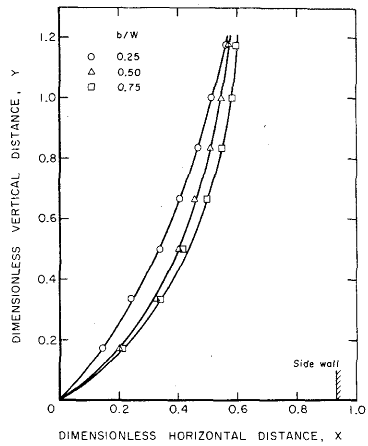


Figure 10. Funnel shapes for three different breadths of the hopper, b = 7.62, 15.24 and 22.86cm with fixed values of $\theta_w = 70°$, H/W = 1.22, W/d = 16 and W = 30.5cm.

the curve for H/W = 1.17 may again represent the beginnings of the transition to type B flow.

Finally in Fig. 10, comparison is made between the funnels for three different breadths, b, of hopper (b/W = 0.25, 0.5 and 0.75). The funnels for the two larger breadths are quite close. However, the funnel for the smallest breadth (b/W = 0.25) seems significantly narrower throughout its length. As mentioned previously, we have tentatively concluded from this that friction on the vertical front and back faces begins to alter the flow regime and funnel shape when b/W is less than some value between 0.25 and 0.5.

In summary, these studies suggest that the type C funnel shapes are primarily dimensioned by the width, W, of the vertical bin. The hopper angle, θ_w, has a fairly simple effect shifting the profile outward as θ_w is decreased. Furthermore, provided the conditions are not close to the critical value of H_{BC}/W for transition to another regime and provided b/W is 0.5 or larger, the funnel shape appears to be relatively independent of H/W, W/d and b/W.

6. CONCLUSIONS

It is clear that a continuation of the studies presented here is necessary; the parametric variations of flow regimes and funnel shapes have not been fully explored as yet. Further research is needed for different materials and for different wall friction conditions; the roles played by small amounts of material cohesion and by the packing conditions are not yet documented. Nevertheless a fairly clear picture is beginning to emerge in which a number of distinct flow regimes can be identified and in which the transitional conditions between regimes depend not only on the hopper angle, θ_w, but also on the geometric ratios H/W, W/d and b/W.

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