Granular State Effects on Wave Propagation

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ABSTRACT

Sound and pressure wave propagation in a granular material is of interest not only for its intrinsic and practical value, but also because it provides a non-intrusive means of probing the state of a granular material. By examining wave speeds and attenuation, insight can be gained into the nature of the contacts between the particles. In the present paper, wave speeds and attenuation rates are first examined for a static granular bed for a variety of system parameters including particle size, composition and the overburden of the material above the measuring transducers. Agitation of the bed is then introduced by shaking the material vertically. This causes the bed to transition from a static granular state to a vibrofluidized state. The dilation of the bed allows for relative particle motion and this has a significant effect on the measured wave speeds and attenuation. Further, the fluid-like characteristics of the agitated bed distort the force-chain framework through which the waves are thought to travel. The consequences of bed consolidation, a natural result of shaking, are also examined.

INTRODUCTION

The characteristics of wave propagation in a granular material are of interest not only on a fundamental level, but also because they provide a means of probing otherwise unobservable quantities. Fundamentally, the wave speed can be related to the stiffness and density of a material.

Early work established the critical dependence of the interaction at particle contacts on wave propagation in a granular material [1,2]. These works viewed the granular assembly as a continuum and formulated bulk properties based on contact properties such as the pressure and coordination number [3]. The continuum approach is appropriate when the characteristic wavelength of the sound is larger than the grain size, but as the wavelength approaches and becomes smaller than the granularity of the system, a different formulation is required [4]. In this limit, more recent work has looked at the details of the microstructure to examine the paths of wave propagation. In this view, waves travel through relatively few stressed particles chains that connect the source to the detector. It has been found that even minute displacements of particles can lead to breaking and reconnection of these chains, thus affecting the path through which waves travel [5]. Time fluctuations in the detected signal have been attributed to this rearrangement of the microstructure even by agents as weak as the wave itself [6,7].

The majority of work done on wave propagation in agitated granular material beds has been done in fluidized beds. In a summary of such experiments, the wave speed was found to decrease abruptly and then gradually increase with increasing agitation (excess fluidization velocity) and attenuation was found to primarily decrease. By comparing the experimental results to a pseudo-homogeneous analysis, the authors conclude that the observed waves are dynamic waves primarily propagated along particle contact and not compression waves [8].

Vertical shaking of a granular material provides another means of changing its granular state/granular temperature. By examining the characteristics of wave propagation at different levels of agitation, we hope to be able to diagnose the state of the material based on the information on the wave speed and attenuation.

EXPERIMENTS

The experiments were conducted in a square box (25 cm by 10 cm tall) filled with a granular material as shown in Figure 1. A 10 cm diameter piston is inserted through one of the side walls and connected to an electromechanical shaker. The piston motion is monitored by an accelerometer fixed to its back surface. The box is mounted atop another electromechanical shaker which is used to vibrate the bed vertically. Two identical pressure transducers are buried in the bed at different distances from the piston and held fixed from above by lab clamps. These distances were fixed for all the experiments described herein. Wave speeds are measured by cross-correlating the signals from the transducers and attenuation data is taken by comparing the magnitude of the pressure recorded at the two transducers. The box was lined to minimize the effect of wave reflection from the walls. Results were taken with no lining, with ½" polystyrene sheet, and ½" foam rubber sheet.

Filling of the box was done in a way that minimized compaction of the material. The box was filled to the same height for every experiment. This defines what we consider our unconsolidated state. For some experiments, the bed was consolidated. This was accomplished by shaking the box for several minutes at ±2g. The consolidation resulted in a drop in the particle free surface of around 1 cm and a visibly obvious increase in the regularity of the particle packing.

Static bed experiments

The first of the static bed experiments was carried out by holding the acceleration of the piston constant and sweeping a frequency range from 150-2000 Hz. This experiment was repeated for different granular materials, for both unconsolidated and consolidated beds, and for an increased overburden of particles. Typical results from these constant acceleration experiments are shown in Figure 2. In general, the phase shift between the two transducer measurements varies linearly with frequency, consistent with nondispersive wave propagation

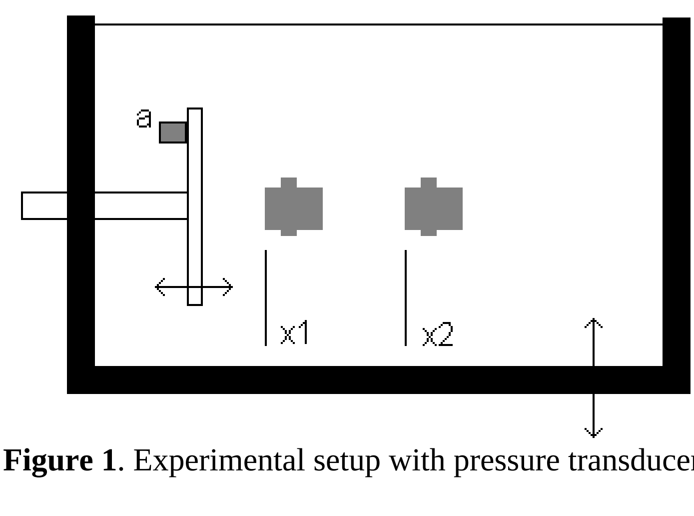

Figure 1. Experimental setup with pressure transducers at positions x_1, x_2 and accelerometer (a).

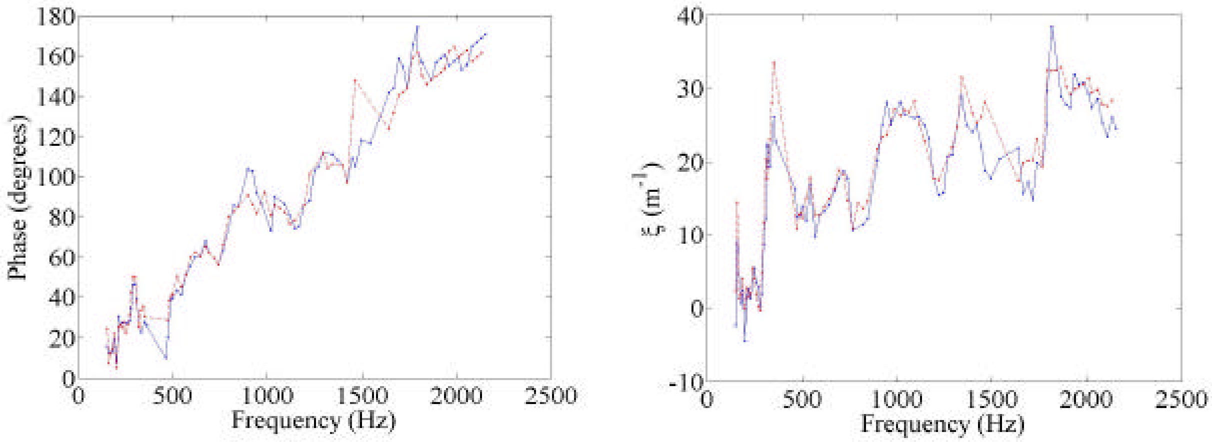

Figure 2. Constant acceleration results at sound acceleration levels of 0.20g (solid blue lines) and 0.30g (dashed red lines) for 4 mm glass particles.

that was observed previously [6,7]. Both of the accelerations shown were taken at a particular frequency before moving to the next frequency. Little difference can be seen in the phase or slope of the trend for these two acceleration levels. Some of the peaks in the plot are attributable to resonances of the box, especially in the range around 400 Hz. The other deviations from strictly linear behavior are as of yet unexplained. The attenuation ratio, $\xi=\ln(p_2/p_1)/x_1-x_2$, is also plotted against frequency in Figure 2. Attenuation increases roughly linearly with frequency. This trend is consistent across all similar plots. Again, the slight increase in the acceleration does not have much of an effect and does little to change the characteristic peaks of the plot. The irregular peaks, particularly those above about 500 Hz, were not reproduced in repeated experiments. In general, the attenuation ratio plots are much less repeatable than the corresponding phase plots.

The constant acceleration experiments are summarized in Figure 3 where the group velocity and mean attenuation ratio are shown. The group velocity, V_g, is deduced from a straight line fit of the data in all graphs such as Figure 2 (left). The only parameters that appear to have an effect on the group velocity are bed consolidation, particle composition, and the overburden of particles. Different box linings have little effect and for the most part, little change occurs with particle size. The exception to this is the case of the 5 mm particles where the results diverge for the two different linings. Geometrical considerations are probably to blame as the 5 mm particle diameter is a significant fraction of both the transducer face diameter and the distance between the piston and first transducer. The result of this is that comparatively few particles contact the transducer face and this number of contacts is likely to be different between experiments.

Both consolidation and an increased overburden of particles increase V_g as would be expected. Consolidation seems to have more of an effect than the increase in overburden. The overburden was increased by more than a factor of two for the data point shown and yet the increase in the group velocity is roughly half the increase due to consolidation. Additionally, material composition has a marked effect. The group velocity for PVC is roughly half that of the glass particles, which should also be expected as $(E/\rho)^{1/2}$ for PVC is 1/3 of that for glass where E is the elastic modulus and ρ is the density.

Conclusions are harder to draw for the attenuation data. For the polystyrene-lined box, the attenuation is relatively independent of particle size, but for the foam rubber lined box, it appears to decrease with increasing particle diameter. Again, there is a large disparity in the value of the

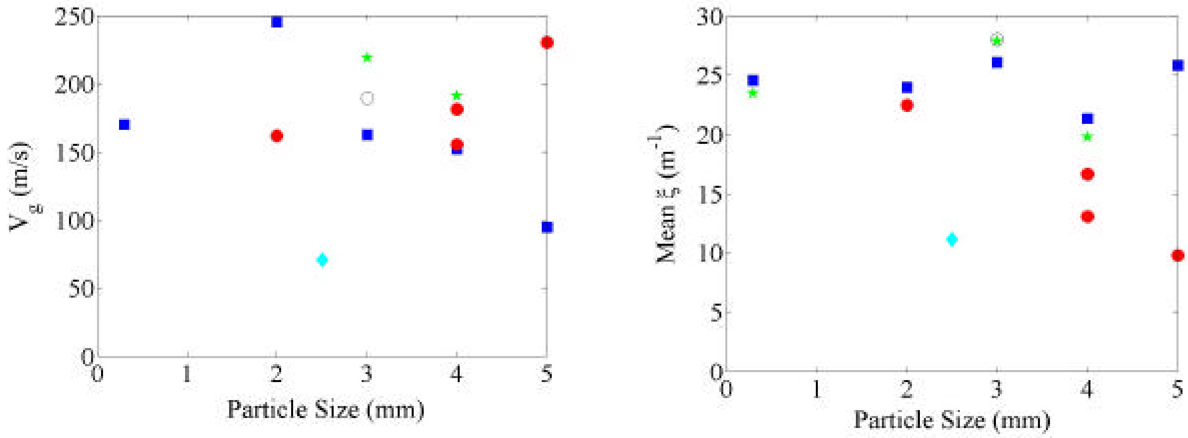

Figure 3. Summary of constant acceleration results for the following conditions: PVC: (♦); Glass: polystyrene lined box (■), foam rubber lined box (●), increased overburden (○), consolidated (★).

attenuation ratio for the 5 mm particles. No apparent trend is discernable for the effect of consolidation and the one data point for the increased overburden suggests increased attenuation, but this is hardly convincing. Material composition is the one clear parameter that affects the attenuation. The attenuation ratio for the PVC particles is roughly half the typical values for the glass particles.

The effect of consolidation is more readily observed by fixing the frequency and varying the piston amplitude. In these tests, multiple data points were taken for each operating point and then averaged to quantify the scatter of the measurements. Representative results of these experiments are shown in Figure 4. The scatter generally increases with the input acceleration, but is substantially decreased by consolidation. Strong hysteresis is seen in both the wave speed and attenuation for the unconsolidated bed. This is presumably due to rearrangement of the microstructure which occurs quite easily before consolidation. After consolidation, this hysteresis mostly vanishes. Consolidation generally leads to increased wave speeds, echoing what was seen in the group velocity measurements. It also decreases the attenuation, a point that was inconclusive in the constant acceleration test results.

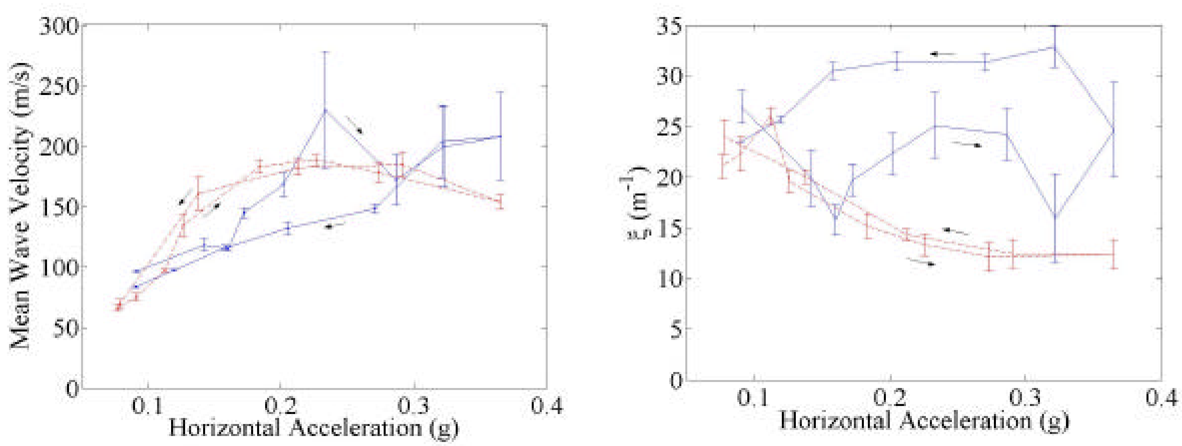

Figure 4. Constant acceleration test for unconsolidated (solid blue lines) and consolidated (dashed red lines) material for increasing and decreasing amplitude (shown by arrows).

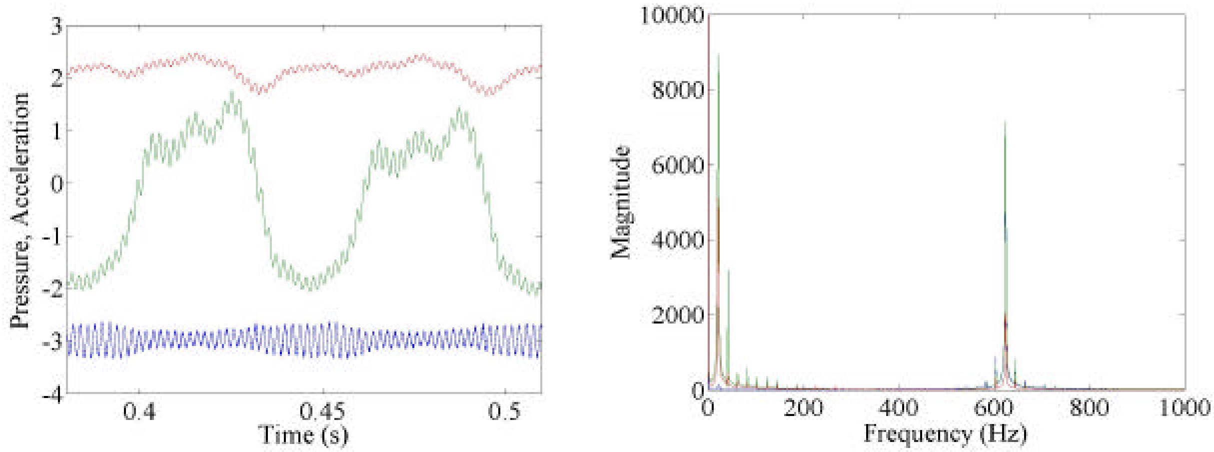

Figure 5. Left: Characteristics of combined shaking: raw signals from the accelerometer (blue/bottom), the near transducer (green/middle), and the far transducer (red/top). Right: Fourier spectra for these signals.

Agitated bed experiments

In this set of experiments, we held the input wave characteristics fixed at 620 Hz and ±0.30g amplitude and varied the amplitude of the vertical shaking. As in the constant frequency tests, multiple measurements were made at each experimental condition. Before taking measurements, the bed was consolidated. In the graphs we present averages and standard deviations (shown in the error bars).

Figure 5 shows the nature of the signals observed. The high frequency wave signal is readily superimposed on the low frequency shaking. The spectra of these signals show sharp peaks at the frequencies input to the system with distinctive beating around the high frequency peak. This beating is suggestive of some as of yet unidentified quadratic nonlinearity.

Typical results from the combined shaking experiments are shown in Figure 6. The wave speed is multi-valued up to an acceleration level of about 0.6g. After this point, the layer reaches some quasi-uniform state where the speed takes the same value when both increasing and decreasing the shaking. Below 0.6g, the wave speeds vary significantly on each path probably

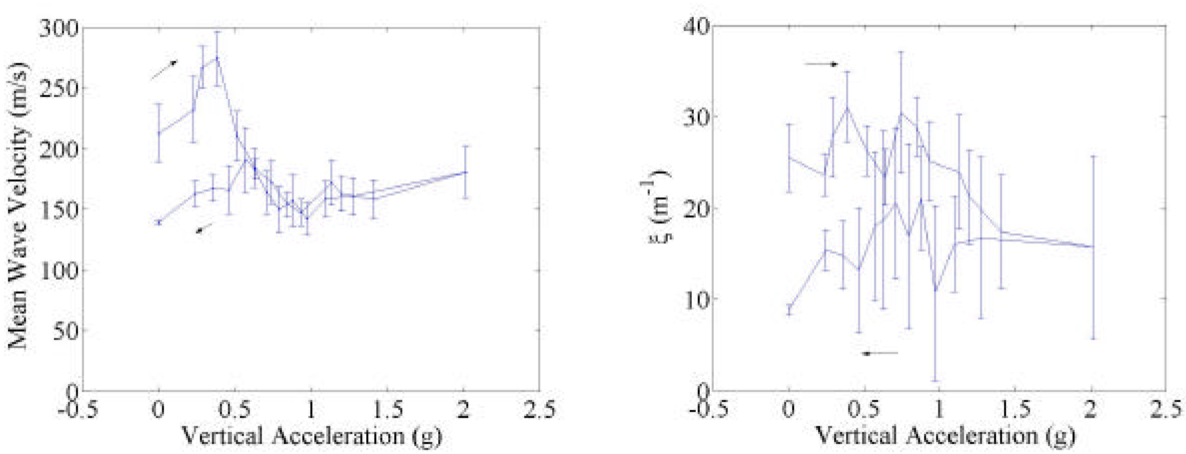

Figure 6. Results in a vertically shaken bed at a fixed input frequency (600 Hz) and acceleration amplitude (0.30g) for increasing and decreasing amplitude (shown by arrows).

due to some relatively large scale rearrangement of the bed. These patterns and values vary widely over the different materials and even between identical tests. Aside from last data point (static bed), the scatter in the data is fairly uniform across all accelerations and relatively large compared to the value for the static bed. This behavior can also be seen in the attenuation data with the scatter being even more pronounced. Unlike the wave speed, the attenuation is not single-valued over any part of the swept acceleration range. Apparently the attenuation is much more sensitive to changes in the bed. The bed never takes on a quasi-uniform appearance in terms of attenuation.

CONCLUSIONS

For static beds, we confirm that wave propagation is nondispersive as noted by Liu and Nagel. We found the group velocity to be roughly twice the value they obtained, though. Our value is more in line with the results of Hardin and Richart. The dependence of the wave speed and attenuation on several experiment conditions was explored. The material composition, as characterized by its sound speed, was found to have the most profound effect. The overburden of particles and the compaction of the layer was also found to have some effect. Preliminary work on agitated beds shows that measurements are possible under these conditions although the results are characterized by a large amount of scatter. After increasing the agitation past some threshold, found to be around 0.6g, the wave speed becomes single-valued. No such quasi-uniform state is observed for the attenuation.

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