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PARTIAL CAVITY INSTABILITIES

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ABSTRACT

This paper reviews some of the literature on partial cavity instabilities on single hydrofoils and then summarizes the striking differences in the appearance and behavior of partial cavities on swept foils (as opposed to two-dimensional, unswept foils) as recently highlighted by de Lange *et al.* (1994) and Laberteaux and Ceccio (1998). These demonstrate the importance of the spanwise evolution of the re-entrant jet and the consequences for the characteristics of the cavity closure flow. It is suggested in this paper that several variants of this evolution can be seen in the photographs of cavitation on single hydrofoils foils and on propellers.

What is common to many of these variants is that the spanwise evolution of the cavity and the re-entrant jet can give rise to conditions at some particular spanwise location(s) which initiate partial cavity instability. In this paper we present information on an instability that was observed to occur on a cavitating propeller of modern US Navy design. Detailed photographic examinations show that the instability oscillations involve spanwise development of a re-entrant jet and behavior similar to that of the partial cavity oscillations previously observed on two-dimensional foils.

NOMENCLATURE

c	Chord of the hydrofoil
C_L	Lift coefficient
C_p	Pressure coefficient, $(p - p_\infty)/\frac{1}{2}\rho U^2$
f	Frequency (Hz)
J	Advance ratio, $U\pi/\Omega R$
J_0	Advance ratio for zero angle of attack
ℓ	Cavity length
p	Liquid pressure
p_v	Vapor pressure
p_∞	Upstream fluid pressure
R	Propeller blade tip radius
u	Fluid velocity
U	Upstream fluid velocity
U_F	Flutter speed
V	Volume of the cavity
α	Angle of attack
μ	Dynamic viscosity of the liquid
ρ	Liquid density
σ	Cavitation number, $(p_\infty - p_v)/\frac{1}{2}\rho U^2$
$\sigma *$	Propeller tip cavitation number, $(p_\infty - p_v)/\frac{1}{2}\rho\Omega^2 R^2$
τ	Fractional time during period
ω	Frequency (rad/s)
ω_F	Natural frequency of vibration (rad/s)
Ω	Propeller rotational frequency (rad/s)

1 INTRODUCTION

In many flows of practical interest, clouds of cavitation bubbles are periodically formed and then collapse when they are convected into regions of higher pressure. It is now widely recognized that the coherent collapse of these clouds can cause a substantial increase in the radiated noise and in the potential for cavitation damage (see, for example, Soyama *et al.* 1992). The mechanics and acoustics of such clouds have consequently attracted much research attention in recent years, both experimental (Bark and van Berlekom 1978, Shen and Peterson 1978, 1980, Franc and Michel 1988, Hart *et al.* 1990, Kubota *et al.* 1989, McKenney and Brennen 1994, Reisman *et al.* 1998) and analytical (van Wijngaarden 1964, Hanson *et al.* 1981, Chahine 1982, d'Agostino and Brennen 1983, 1989, Reisman *et al.* 1998).

The temporal periodicity which is implicit in the phenomenon may occur naturally as a result of a flow instability or it may be the response to a periodic disturbance imposed on the flow such as that experienced by a propeller blade operating in the non-uniform wake of a ship. We focus in this paper on the former, namely the several flow instabilities which may lead to cloud cavitation. One which can readily be discerned in the existing literature is the process by which cavitation bubbles are entrained in periodically shed vortices which then become clouds with circulation. Another is the instability of partial cavities (Wade and Acosta 1966, Brennen 1995). Though we focus in this paper on the latter, it should be noted that there may be other unrecognized instabilities in these complex multiphase flows which lead to larger scale flow oscillations and therefore to cloud cavitation.

2 PARTIAL CAVITY INSTABILITY

The partial cavity instability on a single hydrofoil has been known for many years and documented by many authors. Very briefly, it is observed (for example by Wade and Acosta 1966) that a partial attached cavity on a single hydrofoil is relatively stable provided the cavity length is less than about 3/4 of a chord and that a supercavity is also stable provided it is longer than about 4/3 of a chord. However, for intermediate cavity lengths close to a chord in length, the cavity becomes quite unstable, and the size of the cavity fluctuates quite violently. During a fluctuation cycle, the cavity lengthens fairly smoothly. As it reaches the maximum size, a re-entrant jet is formed in the region of cavity closure and propagates forward filling a substantial fraction of the cavity with a bubbly mixture. Then a process of "pinching-off" of a large cloud of bubbles from the rear causes a sudden decrease in the attached cavity length. The separated cloud can collapse quite violently (Franc and Michel 1988, Hart *et al.* 1990, Kubota *et al.* 1989, McKenney and Brennen 1994, Reisman *et al.* 1998). As Wade and Acosta (1966) point out this process is very similar to the cavitation cycle originally described by Knapp (1955) except that the re-entrant jet does not fill the entire cavity prior to pinch-off.

Wade and Acosta (1966) observed that the frequency of the instability, f, corresponded to a reduced frequency, fc/U, in the range $0.07 - 0.14$. Similar values have been measured recently by Tsujimoto *et al.* (1998). Other investigators, such as Le *et al.* (1993) and de Lange *et al.* (1994), encountered somewhat higher values of the order of 0.34. It seems likely that the monopole excitation of the facility by the oscillation of the cavity volume, $V(t)$, would lead to facility dependent effects and this may account for some of this discrepancy. Most recently, Tsujimoto *et al.* (1998) remark that it may be important to distinguish between the phenomena of vortex shedding from the rear of an otherwise stable fully-developed cavity and the partial cavity instability. We focus here on the latter because of the violence of the phenomenon.

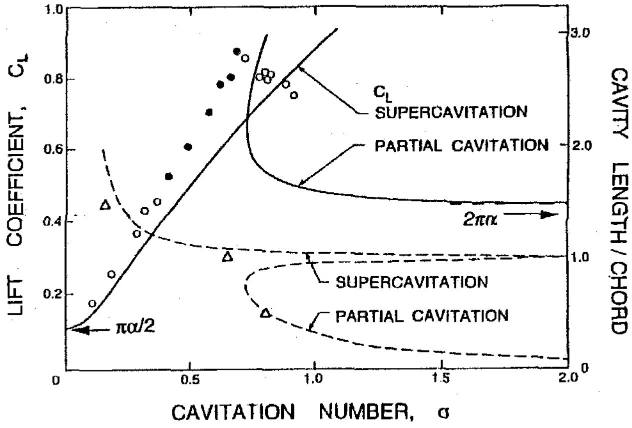


Figure 1: Typical results from the linearized theories for a cavitating flat plate at an angle of attack of 4°. The lift coefficients, C_L (solid lines), and the ratios of cavity length, ℓ, to chord, c (dashed lines), are from the supercavitation theory of Tulin (1953) and the partial cavitation theory of Acosta (1955). Also shown are the experimental results of Wade and Acosta (1966) for ℓ/c ($\triangle$) and for C_L ($\circ$ and $\bullet$) where the open symbols represent points of stable operation and the solid symbols denote points of unstable cavity operation.

The linearized free streamline analyses of these flows by Tulin (1953) in the supercavitating case and by Acosta (1955) in the partial cavitating case offer some insights into the cause of this partial cavity instability (see Brennen 1995). These analyses lead to equilibrium solutions for which the lift coefficient on the foil and the cavity length vary with cavitation number as shown in figure 1. Note that the data from Wade and Acosta (1966) agrees quite well with the results of the steady, linear theory. But here we are addressing the instability which occurs at the solid points in figure 1. Acosta (1981) has previously pointed out (see also Brennen 1995) that this region of instability (specifically a cavity length between 3/4 and 4/3 chords) corresponds closely to the region in which the theories yield a negative lift slope ($dC_L/d\alpha < 0$) and that it could be heuristically argued that such a flow would be unstable. Such a hypothesis would clearly have some merit for a flexibly mounted hydrofoil but since the instability occurs with quite rigidly supported foils, a more fundamental explanation must be sought. Tsujimoto *et al.* argue that the flow is unstable in this region because the cavitation compliance (equal to $-\rho dV/dp_\infty$ (Brennen 1994)) is negative there. This property is also apparent in figure 1 since a positive gradient in the cavity length versus cavitation number graph implies a negative compliance. This is a more plausible explanation for the instability even though the linear stability analysis based on this hypothesis yields oscillation frequencies which do not agree with the experimental measurements. However, the experiments strongly suggest that the instability leads to a limit cycle oscillation and, in such circumstances, the frequency is often radically less than the most unstable frequency predicted by linear stability analysis.

In most of this paper we will be concerned about the radical effects of foil sweep on the evolution of the re-entrant jet and the development of partial cavity instability. However, before embarking on that, it is instructive to review the relationship of the partial cavity instability to another potential instability, namely the leading edge flutter of cavitating foils.

3 CONNECTION WITH LEADING EDGE FLUTTER

Brennen, Oey and Babcock (1980) carried out both experimental and theoretical analyses of leading edge flutter of single cavitating foils, and this shed some particular light on the partial cavitation instability which is worth recounting. The water tunnel experiments involved a number of thin, flexible foils supported near the trailing edge and operated at an angle of attack and at a sufficiently low cavitation number that they had large attached cavities which normally closed well downstream of the trailing edge. As the tunnel velocity, U, was increased these foils would exhibit a dramatic onset of flutter at a particular flutter speed, U_F, given approximately by $U_F/\omega_F c = 0.15 - 0.23$. Here ω_F is the natural frequency of underwater vibration of the foil. Small variations occurred with angle of attack and mass ratio but these are not pertinent to the present discussion. What is pertinent is how this flutter speed and the other oscillation characteristics varied with cavity length. As the cavity length was decreased the flutter speed would decrease, initially slowly, but then dramatically as the cavity length was decreased below $2c$. Since the reduced flutter speed for long cavities, namely $U_F/\omega_F c = 0.15 - 0.23$, corresponds to a reduced frequency of $fc/U = 0.7 - 1.0$, this reduction in frequency is consistent with a transition to partial cavity instability at the substantially lower reduced frequencies described above.

As the cavity length was decreased in those flutter experiments, the amplitude of the vibration and the amplitude of the pressure oscillations in the cavity also increased dramatically. What these observations document is the systematic decline in the stability of a foil with a fully developed cavity when the length of the cavity is reduced below about two chord lengths, and that the partial cavity instability is likely to be the dominant observation for rigid foils when the cavity length is close to the chord of the foil.

4 EFFECT OF FOIL SWEEP

Until very recently, all of the experimental observations of partial cavity instability (and most of the observations for cavitating hydrofoils in general) involved experiments with two-dimensional, unswept foils mounted in water tunnels. However, the recent observations of attached cavities on hydrofoils with sweep by de Lange *et al.* (1994), Jessup (1997) and Laberteaux and Ceccio (1998) have brought to our attention some singular differences between cavity closure with three dimensional swept foils and the traditional two-dimensional foils. Remarkably, closure along a substantial fraction of the span of a swept foil is strikingly steady compared with the corresponding two-dimensional flow. The difference is illustrated in the photographs of Laberteaux and Ceccio, two of which are reproduced in figure 2. As Laberteaux and Ceccio point out, the difference results from changes in the direction of the re-entrant jet. As de Lange *et al* (1994) observe, when the line of closure is no longer normal to the prevailing direction of the fluid flow, the re-entrant jet is no longer directed upstream as it would be in a purely two-dimensional flow but is inclined at a "reflected" angle (as sketched in figure 3) because the momentum tangent to the closure line will be carried through into the re-entrant jet.

One speculation of the consequences of this reflection is included in figure 4 where the probable re-entrant jet streamlines in the Laberteaux and Ceccio swept hydrofoil case (figure 2 lower) are sketched using the jet velocity vectors just after reflection. Note that the cavity closure downstream of the point at which the re-entrant jet reaches the leading edge is radically different and much more unsteady than the closure upstream of this point. A similar phenomenon is seen in the photographs of Jessup (1997). As far as the author is aware, there is no available method which could be used to predict this evolution of the re-entrant jet. Clearly such a method is necessary for the prediction of cavitation on a swept foil or propeller blade.

5 OTHER FOIL GEOMETRIES

The de Lange *et al.* (1994) and Laberteaux and Ceccio (1998) observations were made with swept foils of uniform chord. Foils with spanwise varying chord (*i.e.* non-rectangular planforms), spanwise varying cavitation numbers or incidence angles (as would occur with

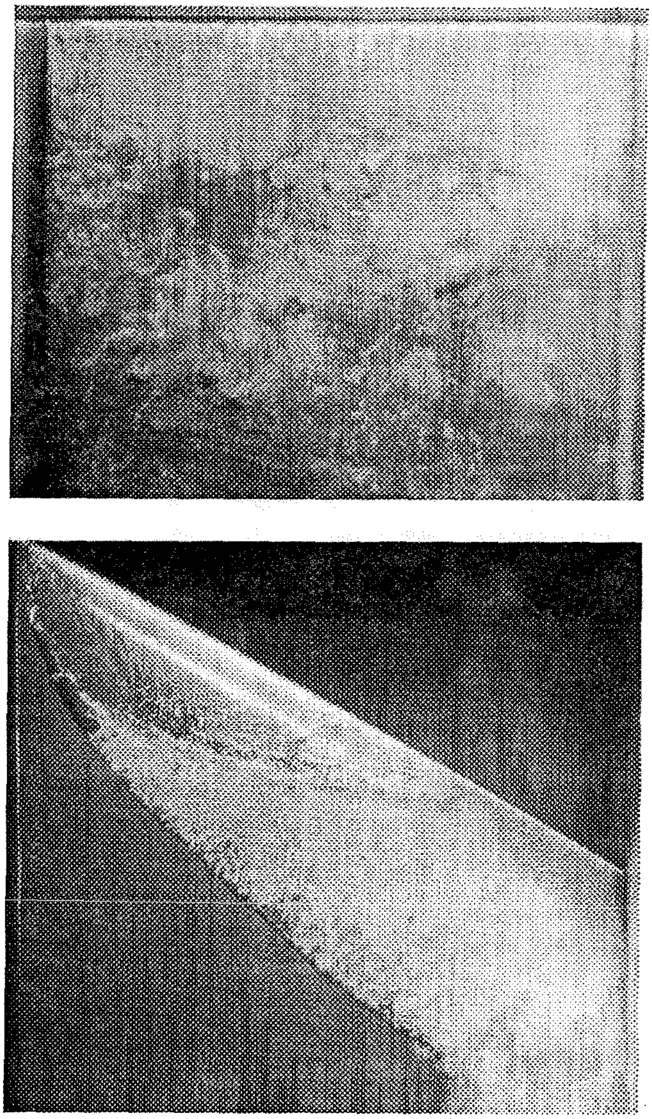


Figure 2: Photographs of the cavitation on the suction surface of single hydrofoils in a water tunnel. The direction of the incident flow is downward. The leading edge is close to the top of the upper photograph and is the inclined line in the lower photograph. The tunnel sidewalls are on the sides of the photographs. Upper photograph: Unswept foil at 2° angle of attack with $U = 9.4m/s$ and $\sigma = 0.6$. Lower photograph: Foil with 30° of sweep at 2° angle of attack with $U = 10.1m/s$ and $\sigma = 0.7$. Reproduced from Laberteaux and Ceccio (1998) with the author's permission.

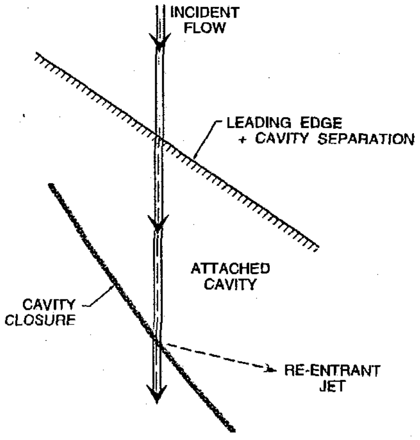


Figure 3: Sketch of the reflection of a re-entrant jet at an inclined cavity closure. Adapted from de Lange *et al.* (1994).

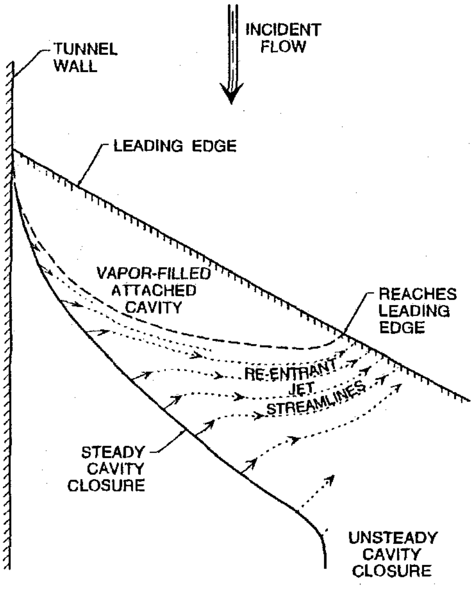


Figure 4: Sketch of the orientation and evolution of the re-entrant jet in figure 2 (lower).

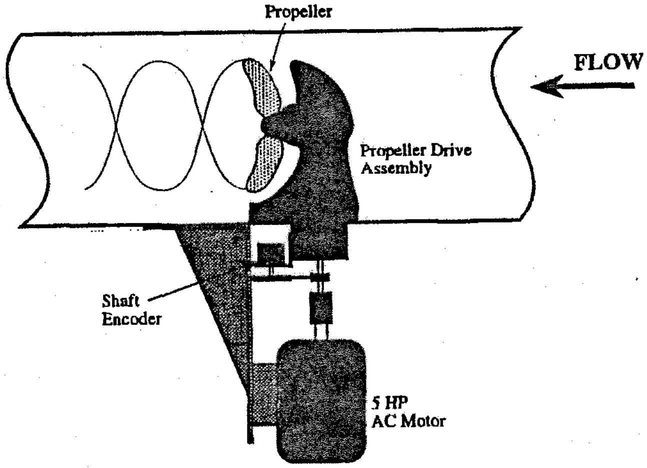


Figure 5: Schematic of the propeller installation in the water tunnel. Adapted from McKenney (1994,95).

a propeller) may exhibit other spanwise developments of the re-entrant jet. Thus many of the instabilities of cavitating propeller blades described by Bark (1986) may result from the spanwise development of the re-entrant jet. Consider, specifically, the flow near the tip of a cavitating propeller blade. The local cavitation number is decreasing and the chord is decreasing with radial position. Thus, at some spanwise location, the flow may become locally unstable to partial cavity instability as the cavity length approaches the local chord length. Though this flow is often further complicated by the interaction with the tip vortex, such a local partial cavity instability may be present in the observations of Higuchi *et al.* (1986) on a finite aspect ratio foil of elliptical planform. In the present paper we describe the occurrence of such an instability on the blades of a propeller.

6 PROPELLER INSTABILITY

Recent experimental observations of cavitation on a typical US Navy propeller (Duttweiler and Brennen 1998) have revealed a substantial instability which appears to be triggered by partial cavity instability near the tip of the blades. We outline this here as an example of the phenomena under discussion.

The US Navy model propeller designated 5236 (diameter $7in$, design advance ratio, $J = 1.15$) was mounted in the Low Turbulence Water Tunnel at Caltech using the drive system developed by McKenney (1994,95) and illustrated in figure 5. The support strut which houses the drive can be rotated so that the propeller can be operated (1) at a yaw angle and/or (2) either upstream or downstream of the strut. The observations described here focus only on the configuration in which the propeller is operating downstream and therefore in the wake of the support strut. In this configuration, the propeller exhibited a flow instability at advance ratios, J, below design when the cavitation number was reduced below a certain critical value. The unstable operating points are presented graphically in figure 6. Note that the instability was not observed at the design advance ratio since the cavitation number could not be decreased below 0.14. Also note that the instability occurs at increasingly large cavitation numbers as the advance ratio is decreased. The instability boundary was quite repeatable, but, when it occured, the magnitude of the oscillations were quite variable. The boundary also showed little hysteresis. Moreover, the instability was relatively insensitive to propeller yaw; figure 7 demonstrates that the instability boundary at a yaw angle of 10° was not much different from that at zero yaw angle, though yaw did appear to be somewhat stabilizing.

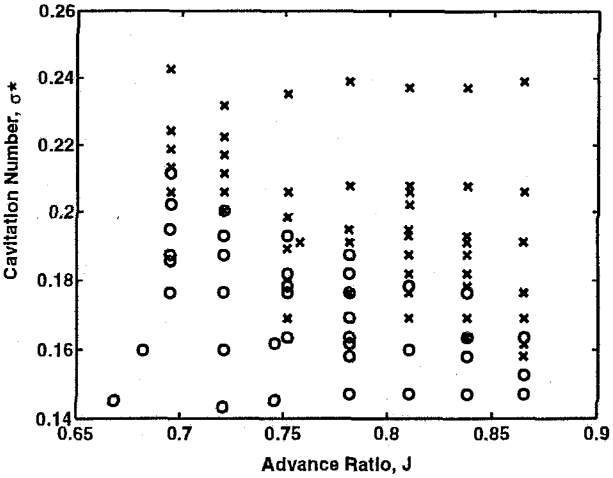


Figure 6: Unstable ($\circ$ symbols), marginally unstable ($\otimes$ symbols) and stable ($\times$ symbols) operating points in $(J, \sigma *)$ space for the 5236 propeller operating at $1850 rpm$ and at zero yaw.

The instability consisted of synchronous oscillation in the size of the cavities on all blades, including both the tip vortex cavities and the blade cavities. These cavity size oscillations were readily visible to the naked eye and readily apparent in the signals from hydrophones installed in the tunnel. Figure 8 presents typical noise spectra from one particular hydrophone installed in the tunnel wall a short distance downstream of the propeller. The figure shows the evolution of the spectrum

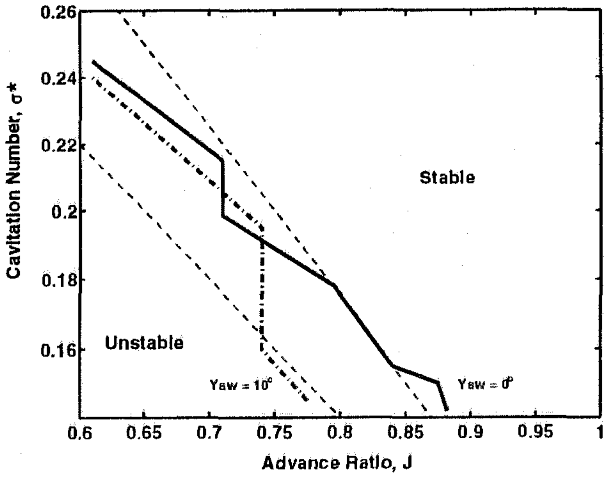


Figure 7: Supplement to figure 6 showing the effect of propeller yaw angle on the instability boundary. Solid line is taken from figure 6 for zero yaw while the dash-dot line is for a yaw angle of 10°. The straight, dashed lines are described in the text.

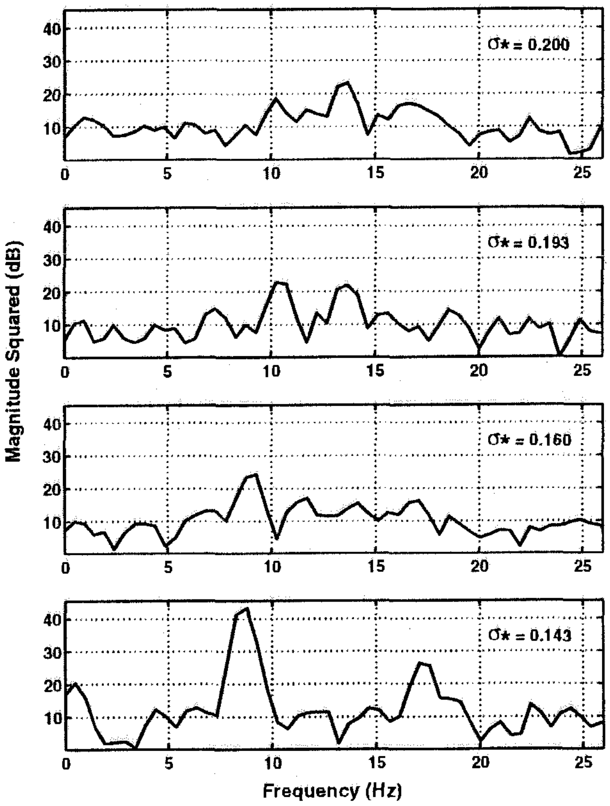


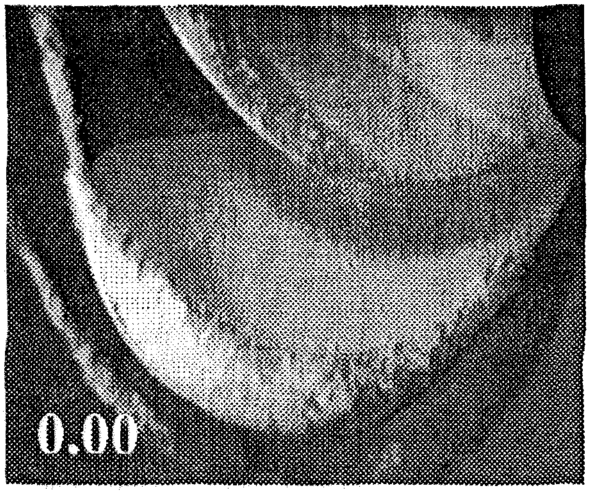
Figure 8: Series of power spectral density plots for the radiated noise from the cavitating propeller operating at $J = 0.72$ and decreasing cavitation number.

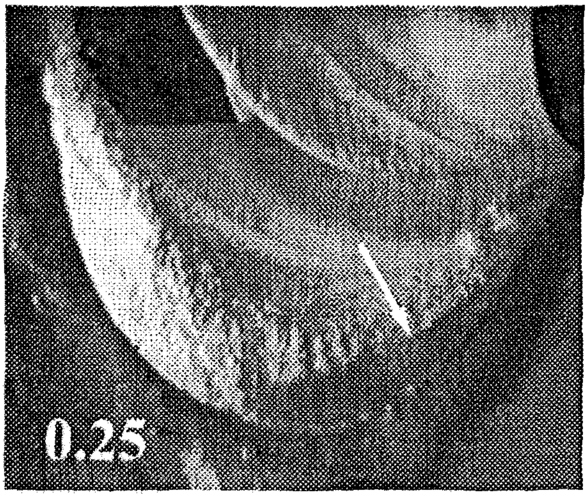
as the cavitation number is reduced at a fixed advance ratio of $J = 0.72$. As $\sigma*$ is reduced and the region of instability is entered, a transition occurs from one dominant peak at $14Hz$ to another at $9Hz$. The growth of the lower frequency peak is what characterized the onset of the instability. For simplicity we refer to the lower frequency as the instability frequency. This frequency was relatively insensitive to cavitation number or advance ratio, and was almost always in the range of $9Hz$ to $10Hz$. The dependence on propeller rotational speed could not be definitively established since similar operating points could only be established for a quite limited range of speeds from $1700rpm$ to $1900rpm$.

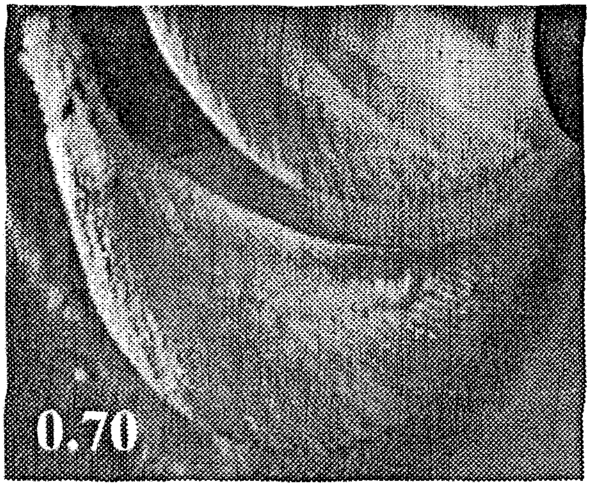
7 INSTABILITY ANALYSIS

Figure 9 presents a series of frames from a digital high speed video recording showing the cavitation on the propeller blades at various moments in the instability oscillation cycle. The numbers are the fraction, τ, of the instability cycle elapsed. At $\tau = 0$ the attached cavity is at its smallest volume and the re-entrant jet penetrates almost to the leading edge. At $\tau = 0.25$ the cavity is growing and the re-entrant jet is being swept downstream; its forward limit can clearly be seen and is identified by the white arrow. At $\tau = 0.70$ the cavity is close to its maximum extent and the re-entrant jet has been swept off the rear of the blade over a substantial part of the span near the tip. Consequently the cavity length near the blade tip is now longer than the blade chord over a substantial part of the blade span. After this the cavity collapses much more quickly than it grew. As shown at $\tau = 0.92$, the re-entrant jet reforms and quickly invades the cavity from the rear. From the high speed video it appears that the jet is also most active at the tip. Eventually, almost the entire cavity is filled as the re-entrant jet approaches the leading edge again at $\tau = 0$.

These observations strongly suggest a limit cycle behavior which is a three-dimensional version of the partial cavity instability previously described for two-dimensional foils. Further perspective on this can be gained by considering the onset boundaries in figures 6 and 7. To do this, consider the approximate functional dependence of the cavity length, ℓ, on the angle of attack, α, and cavitation number, σ, which emerges from the studies of single, two-dimensional foils. These show that ℓ/c is approximately a function of the single variable α/σ. Now, in the context of a propeller, the angle of attack in the vicinity of the blade tip is approximately proportional to $J_O - J$. It follows that, if the instability is similar to the two-dimensional partial







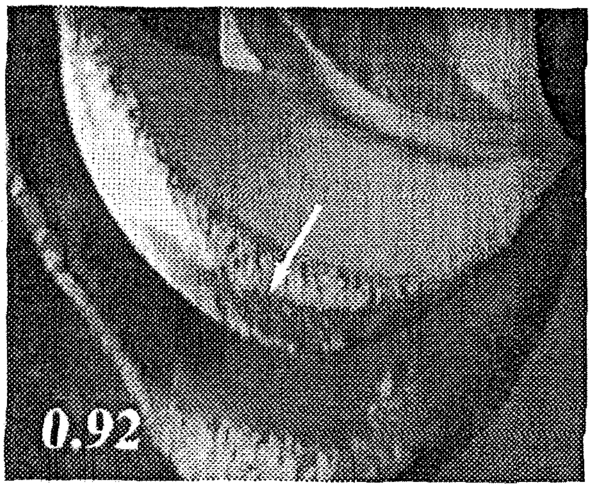


Figure 9: Series of frames from a digital high speed video recording showing the variation in cavitation and position of the re-entrant jet through one instability cycle. The numbers indicate the fraction, τ, of the instability cycle elapsed where the zero is arbitrary but coincides with the smallest extent of cavitation. The white arrows indicate the front of the re-entrant jet.

cavity instability and occurs at a particular value of ℓ/c then the instability boundary should occur along a line of constant $(J_0 - J)/\sigma^*$ in figures 6 and 7. Two examples of lines of constant $(J_0 - J)/\sigma^*$ are plotted in figure 7 to demonstrate that the instability boundary does indeed conform quite closely to such a construct. This further strengthens the explanation of the propeller observations as a partial cavity instability.

When the observed limit cycle frequency of about $9Hz$ is non-dimensionalized using the relative incident velocity at the tip and the maximum propeller chord, *the resulting reduced frequency is about 0.04. This is* significantly smaller than the range of reduced frequencies observed for two-dimensional foils, for example the values of $0.07 - 0.14$ measured by Wade and Acosta (1966). The reason for this discrepancy is not clear but then the mechanism which determines the frequency in either case is not understood.

Finally, we note that this propeller instability investigation is on-going and further information will be provided later.

8 CONCLUDING REMARKS

This paper reviews the literature on partial cavity instability and the dynamics of re-entrant jets for two-dimensional foils. Then we add the recent observations of de Lange *et al.* (1994) and Laberteaux and Ceccio (1998) which highlight the marked differences in the appearance of cavity closure on a foil with sweep as compared to a foil without sweep. These observations demonstrate the importance of the spanwise evolution of cavity closure and the re-entrant jet. These, in turn, have a major impact on the steadiness of the flow and on the formation of cavity clouds in the closure region.

With this as background, observations of an instability of a cavitating propeller are described. High speed video and movie film of this phenomena show that it involves a cycle of cavity growth and collapse similar to that of partial cavity instability. The partial cavities on the propeller blades grow in unison until a substantial region near the tip has a cavity length greater than the local chord. Then the cavity collapses as the re-entrant jet fills the cavity and penetrates almost to the leading edge. The instability is primarily observed at advance ratios below design and at progressively higher cavitation numbers at lower J. It is shown that the instability boundary seems to correspond with a particular cavity size or length at which a substantial spanwise fraction near the tip has a cavity length which oscillates about the chord length. The effect of propeller yaw was also examined and appeared to be minor.

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