Fabrizio d'Auria
Graduate Student.

Luca d'Agostino
Associate Professor.

Dipartimento di Ingegneria Aerospaziale,
Università degli Studi di Pisa,
Via Diotisalvi 2,
56126 Pisa, Italy

Christopher E. Brennen
Professor,
California Institute of Technology,
Mechanical Engineering Dept. 104-44,
1201 E. California Blvd.
Pasadena, CA 91125

Dynamic Response of Ducted Bubbly Flows to Turbomachinery-Induced Perturbations

The present work investigates the dynamics of the three-dimensional, unsteady flow of a bubbly mixture in a cylindrical duct subject to a periodic pressure excitation at one end. One of the purposes is to investigate the bubbly or cavitating flow at inlet to or discharge from a pump whose blade motions would provide such excitation. The flow displays various regimes with radically different wave propagation characteristics. The dynamic effects due to the bubble response may radically alter the fluid behavior depending on the void fraction of the bubbly mixture, the mean bubble size, the pipe diameter, the angular speed of the turbomachine and the mean flow Mach number. This simple linearized analysis illustrates the importance of the complex interactions of the dynamics of the bubbles with the average flow, and provides information on the propagation and growth of the turbopump-induced disturbances in the feed lines operating with bubbly or cavitating liquids. Examples are presented to illustrate the influence of the relevant flow parameters. Finally, the limitations of the theory are outlined.

1 Introduction

Unsteady phenomena in liquid/gaseous mixtures in ducts are relevant to a number of technological applications (Brennen, 1994). Typical in this respect are modern cryogenic liquid propellant rockets in which the propellant storage pressure is close to the saturation value. This inevitably increases the possibility of cavitation in the propellant feed turbopumps, which often extends into the supply lines producing a bubbly two-phase mixture in the inlet line. This can lead to the onset of operational instabilities of the turbopump similar to the rotating stall and surge phenomena commonly observed in compressors (Brennen, 1994).

Extensive efforts have been made to include the effects of bubble dynamics, as well as liquid compressibility and relative motion, in the analysis of dispersed bubbly flow mixtures (van Wijngaarden, 1964, 1968, 1972; Stewart and Wendroff, 1984). Of particular relevance here are the studies of the dynamics of clusters of bubbles by Chahine (1982a, 1982b), Pylkkänen (1986), Omta (1987), d'Agostino and Brennen (1983, 1988, 1989), Kumar and Brennen (1990), Chahine et al. (1991), Mørch (1980, 1981, 1982), and Hansson et al. (1981). These investigations uniformly indicate that, even at relatively low void fractions, the complex interaction of a large number of bubbles with the pressure field drastically modifies the propagation of disturbances in the bubbly mixture and the spectrum of the internal oscillations of the flow (d'Agostino and Brennen, 1988, 1989).

The present paper is an extension of previous research efforts on the dynamics of bubbly and cavitating flows (d'Agostino and Brennen, 1983, 1988, 1989; d'Agostino et al., 1988; d'Auria et al., 1994) all of which were initiated by the paper by d'Agostino and Brennen (1983) in which the expression for the natural frequency of a cloud of bubbles was first derived. Here, a linear perturbation approach is applied to the more complex case of the three-dimensional unsteady flow of a bubbly mixture in a cylindrical duct subject to a periodic pressure excitation at one of the ends. Different wave propagation characteristics are observed which correspond to the various flow regimes previously defined (d'Agostino et al., 1988). The bubble dynamic effects strongly depend on the void fraction of the bubbly mixture, the mean bubble size, the pipe diameter, the angular speed of the turbomachine and the mean flow Mach number. Despite the inherent limitations of the linear approximation, this analysis illustrates some of the dynamic properties and fundamental phenomena of real bubbly liquids and contributes to the understanding of the flow instabilities occurring in several important engineering applications.

2 Basic Equations and Linearization

The basic equations employed are identical to those previously used by d'Agostino et al. (1983, 1988, 1989) and d'Auria et al. (1994). Relative motion between the liquid and the bubbles has a negligible effect on the results for this kind of flows (d'Agostino and Brennen, 1988) and will not be included here. Then, if $\mathbf{u}$ is the velocity of the mixture, with pressure p, unperturbed density ρ, speed of sound c, and bubble concentration β per unit liquid volume, the continuity equation for the mixture, neglecting the mass of the bubbles, becomes:

$$\nabla \cdot \mathbf{u} = \frac{1}{1+\beta\tau}\frac{D(\beta\tau)}{Dt} - \frac{1}{\rho c^2}\frac{Dp}{Dt} \tag{1}$$

where $D/Dt = \partial/\partial t + \mathbf{u}\cdot\nabla$ is the Lagrangian time derivative, and $\tau = 4\pi R^3/3$ is the volume of the bubbles, assumed spherical with radius R. The void fraction, $\alpha = \beta\tau/(1+\beta\tau)$, is assumed to be very small compared with unity. It is also assumed that no bubbles are created or destroyed so that β is a constant in the bubbly fluid. Neglecting body forces and viscous effects in the large-scale flow (viscous effects are included in the bubble dynamics), the momentum equation for the liquid becomes:

$$\frac{\rho}{1+\beta\tau}\frac{D_u\mathbf{u}}{Dt} = -\nabla p \tag{2}$$

The bubble radius is assumed given by the Rayleigh-Plesset

Contributed by the Fluids Engineering Division for publication in the JOURNAL OF FLUIDS ENGINEERING. Manuscript received by the Fluids Engineering Division August 31, 1995; revised manuscript received March 20, 1996. Associate Technical Editor: O. C. Jones.

equation (Plesset and Prosperetti, 1977; Knapp et al., 1970) modified as indicated by Keller et al. (see Prosperetti, 1984) to account for dissipation effects in the bubble dynamics (including liquid compressibility):

$$\left(1-\frac{1}{c}\frac{DR}{Dt}\right)R\frac{D^2R}{Dt^2}+\frac{3}{2}\left(\frac{DR}{Dt}\right)^2\left(1-\frac{1}{3c}\frac{DR}{Dt}\right)$$

$$=\left(1+\frac{1}{c}\frac{DR}{Dt}\right)\frac{p_R(t)-p(t+R/c)}{\rho}+\frac{R}{\rho c}\frac{dp_R(t)}{dt} \quad (3)$$

Here $p_R(t)$ is the liquid pressure at the bubble surface, related to the bubble internal pressure p_B (assumed uniform) by:

$$p_B(t)=p_R(t)+\frac{2S}{R}+4\mu\frac{1}{R}\frac{DR}{Dt}$$

where $p_B(t)$ is the sum of the liquid vapor pressure and the pressure of a fixed mass of noncondensable gas, and S is the surface tension of the liquid. Clearly, for the closure of the problem, the above equations must be supplemented by the mechanical and thermal equations of state and by the energy conservation equations for the two phases and the relevant boundary conditions.

The governing equations are now linearized by assuming decomposition into steady flow components (denoted by the subscript o) and small, time-harmonic complex fluctuations (indicated by the tilde) of frequency, ω. Thus, for instance:

$$p-p_o=\mathrm{Re}\{\tilde{p}\} \quad \text{where} \quad \tilde{p}=\hat{p}e^{-i\omega t},$$

and so on for the other flow variables. In the most general case ω is complex and the fluctuations consist of damped or amplified oscillations with amplification rate given by $\mathrm{Im}(\omega)$. For the sake of simplicity we first consider the case of zero mean flow ($\mathbf{u}_o = 0$). Linearization of the momentum and energy equations for a bubble containing a perfect gas of uniform properties leads to a harmonic oscillator equation for each individual bubble (Prosperetti, 1984, 1977):

$$(-\omega^2-i\omega 2\lambda+\omega_B^2)\hat{R}=-\left(1+i\omega\frac{R_o}{c}\right)\frac{\hat{p}}{\rho R_o}$$

where $\omega_B=\omega_B(\omega)$ is the bubble natural frequency and the damping coefficient, $\lambda=\lambda(\omega)$, is given by the sum of three terms accounting for the viscous, acoustical, and thermal contributions to dissipation (Chapman and Plesset, 1972; d'Agostino and Brennen, 1988).

Elimination of $\hat{\mathbf{u}}$, $\hat{\beta}$, and $\hat{R}$ from the linearized equations yields the following Helmholtz equation for $\hat{p}$:

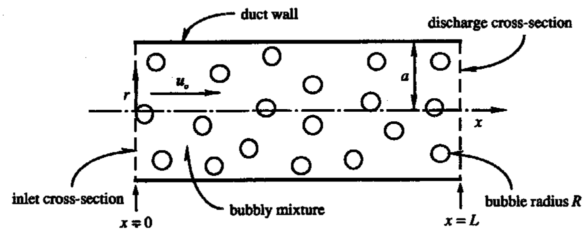

Fig. 1 Schematic of a bubbly flow in a cylindrical duct

$$\nabla^2\hat{p}+k^2(\omega)\hat{p}=0 \quad (4)$$

with the free-space wave number k determined by the dispersion relation:

$$\frac{1}{c_M^2(\omega)}=\frac{k^2(\omega)}{\omega^2}$$

$$=\frac{1}{c_{Mo}^2}\left(\frac{\omega_{Bo}^2(1+i\omega R_o/c)}{\omega_B^2-\omega^2-i\omega 2\lambda}\right)+\frac{(1-\alpha)^2}{c^2} \quad (5)$$

Here $c_M(\omega)$ is the complex and dispersive (frequency dependent) speed of propagation of an harmonic disturbance of angular frequency ω in the free bubbly mixture, while:

$$\omega_{Bo}^2=\frac{3p_{Bo}}{\rho R_o^2}-\frac{2S}{\rho R_o^3} \quad \text{and} \quad c_{Mo}^2=\frac{\omega_{Bo}^2R_o^2}{3\alpha(1-\alpha)}$$

are, respectively, the natural frequency of oscillation of a single bubble at isothermal conditions in an unbounded liquid (Plesset and Prosperetti, 1977; Knapp et al. 1970) and the low-frequency sound speed in a free bubbly flow with incompressible liquid ($\omega \to 0$ and $c \to \infty$).

3 Dynamics of a Bubbly Flow in a Cylindrical Duct

Now examine the three-dimensional, unsteady perturbation of a bubbly mixture in a cylindrical duct of length L and radius a, with rigid walls and arbitrary pressure excitation at the inlet cross section, $x = 0$, as shown in Fig. 1. We shall see that the case of finite mean flow ($\mathbf{u}_o \neq 0$) can readily be obtained by an extension of the zero mean flow solution ($\mathbf{u}_o = 0$) and we thereby begin with the latter. We examine the simple case of a finite length duct, $0 \le x \le L$. As an example, consider the case of a duct connected to a constant pressure reservoir, so that $\hat{p}|_{x=L} = 0$. The other relevant boundary conditions are given by $\hat{u}_r|_{r=a} = 0$ together with the regularity of the solution on the centerline and its periodicity in the azimuthal direction. By

Nomenclature

a = duct radius
c = speed of sound, Fourier coefficient
i = imaginary unit
J = Bessel function of the first kind
k = wave number
L = duct length
M = Mach number
N = number of blades
p = pressure
p_R = liquid pressure at bubble surface
R = bubble radius
r = radial coordinate
S = surface tension
t = time
$\mathbf{u}$ = fluid velocity vector
u = axial velocity
x = axial coordinate
$z_{m,n}$ = nth root of $J'_m(z)$
α = void fraction
β = bubble concentration per unit liquid volume
γ = specific heat ratio
λ = damping coefficient
μ = liquid viscosity
ϑ = angular coordinate
ρ = liquid density
τ = bubble volume
Ω = rotor angular speed
ω = frequency

Subscripts

B = bubble
L = Lagrangian
l = axial mode number
m = order of Bessel function J
n = order of $J'_m(z)$ roots
o = mean flow
s = Fourier index
x = axial

Superscripts

~ = perturbation quantity
∧ = complex amplitude of perturbation
* = complex conjugate
( )′ = differentiation

standard methods (Lebedev, 1965), the separable solution (normalized, for convenience, at $x = 0$) for the finite length duct is found to be:

$$\tilde{p}_{m,n}(\omega) \approx J_m(z_{m,n}r/a)e^{\pm im\vartheta - i\omega t}\frac{\sin k_x(L-x)}{\sin k_x L} \quad (6)$$

(or its complex conjugate), where $z_{m,n}$ is the nth non-negative root of $J'_m(z) = 0$, and the axial wave number is: $k_x(\omega) = \sqrt{k^2(\omega) - z_{m,n}^2/a^2}$ (principal branch). For a semi-infinite duct the radiation condition at $x = 0$ replaces the condition at $x = L$, and, with earlier notations, the corresponding solution is:

$$\tilde{p}_{m,n}(\omega) \approx J_m(z_{m,n}r/a)e^{i(\pm m\vartheta - \omega t + k_x x)} \quad (7)$$

We consider, in particular, the solution for the idealized excitation generated by a turbomachine rotor with N blades and angular speed, Ω, located at the inlet cross-section ($x = 0$). The pressure excitation is assumed $2\pi/N$-periodic in the rotating angular coordinate $\vartheta' = \vartheta - \Omega t$ and can therefore be decomposed as:

$$p(r, \vartheta, 0, t) - p_o = \sum_{s=-\infty}^{\infty}\sum_{n=0}^{\infty} c_{sN,n}J_{\pm sN}(z_{\pm sN,n}r/a)e^{isN(\vartheta - \Omega t)}$$

where s is the harmonic index of the Fourier decomposition, $m = \pm sN \geq 0$ is the azimuthal mode number and $\omega = \pm sN\Omega \geq 0$ is the blade excitation frequency, with the upper and lower signs for $s \geq 0$ and $s < 0$, respectively. The coefficients:

$$c_{0,0} = \frac{N}{\pi a^2}\int_0^a rdr\int_{-\pi/N}^{\pi/N} p(r, \vartheta, t)d\vartheta$$

$$c_{sN,n} = \frac{N\int_0^a rJ_{\pm sN}(z_{\pm sN,n}r/a)dr\int_{-\pi/N}^{\pi/N} p(r, \vartheta, t)e^{-isN\vartheta}d\vartheta}{\pi a^2(1 - s^2N^2/z_{\pm sN,n}^2)J_{\pm sN}^2(z_{\pm sN,n})} \quad \text{for} \quad s, n \neq 0, 0$$

are readily obtained from the orthogonality properties of $e^{-isN\vartheta}$ and $J_{\pm sN}(z_{\pm sN,n}r/a)$ on the duct cross-section. Consistent with the linearization, the solution for the assigned pressure excitation $p(r, \vartheta, t)$ may be expressed by the series:

$$p(r, \vartheta, x, t) - p_o = \sum_{s=-\infty}^{\infty}\sum_{n=0}^{\infty} c_{sN,n}\frac{\tilde{p}_{sN,n}(sN\Omega)}{\tilde{p}^*_{-sN,n}(-sN\Omega)}$$

with the upper conjugate solution valid for $s \geq 0$ and the lower valid for $s < 0$. The remaining flow variables are then readily obtained from the linearized governing equations.

The above treatment is easily extended to the case of nonzero mean flow velocity ($\mathbf{u}_o \neq 0$) by means of the Galilean transformation $x = x_L + u_o t$ between the absolute (Eulerian) frame and the Lagrangian frame (subscript L) moving with the bubbly mixture. In this frame the unperturbed fluid is at rest and the general solution for a finite length duct has the form:

$$\tilde{p}_{m,n}(\omega_L) \approx J_m(z_{m,n}r_L/a)e^{\pm im\vartheta_L - i\omega_L t}\frac{\sin k_x(L - x_L)}{\sin k_x L} \quad (8)$$

where $\omega_L = \omega - u_o k_x$ is the Lagrangian frequency experienced by the bubbles in their trajectory and $k_x = k_x(\omega_L)$. Substituting the transformed coordinates $x_L = x - u_o t$, $r_L = r$ and $\vartheta_L = \vartheta$ we can confirm that the solution $\tilde{p}_{m,n}(\omega)$ for a stationary fluid given by equations (6) and (7) remains valid in the absolute frame provided that the excitation frequency is re-defined as $\omega = \omega_L + u_o k_x$. Therefore, for any assigned value of the Eulerian frequency, ω, the axial wave number k_x is the (generally complex) solution of the equation $k_x = k_x(\omega_L)$ with Lagrangian frequency $\omega_L = \omega - u_o k_x$.

The entire flow has therefore been determined in terms of the material properties of the two phases, the geometry of the duct, the nature of the excitation, and the quantities, R_o, p_o, $\mathbf{u}_o$, and α.

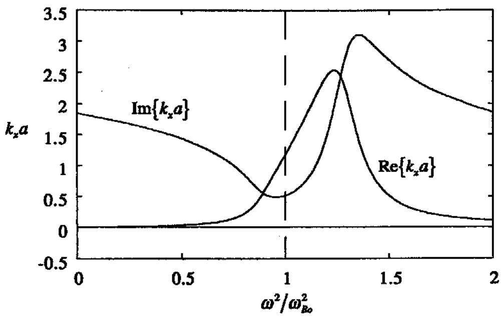

Fig. 2 Real part, Re($k_x a$), and imaginary part, Im($k_x a$), of the normalized damped axial wave number as a function of the square of the reduced frequency, ω/ω_{Bo}. The results shown are for the fundamental radial mode of the first azimuthal harmonic ($m = 1$, $n = 0$, $z_{m,n} = z_{1,0} = 1.8412$) and for $3\alpha(1 - \alpha)a^2/R_o^2 = 1$.

4 Results and Discussion

The results presented are intended to illustrate the most significant features and phenomena implicit in the above solution. We choose a duct of radius $a = 0.15$ m and length $L = 1$ m, containing air bubbles ($R_o = 0.001$ m, $\gamma = 1.4$, $\chi_G = 0.0002$ m^2/s) in water ($\rho = 1000$ kg/m^3, $\mu = 0.001$ Ns/m^2, $S = 0.0728$ N/m, $c = 1485$ m/s) at atmospheric pressure ($p_o = 10^5$ Pa). In addition, we must specify the void fraction, α. Note that the bubble interaction parameter, $3\alpha(1 - \alpha)L^2/R_o^2$, which could be much larger or smaller than unity, appears in all of these bubbly flow analyses. The dynamic response of the flow can be very different depending on $3\alpha(1 - \alpha)L^2/R_o^2$. The mean flow the Mach number $M_o = u_o/c_{Mo}$ also occurs as a parameter in the present solutions.

We first consider the general features of the propagation of disturbances of real frequency ω in the absence of mean flow. Note that the solution for a semi-infinite duct is oscillatory in ϑ, in t, and, in complex sense (damped or amplified), also in x, while its behavior in the radial direction is expressed by Bessel functions of integer order m, scaled by the factor $z_{m,n}/a$ in order to satisfy the kinematic boundary condition at the duct wall. In particular, the solution for $m = n = 0$ corresponds to plane axial waves. Wave propagation along the duct is regulated by the axial wave number, $k_x = k_x(\omega)$. In the undamped case k_x^2 is real (positive, zero, or negative) and the axial modes are either purely harmonic or exponential in space, with wavelength and amplification rate respectively determined by the real and imaginary parts of k_x. In the presence of dissipation k_x is neither real nor purely imaginary (see Fig. 2 for the sample case, $m = 1$ and $n = 0$) and the axial modes consist of harmonic oscillations with amplitude changing in the axial direction. Notice that k_x varies with ω and is generally different from the free-space wave number k because of the presence of the duct boundaries (except for the simple case of plane axial waves where $z_{m,n} = z_{0,0}$ vanishes). Thus, for any particular bubbly mixture, the axial modes will depend on the oscillation frequency and the duct geometry.

The behavior of k_x as a function of the frequency ω is most readily illustrated in the absence of damping (when the argument of the square root is real). Then the axial wave number is either real or purely imaginary depending on the sign of $k^2(\omega) - z_{m,n}^2/a^2$, with a first regular transition at the cut-off frequency:

$$\omega_{m,n}^2 = \omega_{Bo}^2 \Big/ \left(1 + \frac{3\alpha(1 - \alpha)a^2/R_o^2}{z_{m,n}^2}\right) \quad (9)$$

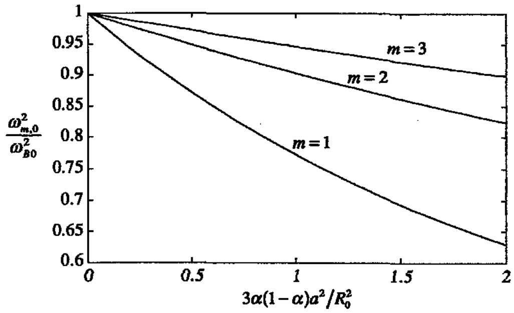

Fig. 3 Normalized cut-off frequency, $\omega^2_{m,0}/\omega^2_{Bo}$, as a function of the bubble interaction parameter, $3\alpha(1-\alpha)a^2/R_o^2$, for the fundamental radial mode ($n=0$) of the lowest azimuthal harmonics ($m=1, 2, 3$)

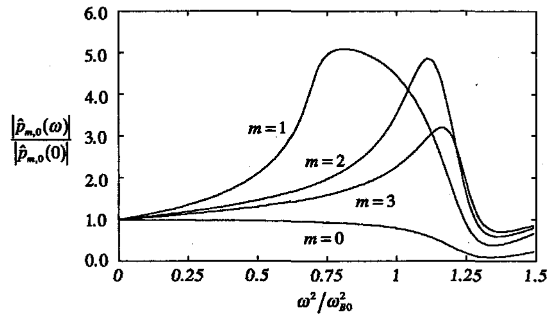

Fig. 4 Normalized amplitude of the pressure oscillations, $|\hat{p}_{m,0}(\omega)|/|\hat{p}_{m,0}(0)|$, as a function of the square of the reduced frequency, ω/ω_{Bo}, for the fundamental radial mode ($n=0$) of the lowest azimuthal harmonics ($m=0, 1, 2, 3$) in a semi-infinite duct with $3\alpha(1-\alpha)a^2/R_o^2=1$

and a second, singular transition at the natural frequency, ω_{Bo}, of an individual bubble in an infinite liquid (bubble resonance condition), where $k^2(\omega)$ has a simple pole. Notice that the cut-off frequencies are never greater than ω_{Bo} and that they increase with the radial mode number, n, for any given azimuthal harmonic m. In addition, the cut-off frequencies decrease with the bubble interaction parameter $3\alpha(1-\alpha)a^2/R_o^2$ as shown in Fig. 3.

In the present problem the perturbations decay exponentially with attenuation rate $\text{Im}(k_x a)$ when their frequency ω is either lower than $\omega_{m,n}$ or greater than ω_{Bo}. On the other hand, they propagate harmonically with wave length $2\pi/\text{Re}(k_x)$ for frequencies in the range $\omega_{m,n} \le \omega \le \omega_{Bo}$. These three regimes are similar to the subsonic, supersonic and super-resonant regimes of flow identified by d'Agostino et al. (1988). As a result of the higher cut-off frequencies for the higher radial modes, appreciable wave-like propagation of m-lobed azimuthal excitation sources occurs when their frequency falls between the cut-off frequency $\omega_{m,0}$ of the fundamental radial mode ($n=0$) and the bubble resonance frequency $\omega=\omega_{Bo}$.

When applied to the perturbation generated by a turbomachine with N blades and angular speed Ω, from earlier expressions of k^2, appreciable propagation of the sth harmonic disturbance will therefore occur when:

$$M_{|sN|} \ge \frac{z_{|sN|,0}}{|sN|} \tag{10}$$

where $z_{|sN|,0}/|sN|$ is the cut-off value of the blade tip Mach number $M_{|sN|}=\Omega a/c_M(|sN\Omega|)$. The values of $z_{|sN|,0}/|sN|$ are always slightly supersonic and approach unity as the azimuthal mode number sN tends to infinity. Values for $sN=1, 2, 3, 4$, and 5 are, respectively, 1.36, 1.24, 1.18, 1.15, and 1.14.

In more familiar terms, we have determined that effective propagation of the disturbances generated by a turbomachine operating with bubbly flows is limited to the excitation from supersonic rotors not exceeding the bubble resonance condition $|sN\Omega| \le \omega_{Bo}$. This phenomenon is in line with well-established results for compressible nondispersive barotropic fluids (Tyler and Sofrin, 1962; Benzakein, 1972) and may have important implications for the onset and stability of rotating stall and cavitation in the suction lines of pumping systems operating with bubbly or cavitating flows. Note that, since the sonic speed in a bubbly mixture can be very small, it is not implausible to have a supersonic condition in a quite conventional pump.

The presence of a second region of exponential decay of the solution beyond the bubble natural frequency (termed super-resonant condition by d'Agostino et al., 1988) is the direct consequence of the dominance of the inertial forces, which prevent the bubbles from effectively responding to the excitation. Therefore super-resonant flows tend to behave in an essentially incompressible way, not dissimilar to subsonic flows.

From the relevant expression for $\tilde{p}$ note that free oscillations ($c_{m,n}=0$) of bubbly flows in finite-length ducts can only occur when $\sin k_x L = 0$, a condition that, together with the dispersion relation, determines the natural frequencies $\omega_{m,n,l}$ and mode shapes $\tilde{p}_{m,n,l}$. In the absence of damping:

$$\omega^2_{m,n,l} = \omega^2_{Bo} \Big/ \left(1 + \frac{3\alpha(1-\alpha)a^2/R_o^2}{z^2_{m,n} + l^2\pi^2a^2/L^2}\right) \tag{11}$$

$$\tilde{p}_{m,n,l} \approx J_m(z_{m,n}r/a)e^{\pm im\vartheta - i\omega_{m,n,l}t} \sin\frac{l\pi x}{L} \tag{12}$$

where l is a positive integer. Notice that the natural frequencies $\omega_{m,n,l}$ always lie between the cut-off frequencies $\omega_{m,n}$ and ω_{Bo}. They also increase with the axial mode number l, and converge to ω_{Bo} as $l \to +\infty$. Just like the cut-off frequencies, the natural frequencies decrease with the increase in the parameter $3\alpha(1-\alpha)a^2/R_o^2$ and are substantially smaller than ω_{Bo} when it is of order unity or larger.

Now let us consider the effect of damping. The inclusion of damping (see Fig. 2) makes the axial wave number k_x (and therefore also the cut-off frequencies $\omega_{m,n}$) complex and eliminates the singularity at $\omega=\omega_{Bo}$, thereby blurring the transitions between the three propagation regimes. In addition, the higher frequencies are more severely damped than the lower frequencies so that the lower modes will predominate in any application. Except for these aspects, the general propagation features of the solution remain essentially unchanged for moderately damped flows like the present sample case of air bubbles in water. The rest of the results presented include damping (Fig. 4 onward).

The relative amplitudes of the pressure and bubble radius oscillations in a semi-infinite duct are shown in Figs. 4 and 5 as a function of frequency for the fundamental radial mode ($n=0$) of the lowest azimuthal harmonics ($m=0, 1, 2, 3$) with

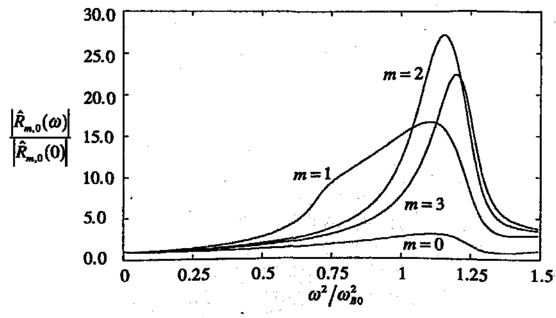

Fig. 5 Normalized amplitude of the bubble radius oscillations, $|\hat{R}_{m,0}(\omega)|/|\hat{R}_{m,0}(0)|$, as a function of the square of the reduced frequency, ω/ω_{Bo}, for the fundamental radial mode ($n=0$) of the lowest azimuthal harmonics ($m=0, 1, 2, 3$) in a semiinfinite duct with $3\alpha(1-\alpha)a^2/R_o^2=1$

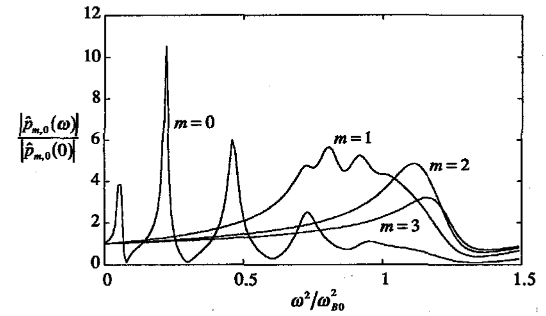

Fig. 6 Normalized amplitude of the pressure oscillations, $|\hat{p}_{m,0}(\omega)|/|\hat{p}_{m,0}(0)|$, as a function of the square of the reduced frequency, ω/ω_{Bo}, for the fundamental radial mode ($n = 0$) of the lowest azimuthal harmonics ($m = 0, 1, 2, 3$) in a finite-length duct with $3\alpha(1 - \alpha)a^2/R_o^2 = 1$

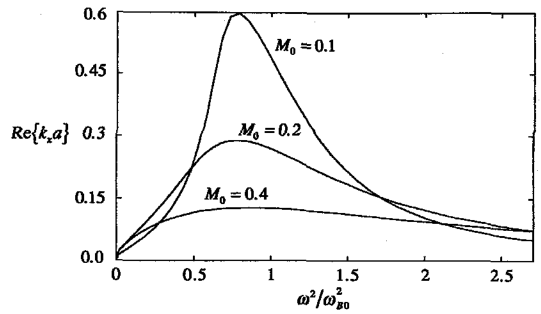

Fig. 8 Real part, Re($k_x a$), of the normalized axial wavenumber for the fundamental radial mode of the first azimuthal harmonic ($m = 1$, $n = 0$, $\alpha_{m,n} = \alpha_{1,0} = 1.8412$) as a function of the square of the reduced frequency, ω/ω_{Bo}, in the absence of damping for $M_o = 0.1, 0.2, 0.4$ and for $3\alpha(1 - \alpha)a^2/R_o^2 = 1$

$3\alpha(1 - \alpha)a^2/R_o^2 = 1$, (corresponding to a void fraction $\alpha \cong 1.5 \times 10^{-5}$ for the assumed values of the duct and bubble radii). The corresponding results for a finite-length duct are shown in Figs. 6 and 7. For all azimuthal harmonics the response of the bubble radius is maximum near the bubble resonance frequency, ω_{Bo}. Despite the rather large value of the bubble interaction parameter $3\alpha(1 - \alpha)a^2/R_o^2$, appreciable oscillations of the pressure and bubble radius are only observed for the zeroth and first azimuthal harmonics in the frequency range from $\omega_{m,0}$ ($\omega_{m,0} = 0$ for plane axial waves) to ω_{Bo}. The solution for the finite-length duct (Figs. 6 and 7) also shows clear evidence of resonant oscillations (amplitude peaks) of the zeroth and first azimuthal modes ($m = 0, 1$) at the corresponding natural frequencies, due to reflection of the flow disturbances by the downstream boundary condition. Additional computations have shown that the higher azimuthal harmonics start displaying appreciable resonant oscillations when the length of the duct is decreased. The same trend has been observed when increasing the duct radius for a given duct length. Thus the aspect ratio, L/a, plays a key role in the dynamics of a bubbly flow in a cylindrical duct.

Next we briefly discuss the extension of previous results to the case of nonzero mean flow velocity ($\mathbf{u}_o \neq 0$) by means of a few simple re-interpretations. In the first place, as shown in Section 3, the presence of a constant mean flow velocity $\mathbf{u}_o$ results in a complex value of the axial wave number even in non-dissipative flows (as illustrated in Figs. 8 and 9) but does not alter the formal expression for the pressure perturbation, given by Eqs. (6) and (7). This implies that the analysis for the zero mean flow case can be easily extended to the case of nonzero mean flow provided that the (·) excitation frequency is redefined as $\omega = \omega_L + u_o k_x$. Moreover the fact that the axial wave number is now given by $k_x = k_x(\omega_L)$ and its imaginary part, Im(k_x), never vanishes (even in the absence of dissipation)

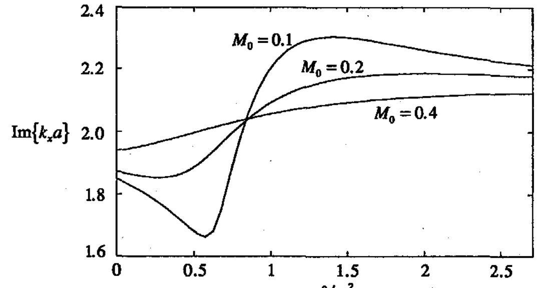

Fig. 9 Imaginary part, Im($k_x a$), of the normalized axial wavenumber for the fundamental radial mode of the first azimuthal harmonic ($m = 1$, $n = 0$, $\alpha_{m,n} = \alpha_{1,0} = 1.8412$) as a function of the square of the reduced frequency, ω/ω_{Bo}, in the absence of damping with $M_o = 0.1, 0.2, 0.4$ and for $3\alpha(1 - \alpha)a^2/R_o^2 = 1$

implies a redefinition of the cut-off frequency: the condition for appreciable propagation of purely harmonic disturbances of real frequency ω through the bubbly mixture (which in the case of zero mean flow was given by Im(k_x) = 0) is now replaced by the condition that Im(k_x) be a minimum. In other words, in the presence of mean flow, the cut-off frequency is given by the value of ω corresponding to the minimum of Im(k_x), as determined by the equation:

$$k_x = \sqrt{\frac{\omega_{Bo}^2}{c_{Mo}^2}\frac{(\omega - u_o k_x)^2}{\omega_{Bo}^2 - (\omega - u_o k_x)^2} - \frac{z_{m,n}^2}{a^2}} \qquad (13)$$

The dependence of the cut-off frequency on the Mach number, $M_o = u_o/c_{Mo}$, for the fundamental radial mode and the lowest azimuthal harmonic is shown in Fig. 10. Notice the rapid decrease in the cut-off frequency for all azimuthal modes as the

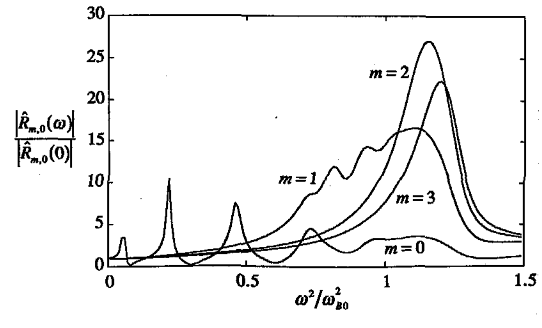

Fig. 7 Normalized amplitude of the bubble radius oscillations, $|\hat{R}_{m,0}(\omega)|/|\hat{R}_{m,0}(0)|$, as a function of the square of the reduced frequency, ω/ω_{Bo}, for the fundamental radial mode ($n = 0$) of the lowest azimuthal harmonics ($m = 0, 1, 2, 3$) in a finite-length duct with $3\alpha(1 - \alpha)a^2/R_o^2 = 1$

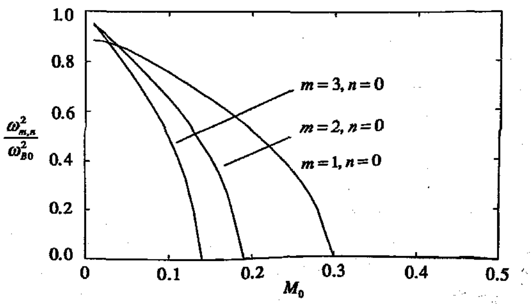

Fig. 10 Normalized cut-off frequency $\omega_{m,0}^2/\omega_{Bo}^2$ as a function of the flow Mach number $M_o = u_o/c_{Mo}$ for the fundamental radial mode ($n = 0$), the lower azimuthal harmonics ($m = 1, 2, 3$) and $3\alpha(1 - \alpha)a^2/R_o^2 = 1$

flow Mach number approaches unity. This feature is also characteristic of compressible flow solutions (Benzakein, 1972).

5 Limitations and Sensitivities

Restrictions to the validity of the present bubbly flow model result from the introduction of the continuum hypothesis, the use of the linear approximation, and the neglect of local interactions between the bubbles. The continuum approach requires the bubble radius, R_o, to be much smaller than any of the macroscopic length scales of the flow, here $a/z_{m,n}$, a/m and $1/|k_x|$ in the radial, azimuthal, and axial directions. This condition is increasingly restrictive for higher and higher harmonics, and therefore only a limited number of Fourier-Bessel components can be realistically included in the solution.

For the linear perturbation approach to hold, the excitation amplitude must not exceed some linear range, especially near resonance. This condition probably represents the most stringent restriction on the present analysis.

The error associated with the neglect of local bubble interactions can be neglected provided that $\alpha^{1/3} \ll |\omega_B^2/\omega^2 - 1|$ (d'Agostino and Brennen, 1988). Far from bubble resonance, this condition is generally satisfied in low void fraction flows.

6 Conclusions

This study reveals a number of important effects occurring in bubbly and cavitating flows in cylindrical ducts as a consequence of the strong coupling between the local dynamics of the bubbles and the global behavior of the flow. The propagation of disturbances along the duct is modified by the large reduction of the sonic speed, which becomes both complex (dissipative) and dispersive (frequency dependent). Additional modifications are introduced by the boundaries, which determine the excitation modes and their cut-off frequencies, and, in finite length ducts, the natural frequencies and mode shapes. The cut-off and natural frequencies never exceed the resonance frequency of individual bubbles, and are very much smaller when the parameter $3\alpha(1 - \alpha)a^2/R_o^2$ is of order unity or larger. Appreciable wave-like propagation of each excitation mode along the duct is limited to the frequency range between cut-off and the bubble resonance condition, and, except for plane waves, is characteristic of supersonic (but subresonant) flows, as defined by d'Agostino et al. (1988). In finite-length ducts, the same frequency range also contains the infinite set of natural frequencies of the resonant modes. The different propagation properties of subsonic, supersonic and super-resonant flows are due to the relative importance of pressure and inertial forces in the bubble dynamics at different excitation frequencies, as already outlined in previous papers (d'Agostino and Brennen, 1988).

In duct flows subject to excitation by a turbomachine, only the perturbations from supersonic rotors propagate effectively and are potentially capable of becoming self-sustaining when effectively reflected by the downstream boundary condition. Given the low sonic speed of bubbly mixtures, the cut-off conditions can readily be exceeded in high-speed turbopumps. This phenomenon is therefore potentially relevant to surge-like auto-oscillations and rotating cavitation instabilities in pumping systems operating with bubbly flows.

Because of the damping in the bubble dynamics the spectral response of the flow is therefore dominated by the lowest resonant frequencies which depend on the bubble interaction parameter, $3\alpha(1 - \alpha)a^2/R_o^2$. The increase of this parameter causes a substantial reduction in the bubble response peaks owing to the greater compliance of the flow, and a decrease in the corresponding frequencies results.

The length-to-radius ratio of the duct also plays an important role in the dynamics of this kind of flow. Higher modes display stronger oscillations as the aspect ratio L/a of the duct increases and the peak frequencies are strongly shifted towards lower values of ω/ω_{Bo}.

Finally, rather drastic modifications of the dynamic behavior of the bubbly mixture also occur when the mean flow velocity of the bubbly mixture becomes comparable to the low-frequency sonic speed. As in more conventional compressible flows, the most important effect of the mean flow arises from the rapid reduction of the cut-off frequencies of all fundamental modes as the Mach number approaches unity.

The present theory has been derived under fairly restrictive linearization assumptions and, therefore, is not expected to provide a quantitative description of unsteady bubbly flows in cylindrical ducts except in the acoustical limit. Bubble radius perturbations are often large in practical applications where the void fraction can be assumed to be small. Therefore the most serious limitation of the present theory is inherent in the linearization of the bubble dynamics.

Acknowledgments

This work has been supported by the Italian Department of Universities and Scientific & Technologic Research under the 1993 grant program for academic research and by the Office of Naval Research under grant number N-00014-91-K-1295. The authors wish to thank Professor T. Colonius for valuable discussions on the subject matter.

References

Brennen, C. E., 1994, "Hydrodynamics of Pumps," Concepts ETI Inc. and Oxford University Press.

Benzakein, M. J., 1972, "Research on Fan Noise Generation," *Journal of the Acoustical Society of America*, Vol. 51, No. 5, Part I, pp. 1427–1438.

Chahine, G. L., 1982a, "Pressure Field Generated by the Collective Collapse of Cavitation Bubbles," *IAHR Symposium on Operating Problems of Pump Stations and Power Plants*, Amsterdam, Netherlands, Vol. 1, Paper No. 2.

Chahine, G. L., 1982b, "Cloud Cavitation Theory," *14th Symposium on Naval Hydrodynamics*, Session I, p. 51.

Chahine, G. L., Duraiswami, R., and Lakshminarasimha, A. N., 1991, "Dynamical Interactions in a Bubble Cloud," ASME *Cavitation and Multiphase Flow Forum*, Portland, OR, pp. 49–54.

Chapman, R. B., and Plesset, M. S., 1972, "Nonlinear Effects in the Collapse of a Nearly Spherical Cavity in a Liquid," ASME *Journal of Basic Engineering*, pp. 172–183.

d'Agostino, L., and Brennen, C. E., 1983, "On the Acoustical Dynamics of Bubble Clouds," ASME *Cavitation and Multiphase Flow Forum*, Houston, TX.

d'Agostino, L., and Brennen, C. E., 1988, "Acoustical Absorption and Scattering Cross-Sections of Spherical Bubble Clouds," *J. Acoust. Soc. Am.*, No. 84 (6), pp. 2126–2134.

d'Agostino, L., and Brennen, C. E., 1989, "Linearized Dynamics of Spherical Bubble Clouds," *Journal of Fluid Mechanics*, Vol. 199, pp. 155–176.

d'Agostino, L., Brennen, C. E., and Acosta, A. J., 1988, "Linearized Dynamics of Two-Dimensional Bubbly and Cavitating Flows over Slender Surfaces," *Journal of Fluid Mechanics*, Vol. 192, pp. 485–509.

d'Auria, F., d'Agostino, L., and Brennen, C. E., 1994, "Linearized Dynamics of Bubbly and Cavitating Flows in Cylindrical Ducts," ASME *Cavitation and Multiphase Flow Forum*, Lake Tahoe, NV, pp. 59–66.

Hansson, I., Kedriniskii, V., and Mørch, K. A., 1981, "On the Dynamics of Cavity Clusters," *Journal of Applied Physics*, Vol. 15, pp. 1725–1734.

Knapp, R. T., Daily, J. W., and Hammit, F. G., 1970, *Cavitation*, McGraw Hill, New York.

Kumar, S., and Brennen, C. E., 1990, "Nonlinear Effects in Cavitation Cloud Dynamics," ASME *Cavitation and Multiphase Flow Forum*, Toronto, Ontario, Canada, pp. 107–113.

Lebedev, N. N., 1965, *Special Functions and Their Applications*, Prentice Hall.

Mørch, K. A., 1980, "On the Collapse of Cavity Cluster in Flow Cavitation," *Proceedings of the 1st International Conference on Cavitation and Inhomogeneities in Underwater Acoustics*, Springer Series in Electrophysics, Vol. 4, pp. 95–100.

Mørch, K. A., 1981, "Cavity Cluster Dynamics and Cavitation Erosion," ASME *Cavitation and Polyphase Flow Forum*, 1981, pp. 1–10.

Mørch, K. A., 1982, "Energy Considerations on the Collapse of Cavity Cluster," *Applied Scientific Research*, Vol. 38, p. 313.

Omta, R., 1987, "Oscillations of a Cloud of Bubbles of Small and Not So Small Amplitude," *Journal of the Acoustical Society of America*, Vol. 82 (3), pp. 1018–1033.

Plesset, M. S., and Prosperetti, A., 1977, "Bubble Dynamics and Cavitation," *Annual Review of Fluid Mechanics*, Vol. 9, pp. 145–185.

Pylkkänen, J. V., 1986, "Characteristics of Spherical Cloud Cavity," *Advancements in Aerodynamics, Fluid Mechanics and Hydraulics*, Arndt et al., eds., ASCE Conference, Minneapolis, MN, pp. 96–103.

Prosperetti, A., 1984, "Bubble Phenomena in Sound Fields: Part One," *Ultrasonics,* Mar. pp. 69–78.

Prosperetti, A., 1977, "Thermal Effects and Damping Mechanisms in the Forced Radial Oscillations of Gas Bubbles in Liquids," *Journal of the Acoustical Society of America,* Vol. 61, No. 1 (6), pp. 17–27.

Stewart, H. B., and Wendroff, B., 1984, "Two-Phase Flows: Models and Methods," *Journal of Computational Physics,* Vol. 56, pp. 363–409.

Tyler, J. M., and Sofrin, T. G., 1962, "Axial Flow Compressor Noise Studies," *SAE Transactions,* Vol. 70, pp. 309–332.

van Wijngaarden, L., 1964, "On the Collective Collapse of a Large Number of Gas Bubbles in Water," *Proceedings of the 11th International Congress on Applied Mechanics,* Springer-Verlag, Berlin, pp. 854–861.

van Wijngaarden, L., 1968, "On the Equations of Motion of Mixtures of Liquid and Gas Bubbles," *Journal of Fluid Mechanics,* Vol. 33, part 3, pp. 465–474.

van Wijngaarden, L., 1972, "One-Dimensional Flow of Liquids Containing Small Gas Bubbles," *Annual Review of Fluid Mechanics,* Vol. 4, pp. 369–396.