Acoustical absorption and scattering cross sections of spherical bubble clouds

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The present work investigates the acoustical absorption and scattering cross sections of spherical bubble clouds subject to harmonic farfield pressure excitation. Bubble dynamics effects and energy dissipation due to viscosity, heat transfer, liquid compressibility, and relative motion of the two phases are included. The equations of motion for the average flow and for the bubble radius are linearized and a closed-form solution is obtained. Due to the presence of natural oscillatory modes and frequencies, the acoustical cross sections of the cloud are very different from those of each individual bubble in the cloud, as well as from the acoustical cross sections of a single large bubble with the same volume of vapor and gas. In general, the acoustical properties of any given volume of the dispersed phase depend strongly on the degree of dispersion because of the complex interactions of the dynamics of the bubbles with the whole flow.

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INTRODUCTION

This paper illustrates part of our current research on the role played by the dynamics of bubble volume changes in the fluid mechanics of bubbly or cavitating flows and represents a natural extension of our previous work on the dynamics of one-dimensional unsteady flows of spherical bubble clouds subject to farfield pressure perturbations (d'Agostino and Brennen, 1988). Among the practical objectives of this study is a better understanding of the global effects of many bubbles in the dynamics and, specifically, in the acoustical behavior of bubbly and cavitating flows. Traditionally the acoustical properties and, in particular, the noise radiation of these flows have been analyzed and interpreted on the basis of single bubble dynamics assuming that the effects of individual bubbles can be algebraically summed. This assumption, for example, is inherent to virtually all commonly accepted scaling laws of noise generation in bubbly and cavitating flows; the void fraction and bubble concentration in the cavitation region never appear as scaling parameters (Blake, 1986). The interactive effects that the bubble volume changes can have in many practical cases on the velocity and pressure distributions (and therefore ultimately on the magnitude of the bubble response itself) are neglected, thus eliminating the effects of any large scale internal motion in the bubbly region of the flow.

The traditional approach may be adequate when the bubble concentration is extremely low, but it clearly loses validity when the bubble concentration becomes larger and the possibility of global motion in the bubbly mixture arises. As early as in 1969, Erdmann and his co-workers noticed a surprising and unexplained sharp decrease of the sound pressure level from traveling bubble cavitation on propeller hydrofoils when the cavitation number dropped below 80% of its inception value and cavitation became more extensive. The optical observations of traveling bubble cavitation on Schiebe headforms in water tunnel tests by Marboe *et al.*, 1986, and the simultaneous sound measurements displayed the tendency of the noise spectrum to shift towards lower frequencies than expected from single bubble dynamics considerations. Marboe and his co-workers suggested the occurrence of asymmetric bubble collapse as a possible cause of this phenomenon. In view of our current results, global bubble interaction effects in the cavitation region when a sufficient concentration of bubbles is present are another possible explanation of the observed reduction of the sound pressure level and of the downward shift of the noise frequency spectrum in heavily cavitating flows. Similar recent experimental results by Arakeri and Shanmuganathan, 1985, and Billet, 1986, have also helped identify bubble interactions in cavitating flows as a likely source of the observed discrepancies. The main purpose of this research is to provide some physical interpretation of the origin of these alterations. Despite the extensive linearizations inherent in the analysis, we are confident that the results convey a qualitative understanding of the dynamic and acoustical properties of real bubbly flows and represent a useful guidance in the study of such flows with nonlinear bubble dynamics.

The last few decades have seen extensive research on the dynamics of bubbly flows (van Wijngaarden, 1968; van Wijngaarden, 1972; Stewart and Wendroff, 1984). Early studies based on space-averaged equations for the mixture in the absence of relative motion between the two phases (Tangren *et al.*, 1949) did not consider bubble dynamic effects. This approach simply leads to an equivalent compressible homogeneous medium. In a classic paper, Foldy, 1945, accounted for the dynamics of individual bubbles by treating them as randomly distributed point scatterers. Assuming that the system is ergodic, the collective effect of bubble dynamic response on the flow is then obtained by taking the ensemble average over all possible configurations. Later, more general equivalent flow models of dispersed two-phase mixtures, which include the effects of bubble dynamics, liquid compressibility, and relative motion, have been developed by ensemble, volume (Biesheuvel and van Wijngaarden, 1984), or time (Ishii, 1975) averaging of the conservation equations for each separate phase. These models have been successfully applied to describe the propagation of both infinitesimal and finite amplitude one-dimensional disturbances through liquids containing small gas bubbles (Carstensen and Foldy, 1947; Fox *et al.*, 1955; Macpherson, 1957; Silberman, 1957; Noordzij, 1973; Noordzij and van Wijngaarden, 1974).

A natural way to account for the effects of bubble dynamics and slip velocity between the two phases is to include the Rayleigh–Plesset and the relative motion equations in the space-averaged equations. However, because of their complexity, there are few reported examples of the application to specific flow geometries of the space-averaged equations that include the effects of bubble response. Recently, Mørch, 1980, 1981, Chahine, 1982a, 1982b, and others have focused their attention on the dynamics of a cloud or cluster of cavitating bubbles and have expanded on the work of van Wijngaarden, 1964. Unfortunately, there appear to be a number of inconsistencies in this recent work which will require further study before a coherent body of knowledge on the dynamics of clouds of bubbles is established. For example, the early work of Chahine, 1982a, does not account for the large scale effects that the bubble volume changes have on the velocity field and, therefore, on the pressure experienced by each individual bubble, though in a later paper, Chahine, 1982b, does begin to consider these global interactions. On the other hand, Mørch and his co-workers, 1980, 1981, 1982, have visualized the collapse of a cloud of cavitating bubbles as involving the inward propagation of a shock wave: it is assumed that the bubbles collapse completely when they encounter the shock. This implies the virtual absence of noncondensable gas in the bubbles and the predominance of vapor. Yet in these circumstances the mixture in the cloud will not have any real sonic speed. As implied by a negative left-hand side of Eq. (13), the fluid motion equations for the mixture would be elliptic not hyperbolic and hence shock wave solutions seem inappropriate. A discussion of the nature of the characteristics of spherical cavity clouds is contained in Pylkkänen, 1986, for various bubbly flow models containing four, five, and six independent variables.

In the present program we focused our attention on one-dimensional steady flows or two-dimensional time-dependent flows. In earlier publications (d'Agostino and Brennen, 1988, d'Agostino *et al.*, 1988) and two previous notes (d'Agostino and Brennen, 1983; d'Agostino *et al.*, 1984) we considered the two-dimensional steady flow of a bubbly liquid over wave-shaped surfaces and the undamped linearized dynamics of a spherical cloud of bubbles subject to an harmonic pressure field. The results clearly show that the fluid motion can be critically controlled by bubble dynamic effects. Specifically, the dominating phenomenon consists of the combined response of the bubbles to the pressure in the surrounding liquid, which results in volume changes leading to a global accelerating velocity field. Associated with this velocity field is a pressure gradient that in turn determines the pressure encountered by each individual bubble in the mixture. Furthermore, it can be shown that such global interactions usually dominate any local pressure perturbations experienced by one bubble due to the growth or collapse of a neighbor (see Sec. IV). In the present work the same approach is applied to derive the acoustical absorption and scattering cross sections of a spherical bubble cloud subject to harmonic farfield pressure perturbations.

During the preparation of this article, the bubble cloud flow problem has been independently addressed by Omta, 1987, using a similar approach. Omta linearized the Biesheuvel–van Wijngaarden homogeneous flow equations for a bubbly mixture (Biesheuvel and van Wijngaarden, 1984) and derived an analytical solution to the flow in a spherical bubble cloud. In his work a number of simplifying assumptions have been introduced with respect to the present analysis. With a slight inconsistency, Omta neglected viscosity and liquid compressibility (and therefore their contributions to damping) in the bubble dynamics, but retained them when considering the relative motion of the two phases and the propagation of pressure disturbances in the liquid. Surface tension has also been neglected and relative motion does not affect the solution explicitly, since in Omta's derivation the slip velocity problem is fully decoupled from the cloud dynamics. Thus thermal damping is, in practice, the only dissipation mechanism accounted for. On the other hand, the above effects are included in the present theory and therefore their relative importance can be easily assessed. Although viscous and acoustic damping in the bubble dynamics and surface tension effects can be important in small bubbles at high excitation frequencies (as indicated by Plesset and Prosperetti, 1977, and confirmed here), the two treatments lead to virtually the same main conclusions on the general characteristics of the flow in the bubble cloud. Despite all its intrinsic limitations, the following linear analysis indicates some of the fundamental phenomena involved and represents a useful basis for the study of such flows with nonlinear bubble dynamics (which we intend to discuss in a later publication).

I. BASIC EQUATIONS

Following the same approach previously indicated in our earlier works (d'Agostino and Brennen, 1988; d'Agostino *et al.*, 1988), we address the problem of the simultaneous solution of the fluid dynamic equations for the two phases with the relevant interaction terms. Let $p(\mathbf{x},t)$ be the liquid pressure, $\mathbf{u}(\mathbf{x},t)$ the velocity of the liquid, $\mathbf{v}(\mathbf{x},t)$ the velocity of the bubbles, and $\mathbf{w}(\mathbf{x},t) = \mathbf{v}(\mathbf{x},t) - \mathbf{u}(\mathbf{x},t)$ the relative velocity of the two phases (defined as the corresponding quantities in the absence of local perturbations due to any neighboring bubbles). Then the liquid, assumed viscous and compressible with viscosity μ, density ρ, speed of sound $c = \sqrt{dp/d\rho}$, and concentration $\beta(\mathbf{x},t)$ of bubbles per unit liquid volume, satisfies the continuity equation in the form:

$$\nabla\cdot\mathbf{u} = \frac{1}{1+\beta\tau}\frac{D_u(\beta\tau)}{D_u t} - \frac{1}{\rho c^2}\frac{D_u p}{D_u t}, \qquad (1)$$

where $D_u/D_u t = \partial/\partial t + \mathbf{u}\cdot\nabla$ indicates the Lagrangian time derivative following the liquid, $\tau(\mathbf{x},t)$ is the individual bubble volume, and β is related to the void fraction by $\beta\tau = \alpha/(1-\alpha)$. Under the hypothesis that no bubbles are created nor destroyed, the bubble number continuity equation gives

$$\nabla\cdot\mathbf{v} = \frac{1}{1+\beta\tau}\frac{D_v(\beta\tau)}{D_v t} - \frac{1}{\beta}\frac{D_v p}{D_v t}. \tag{2}$$

Here, $D_v/D_v t = \partial/\partial t + \mathbf{v}\cdot\nabla$ indicates the Lagrangian time derivative following the bubbles. Furthermore, if external body forces are unimportant, the momentum equation for the liquid phase is

$$\rho\frac{D_u\mathbf{u}}{D_u t} = -(1+\beta\tau)\nabla p. \tag{3}$$

Finally, under the additional hypothesis that the bubbles remain spherical, it follows that $\tau = 4\pi R^3/3$, with the bubble radius $R(\mathbf{x},t)$ determined by the Rayleigh–Plesset equation (Plesset and Prosperetti, 1977; Knapp *et al.*, 1970). In order to account for heat and mass transfer, viscous dissipation, and liquid compressibility effects in the dynamics of the bubbles, the Rayleigh–Plesset equation is modified as indicated by Keller *et al.* (Prosperetti, 1984):

$$\left(1-\frac{\dot R}{c}\right)R\ddot R + \frac{3\dot R^2}{2}\left(1-\frac{\dot R}{3c}\right) = \left(1+\frac{\dot R}{c}\right)\frac{p_R(t)+p(t+R/c)}{\rho} + \frac{R}{\rho c}\frac{dp_R(t)}{dt}, \tag{4}$$

where $p_R(t)$ is the liquid pressure at the bubble surface, related to the bubble internal pressure p_B (assumed uniform) by

$$p_B(t) = p_R(t) + \frac{2S}{R} + 4\mu\frac{\dot R}{R}. \tag{5}$$

Here dots denote Lagrangian time derivatives following the bubbles and S is the surface tension at the bubble interface. Finally, the momentum balance for the dispersed phase, as required for the closure of the problem, is given by the relative motion equation for a spherical bubble of negligible mass in a viscous liquid with Stokes' drag:

$$\frac{\rho\tau}{2}\frac{D_v\mathbf{v}}{D_v t} - \frac{\rho\tau}{2}\frac{D_u\mathbf{u}}{D_u t} + 6\pi\mu R(\mathbf{v}-\mathbf{u}) + \frac{\rho(\mathbf{v}-\mathbf{u})}{2}\frac{D_v\tau}{D_v t} - \tau\nabla p = 0. \tag{6}$$

The above equations, together with suitable boundary conditions, represent in theory a complete system of equations for $p(\mathbf{x},t)$, $\mathbf{u}(\mathbf{x},t)$, $\mathbf{v}(\mathbf{x},t)$, and $\tau(\mathbf{x},t)$. However, in practice, their highly nonlinear nature requires further simplifications for a closed-form solution to be attained even for very simple flows.

II. DYNAMICS OF SPHERICAL BUBBLE CLOUDS

We consider the problem of a one-dimensional flow in a spherical bubble cloud of radius A and void fraction α located at $\mathbf{x}=0$ in an unbounded liquid at rest at infinity, as shown in Fig. 1. Let the perturbation of the farfield pressure be represented by a one-dimensional plane acoustic wave

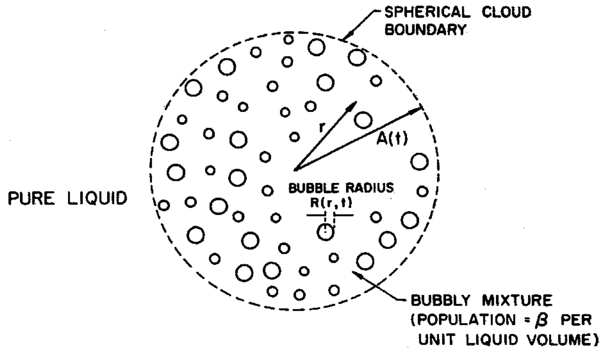

FIG. 1. Schematic of a spherical cloud of bubbles.

$p_\infty(t) = p_0[1+\epsilon\exp i(\omega t - hx)]$ of wavenumber $h=\omega/c \ll 1/A_0$ with $\epsilon \ll 1$ and let the subscript 0 indicate the unperturbed conditions corresponding to $\epsilon = 0$. Then, assuming, for simplicity, that all the bubbles have the same equilibrium radius R_0, the undisturbed pressure in the liquid is

$$p_0 = p_{B0} - 2S/R_0. \tag{7}$$

Linearization of Eqs. (4) and (5) for small changes of the bubble radius $R(t) = R_0[1+\varphi\exp(i\omega t)]$ under the action of a periodic pressure perturbation $p(t) = p_0(1+\epsilon\exp i\omega t)$ leads to modeling each individual gas bubble as an harmonic oscillator (Prosperetti, 1984):

$$(-\omega^2 + i\omega 2\lambda + \omega_{B\omega}^2)\varphi e^{i\omega t} = -\epsilon\frac{p_0}{\rho R_0^2}\left(1 - i\frac{\omega R_0}{c}\right)e^{i\omega t} \tag{8}$$

with internal pressure $p_B(t) = p_{B0}[1-\phi\varphi\exp(i\omega t)]$, where

$$2\lambda = \frac{4\mu}{\rho R_0^2} + \frac{\omega^2 R_0}{c} + \frac{p_{B0}}{\rho\omega R_0^2}\Im(\phi), \tag{9}$$

$$\omega_{B\omega}^2 = \Re(\phi)\frac{p_{B0}}{\rho R_0^2} - \frac{2S}{\rho R_0^3}, \tag{10}$$

$$\phi = \frac{3\gamma\theta^2}{\theta[\theta + 3(\gamma-1)A_-] - i3(\gamma-1)(\theta A_+ - 2)}, \tag{11}$$

$$A_\pm = \frac{\sinh\theta \pm \sin\theta}{\cosh\theta - \cos\theta}, \tag{12}$$

and $\theta = R_0\sqrt{(2\omega/\chi_G)}$ is the ratio of the bubble radius to the bubble thermal diffusion length. The three terms of the effective damping coefficient λ respectively represent the contributions of the viscous, acoustical, and thermal dissipation, $\omega_{B\omega}$ is the effective natural frequency of the oscillator when excited at frequency ω, and $\Re(\phi)/3$ can be interpreted as the effective polytropic exponent of the gas in the bubble, which respectively tends to 1 and γ in the isothermal and isentropic limits for $\omega \to 0$ and $\omega \to +\infty$ (Prosperetti, 1984).

We limit our analysis to the case of uniform and relatively low void fraction so that the flow velocities $\mathbf{u}$,$\mathbf{v}$, and $\mathbf{w}$ are small and purely radial, with components $u(r,t)$, $v(r,t)$, and $w(r,t)$. Then Eqs. (1), (2), (3), and (6) reduce to

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u) = (1-\alpha_0)\beta_0\frac{\partial \tau}{\partial t} + (1-\alpha_0)\tau_0\frac{\partial \beta}{\partial t} - \frac{1}{\rho c^2}\frac{\partial p}{\partial t}, \quad (13)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 w) = -\frac{1}{\beta_0}\frac{\partial \beta}{\partial t} + \frac{1}{\rho c^2}\frac{\partial p}{\partial t}, \quad (14)$$

$$(1-\alpha_0)\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial r}, \quad (15)$$

$$\frac{\partial w}{\partial t} + \left(\frac{9\mu}{\rho R_0^2} + \frac{1}{R_0^2}\frac{\partial R^3}{\partial t}\right) w + \frac{2}{\rho}\frac{\partial p}{\partial r} = 0. \quad (16)$$

For the incompressible single phase flow outside the cloud $[r \geqslant A(t)]$,

$$u(r,t) = C(t)/r^2,$$
$$p(r,t) = p_\infty(t) + \frac{\rho}{r^2}\frac{dC(t)}{dt} + O(C^2(t)), \quad (17)$$

where $C(t)$ is of perturbation order in low void fraction flows. The boundary condition at the cloud surface simply requires the continuity of $p(r,t)$ and $u(r,t)$ at the interface between the cloud and the pure liquid ($r \simeq A_0$ in the linearized approximation). In addition, at the center of the cloud the flow is required to be regular.

The solution of these equations in the domain $r \leqslant A_0$ for the propagation of small spherical disturbances of the form $[\exp i(kr+\omega t)]/r$ is

$$R(r,t) = R_0 - R_0\epsilon\frac{p_0(1-\alpha_0)/\rho R_0^2}{\omega_{B\omega}^2 - \omega^2 + i\omega 2\lambda}\left(\frac{1 - i\omega R_0/c}{\cos kA_0 - \alpha_0\sin(kA_0)/kA_0}\right)\frac{\sin kr}{kr}e^{i\omega t}, \quad (18)$$

$$p(r,t) = p_0 + p_0\epsilon\frac{1-\alpha_0}{\cos kA_0 - \alpha_0\sin(kA_0)/kA_0}\left(\frac{\sin kr}{kr}\right)e^{i\omega t}, \quad (19)$$

$$u(r,t) = i\epsilon\frac{p_0/\rho\omega r}{\cos kA_0 - \alpha_0\sin(kA_0)/kA_0}\left(\cos kr - \frac{\sin kr}{kr}\right)e^{i\omega t}, \quad (20)$$

$$w(r,t) = i\epsilon\left(\frac{p_0/\rho\omega r}{\cos kA_0 - \alpha_0\sin(kA_0)/kA_0}\right)\frac{2(1-\alpha_0)}{1 + 9\nu/i\omega R_0^2}\left(\cos kr - \frac{\sin kr}{kr}\right)e^{i\omega t}, \quad (21)$$

$$\beta(r,t) = \beta_0 + \beta_0\epsilon\frac{p_0}{\rho}\left(\frac{1}{c^2} + \frac{k^2}{\omega^2}\right)\frac{1-\alpha_0}{\cos kA_0 - \alpha_0\sin(kA_0)/kA_0}\left(\frac{\sin kr}{kr}\right)e^{i\omega t}. \quad (22)$$

Here, k is the principal square root (with non-negative real and imaginary parts) of k^2 given by the dispersion equation for a bubbly flow (van Wijngaarden, 1980) with bubble dynamic damping, liquid compressibility, and relative motion effects (d'Agostino and Brennen, 1987):

$$\frac{k^2}{\omega^2} = \left[\frac{1}{c_m^2}\left(\frac{\omega_B^2(1 - i\omega R_0/c)}{\omega_{B\omega}^2 - \omega^2 + i\omega 2\lambda}\right) + \frac{(1-\alpha_0)^2}{c^2}\right] \times\left(1 + \frac{2\alpha_0(1-\alpha_0)}{1 + 9\mu/i\omega\rho R_0^2}\right)^{-1}, \quad (23)$$

where ω_B is the solution of the implicit equation $\omega_B = \omega_{B\omega}(\omega_B)$ and

$$c_m^2 = \omega_B^2 R_0^2/3\alpha_0(1-\alpha_0) \quad (24)$$

is the low-frequency sound speed in homogeneous bubbly mixtures with incompressible liquid. In the absence of damping, ω_B reduces to the natural frequency of oscillation of a single bubble at isothermal conditions in an unbounded liquid (Plesset and Prosperetti, 1977; Knapp *et al.*, 1970). Similarly, when surface tension and energy dissipation are neglected, c_m reduces to the well-known expression of the low-frequency sound speed for a homogeneous mixture (van Wijngaarden, 1980). If the bubbles are in stable equilibrium in their mean or unperturbed state, then $\Re(\phi)p_{B_0} > 2S/R_0$ and both ω_B and c_m are real.

For comparison, the solution for a single bubble of equilibrium radius R_0 subject to the same acoustic field is (Prosperetti, 1984)

$$p_R(t) = p_0 + \epsilon\frac{p_0}{\rho R_0^2}\left(p_{B0}\phi - \frac{2S}{R_0} + i\omega 4\mu\right) \times\frac{1 - i\omega R_0/c}{\omega_{B\omega}^2 - \omega^2 + i\omega 2\lambda}e^{i\omega t}, \quad (25)$$

$$u_R(t) = \dot{R}(t) = -i\omega\epsilon\left(\frac{p_0}{\rho R_0}\right)\frac{1 - i\omega R_0/c}{\omega_{B\omega}^2 - \omega^2 + i\omega 2\lambda}e^{i\omega t}, \quad (26)$$

where $p_R(t)$ and $u_R(t) = \dot{R}(t)$ are, respectively, the pressure and the velocity of the liquid at the bubble surface.

Now, the average power absorbed by the forced oscillations of a sphere (whether a cloud or a single bubble) with unperturbed radius b_0 during a period $T = 2\pi/\omega$ of the exciting acoustic field is

$$\dot{W}_a = -\frac{1}{T}\int_0^T 4\pi b_0^2 p_b(t)u_b(t)dt, \quad (27)$$

where $p_b(t)t$ and $u_b(t)$ are the pressure and the velocity at the sphere boundary, respectively.

The oscillating sphere also acts as a monopole source that generates the acoustic field:

$$p'(r,t) = i\frac{\rho U_b\omega b_0^2}{(1 + ihb_0)r}e^{i[\omega t - h(r-b_0)]} \quad (28)$$

$$u'(r,t) = i\frac{U_b h b_0^2}{(1 + ihb_0)r}\left(1 + \frac{1}{ihr}\right)e^{i[\omega t - h(r-b_0)]}. \quad (29)$$

Hence the average power radiated by the sphere is

$$W_\sigma = \lim_{r \to +\infty} \frac{1}{T} \int_0^T 4\pi r^2 p'(r,t) u'(r,t) dt. \tag{30}$$

Normalization of $\dot{W}_a$ and $\dot{W}_\sigma$ with the average power density $\epsilon^2 p_0^2/\rho c$ of the excitation wave gives the following expressions for the acoustical absorption and scattering cross sections of an oscillating sphere:

$$a = -4\pi b_0^2 \frac{\Re(P_b)\Re(U_b) + \Im(P_b)\Im(U_b)}{\epsilon^2 p_0^2/\rho c}, \tag{31}$$

$$\sigma = \lim_{r \to +\infty} 4\pi r^2 \frac{\Re(P'(r))\Re(U'(r)) + \Im(P'(r))\Im(U'(r))}{\epsilon^2 p_0^2/\rho c}, \tag{32}$$

where P_b, U_b, $P'(r)$, and $U'(r)$, respectively, indicate the complex amplitudes of $p_b(t)$, $u_b(t)$, $p'(r,t)$, and $u'(r,t)$.

The acoustical cross sections of either a bubble cloud or of a single bubble can then be computed from the above formulas. Clearly, for a bubble cloud $b_0 = A_0$, while $p_b(t)$ and $u_b(t)$ are given by Eqs. (19) and (20) for $r = A_0$. On the other hand, for a single bubble, $b_0 = R_0$, with $p_b(t)$ and $u_b(t)$ expressed by Eqs. (25) and (26), respectively.

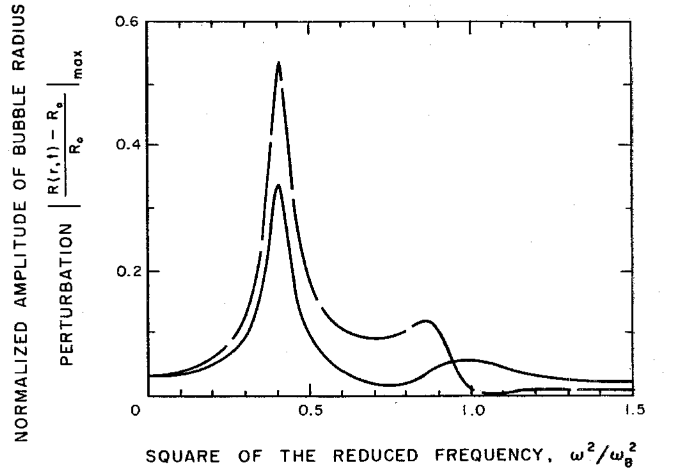

FIG. 2. Normalized amplitude of the bubble radius damped oscillations at the surface ($r = A_0$, solid line) and at the center of the cloud ($r = 0$, broken line) as a function of the square reduced frequency ω^2/ω_B^2 for $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \pi^2/4$.

III. RESULTS AND DISCUSSION

In this section we consider the case of air bubbles ($\gamma = 1.4$, $\chi_G = 0.0002$ m^2/s) in water ($\rho = 1000$ kg/m^3, $\mu = 0.001$ Ns/m^2, $S = 0.0728$ N/m, $c = 1485$ m/s). Unless otherwise specified the remaining flow parameters are: $p_0 = 10^5$ Pa, $R_0 = 0.001$ m, $A_0 = 0.1$ m, and $\epsilon = 0.1$. In most cases the parameter $\omega_B^2 A_0^2/c_m^2 = 3\alpha_0(1-\alpha_0)A_0^2/R_0^2$ is assigned and the void fraction α_0 is determined accordingly.

Free oscillations of the cloud only occur in the absence of damping when the exciting frequency ω experienced by each bubble is equal to the natural frequency ω_B of an individual bubble in an infinite liquid (bubble resonance condition) or to one of the natural frequencies of the bubble cloud. In the limit of low void fraction, the natural frequencies ω_n of the cloud are approximated by the infinite sequence:

$$\omega_n^2 \simeq \omega_B^2 \left(1 + \frac{3\alpha_0(1-\alpha_0)A_0^2}{(n-1/2)^2\pi^2 R_0^2}\right)^{-1}, \quad n = 0,1,2,\ldots. \tag{33}$$

For large n this sequence converges to the frequency ω_B corresponding to the bubble resonance conditions. For small n the behavior of this sequence depends on the value of $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \omega_B^2 A_0^2/c_m^2$. When this parameter is of order unity or larger the lowest natural modes can occur at comparatively low frequency. When the reverse is the case all the natural modes of the system take place with a frequency only slightly lower than the bubble resonance frequency. The above expression (33) corresponds to Eq. (144) of Omta, 1987, for the natural frequencies of a bubble cloud. Direct comparison of these equations is not possible due to the different modeling of bubble dynamics in the two cases. However, it is easily verified that they both reduce, as expected, to the same expression for ω_n when $3\alpha_0 \times (1-\alpha_0)A_0^2/R_0^2 \gg 1$, when surface tension and void fraction are small and when the gas in the bubbles behaves isothermally. In this limiting case the natural frequencies of the cloud are independent on the bubble radius and vary slowly with the cloud radius and the void fraction when the total volume of the gas phase is fixed, as indicated by Omta, 1987. The occurrence of resonances in the cloud also divides the flow solution into three different regimes, namely: subresonant ($0 < \omega < \omega_1$), transresonant ($\omega_1 < \omega < \omega_B$), and superresonant ($\omega > \omega_B$). As we shall see later, this has significant consequences on the behavior of the flow.

The relative amplitudes of the bubble radius oscillations at the center and at the surface of the cloud are shown in Fig. 2 as a function of the normalized square frequency for a typical case of $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \pi^2/4$. At the boundary of the cloud all resonance peaks except the first are virtually eliminated by the presence of damping and replaced by a second much smaller and broader peak around the individual bubble natural frequency. At the center of the cloud, the peak corresponding to the second resonant mode (whose amplitude is larger in the inner regions of the cloud) is still recognizable, although greatly attenuated. On the other hand, the peak at the bubble resonance frequency is absent because it is not associated with any global motion in the flow and because any external disturbance at the bubble natural frequency is quickly attenuated by the resonant response of the bubbles in the outer regions of the cloud. Also note that the amplitude of the bubble radius response is larger at the center of the cloud than at the surface. Violent oscillations of the bubbles near the center of the cloud have also been obtained by Omta, 1987, in his nonlinear computations of the cloud response to a sudden change of the external pressure. The other flow variables behave in a qualitatively similar manner (d'Agostino and Brennen, 1988). Therefore the first natural mode of oscillation of the cloud at a frequency $\omega \simeq \omega_1$ represents the most important component of the cloud response. Its effects also dominate the contributions of individual bubbles at their own natural frequency. These conclusions fully agree with the theory and computations of Omta, 1987. The above results clearly indicate that the acoustical properties of bubble clouds are not adequately

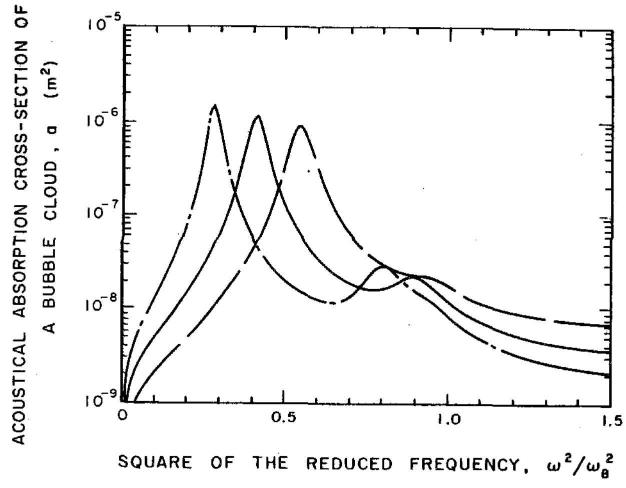

FIG. 4. Acoustical absorption cross sections of a bubble cloud as a function of the reduced frequency ω^2/ω_B^2 for three values of the parameter $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \pi^2/4$ (solid line), $\pi^2/8$ (broken line), and $\pi^2/2$ (dash-dotted line).

described in terms of the independent responses of individual bubbles, at least as long as the parameter $3\alpha_0(1-\alpha_0)A_0^2/R_0^2$ is of order 1 or larger and, therefore, the first natural frequency of the cloud is significantly smaller than ω_B.

The relative amplitudes of the damped bubble radius oscillations throughout the cloud at various frequencies are illustrated in Fig. 3 for $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \pi^2/4$. Note that the bubble response is larger at the center of the cloud for forcing frequencies below the bubble natural frequency, while the reverse is the case for superresonant excitation. In fact, in the subresonant regime the bubbles have ample time to react and therefore behave in a compliant way, with the largest motion concentrated in the interior of the cloud. The pressure change is essentially in phase with the excitation and the bubble response is almost in phase opposition. On the other hand, in superresonant flows the bubbles cannot respond as quickly as the excitation requires because of their inertia and therefore appear to be "stiffer." This effect clearly increases with the excitation frequency and therefore the cloud response, initially concentrated in the outer regions, becomes more uniform at higher frequencies. The pressure and the bubble radius changes are almost in phase with the excitation. Finally, in the transresonant regime the situation is complicated by the presence of more-articulated internal motions of the cloud due to the occurrence of resonances. The phase of the flow parameters with respect to the excitation depends on the dominant oscillation mode in the cloud. Between the first and the second natural frequencies, for example, the bubble radius response is essentially in phase with the excitation, while the pressure is almost in phase opposition.

The effects of different void fractions are illustrated in Figs. 4 and 5, which show the acoustical absorption and scattering cross sections of a bubble cloud as a function of the normalized square frequency for various values of the parameter $3\alpha_0(1-\alpha_0)A_0^2/R_0^2$. Note the presence of two peaks corresponding to the first and the second natural modes of the bubble cloud and the absence of a third peak at bubble resonance conditions. Since the natural frequencies are determined by the parameter $3\alpha_0(1-\alpha_0)A_0^2/R_0^2$ through Eq. (17), the peak frequencies corresponding to the cloud's natural modes of oscillations decrease at higher void fractions. Also note that the maximum values of the acoustical absorption and scattering cross sections increase slightly with void fraction and the second resonant peaks tend to become more pronounced due to the greater compressibility of the cloud.

Comparisons of the acoustical absorption and scattering cross sections of a bubble cloud with those of each indi-

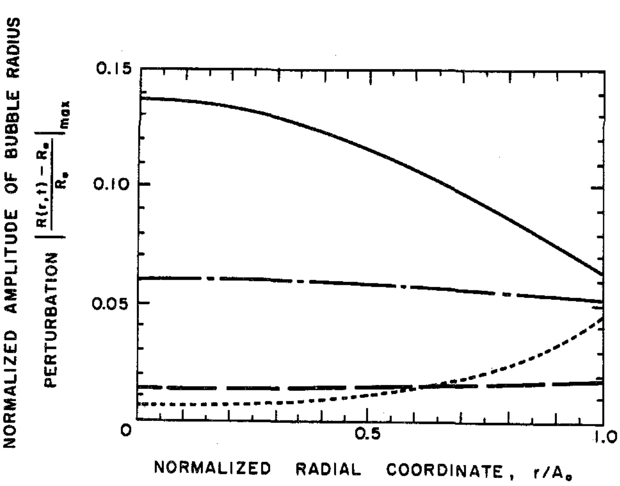

FIG. 3. Normalized amplitude of the bubble radius damped oscillations in a bubble cloud as a function of the normalized radial coordinate r/A_0 for $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \pi^2/4$ and various values of the excitation frequency: $\omega^2 = \omega_1^2/2$ (solid line), $(\omega_1^2+\omega_2^2)/2$ (dash-dotted line), $1.1\,\omega_B^2$ (dotted line), and $2\omega_B^2$ (broken line).

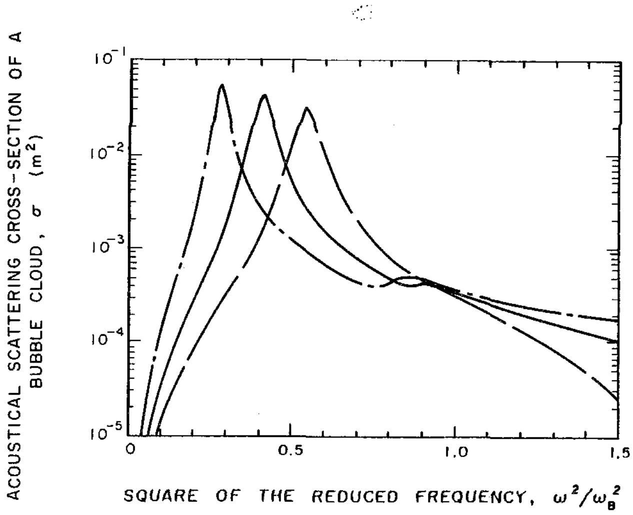

FIG. 5. Acoustical scattering cross sections of a bubble cloud as a function of the reduced frequency ω^2/ω_B^2 for three values of the parameter $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \pi^2/4$ (solid line), $\pi^2/8$ (broken line), and $\pi^2/2$ (dash-dotted line).

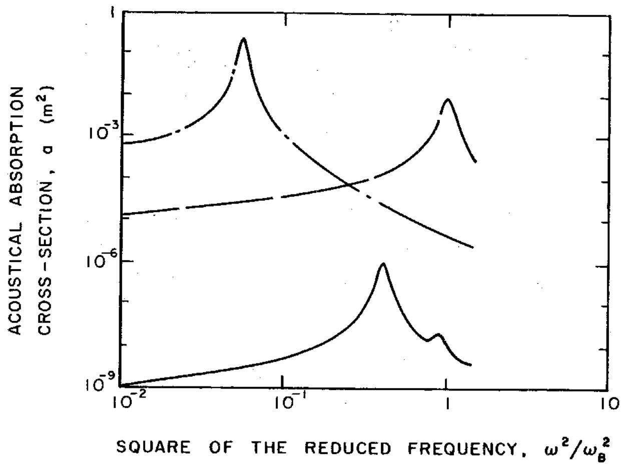

FIG. 6. Acoustical absorption cross sections as a function of the reduced frequency ω^2/ω_B^2. The curves refer to a bubble cloud with $3\alpha_0(1-\alpha_0)A_0^2/R_0^2=\pi^2/4$ (solid line), a single bubble in the cloud (broken line), and a larger single bubble of radius $R_0=A_0\alpha_0^{1/3}$ (dash-dotted line) whose volume is equal to the total volume of the bubbles in the cloud.

vidual bubble in the cloud and of a single large bubble with the same total volume as the bubbles in the cloud are shown in Figs. 6 and 7 for $3\alpha_0(1-\alpha_0)A_0^2/R_0^2=\pi^2/4$. Note that the results in the various cases vary by orders of magnitude in both amplitude and spectral distribution. In particular, the large bubble has the highest acoustical absorption cross section with the lowest peak frequency. On the other hand, the individual bubble has a lower maximum value of the acoustical absorption cross section, but the highest peak frequency, while the bubble cloud is characterized by an intermediate value of the peak frequency and by a much smaller maximum amplitude of the acoustical absorption cross section. The situation for the acoustical scattering cross sections is similar, with the peaks located at the same frequencies. However, the spread in the maximum values is significantly reduced because the scattering cross section of the bubble cloud is larger than that of the individual small bubble and only slightly lower that the scattering cross section of the single large bubble. It appears therefore that the acoustical properties of any given volume of the dispersed phase depend strongly on the degree of dispersion in the bubbly mixture. This has important consequences in the analysis of noise in bubbly and cavitating flows.

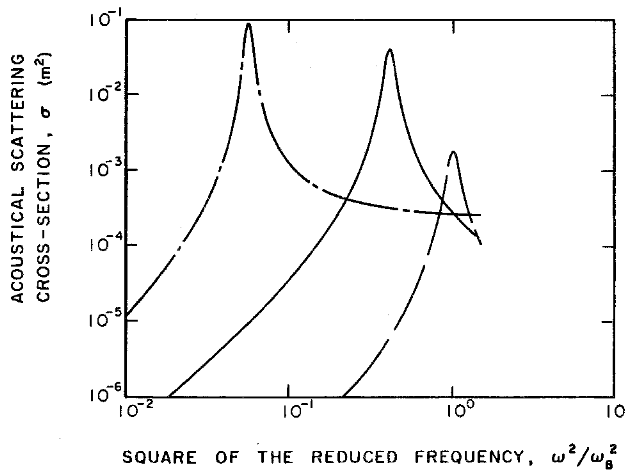

FIG. 7. Acoustical scattering cross sections as a function of the reduced frequency ω^2/ω_B^2. The curves refer to a bubble cloud with $3\alpha_0(1-\alpha_0)A_0^2/R_0^2=\pi^2/4$ (solid line), a single bubble in the cloud (broken line), and a larger single bubble of radius $R_0=A_0\alpha_0^{1/3}$ (dash-dotted line) whose volume is equal to the total volume of the bubbles in the cloud.

IV. LIMITATIONS

We now briefly examine the restrictions imposed on the previous theory by the various simplifying assumptions that have been made. Specifically we will discuss the limitations due to the introduction of the continuum model of the flow, to the use of the linear perturbation approach in deriving the solution, and to the neglect of the local pressure perturbations in the neighborhood of each individual bubble. In what follows we will refer to the solution for harmonic excitation, since it represents the basis of the generalization to arbitrary-shaped farfield forcing pressure.

The perturbation approach simply requires that $\varphi \ll 1$ in Eq. (18), a constraint that can be satisfied far from resonance conditions with proper choice of the excitation relative amplitude ϵ. This is probably the most restrictive limitation of the present analysis.

For the continuum approach to be valid, the two phases must be minutely dispersed with respect to the shortest characteristic length of the flow, here either the cloud radius A_0 or the wavelength $2\pi/k$ of the disturbances in the r direction. Hence the average bubble spacing $s=O(R_0/\alpha_0^{1/3})$ is required to satisfy the most restrictive of the two conditions: $s \ll A_0$ and $ks \ll 1$.

In order to estimate the error associated with the neglect of local pressure effects due to the dynamic response of each individual bubble, we consider the pressure perturbation experienced by one bubble as a consequence of the growth or collapse of a neighbor:

$$\Delta p' = \rho\left\{\frac{R}{s}\left[R\frac{D^2R}{Dt^2}+2\left(\frac{DR}{Dt}\right)^2\right]-\frac{R^4}{2s^4}\left(\frac{DR}{Dt}\right)^2\right\}, \tag{34}$$

where $R=R_0(1+\varphi)$ is given by Eq. (18). To the same order of approximation used to develop the present analysis, comparison with the global pressure change $\Delta p = p(r,t)-p_0$ expressed by Eq. (19) then shows that the local pressure perturbations are unimportant if

$$\frac{R_0}{s}\left|\frac{\omega^2(1-i\omega R_0/c)}{\omega_{B\omega}^2-\omega^2+i\omega 2\lambda}\right| \ll 1. \tag{35}$$

Far from bubble resonance regime, this condition is generally satisfied in low void fraction flows.

V. CONCLUSIONS

The results of this study reveal a number of important effects occurring in confined bubbly and cavitating flows. As anticipated in the introduction and confirmed by the present theory, the dynamics of the bubbles is strongly coupled through the pressure and velocity fields with the global dynamics of the flow in the bubble cloud. The bubbles are responsible for the occurrence of bubble resonance phenomena and for the drastic modification of the sonic speed in the medium, which decreases and becomes dispersive (frequency dependent). Furthermore, internal resonant modes of oscillation are possible at the system's natural frequencies due to the presence of boundaries confining the bubbly region of the flow.

The occurrence of resonances leads in turn to the identification of three different flow regimes, referred to as subresonant, transresonant, and superresonant. These are defined by the relation between the exciting frequency, the first natural frequency of the cloud, and the individual bubble natural frequency. The natural frequencies of the cloud are always lower than the natural frequency of the individual bubbles. In particular, they become significantly smaller than the bubble resonance frequency when the parameter $3\alpha_0(1-\alpha_0)A_0^2/R_0^2 = \omega_B^2 A_0^2/c_m^2$ is of order unity or larger. In the presence of damping the first natural mode of oscillation of the cloud is the most important component of the cloud dynamic response. Its effects dominate those of higher modes and the contributions of individual bubbles at their own natural frequency. Substantial global bubble interactions occur in the flow, with the result that the acoustical properties of bubbly clouds are no longer adequately described in terms of the collective but independent responses of the individual bubbles. In particular, the acoustical absorption and scattering cross sections of a bubble cloud are very significantly different in both amplitude and frequency distribution from the acoustical absorption and scattering cross sections of individual bubbles in the cloud. They are also very different from the cross sections of a single large bubble with the same total volume of vapor and gas. It appears therefore that the acoustical properties of any given volume of the dispersed phase depend strongly on the degree of dispersion of the vapor/gas phase in the bubbly mixture. An increase of the void fraction also causes a substantial reduction in the amplitude of the bubble response. This, in turn, could reduce the acoustic noise in bubbly mixtures or the damage potential in cavitating flows. The above phenomena may help to explain some of the unexpected changes experimentally observed in the noise spectrum of bubbly cavitating flows.

The present theory contains many simplifying assumptions involving the flow geometry and the linearization of both the velocity field and the bubble dynamics. It cannot, therefore, be expected to provide a quantitative description of the unsteady behavior of bubble clouds subject to farfield pressure excitation, except in the acoustical limit. Large bubble radius perturbations occur in most flows of practical interest; hence the most crucial limitation in the present paper is the linearization of the bubble dynamics, while the assumption of small velocity perturbations is likely to be more widely justified. If all the above linearizations were omitted, only numerical solutions could be realistically attempted. However, if only the hypothesis of linear bubble dynamics is relaxed, the development of quasilinear theories might be possible and would have a much broader applicability.

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