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Short Communication

Flow Regimes in Inclined Open-Channel Flows of Granular Materials

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Regimes of inclined granular chute flow

Open-channel flows of fluids may be classified as supercritical or subcritical, depending on whether their average velocity $\bar{u}$ is larger, equal to or smaller than the propagation rate of small disturbances $\sqrt{gh}\cos\alpha$ (where g is the gravitational acceleration, h is the flow depth and α is the channel inclination). Typically, the flow type is classified by the magnitude of the Froude number, $Fr = \bar{u}/\sqrt{gh}$, relative to its value under critical conditions $Fr_c = \sqrt{\cos\alpha}$. Supercritical and subcritical flow represent conjugate states of open-channel flows; that is, a given supercritical flow will transition via an hydraulic jump into a unique subcritical flow. (However, energy considerations prohibit a reverse hydraulic jump that will transition a subcritical flow back to its corresponding supercritical value.) Supercritical flows are unaffected by downstream conditions, as they move faster than the downstream information can propagate upstream. Subcritical flows are strongly affected by downstream conditions. If downstream conditions are relaxed, a subcritical flow may transition back to supercritical flow (although not its conjugate state) via an expansion wave propagating upstream. The existence of a subcritical flow requires that the flow must be critical somewhere downstream before any abrupt expansion of the channel (such as the drop-off at the channel's end). The critical state prevents an expansion wave from propagating upstream from the expansion and causing a transition to supercritical flow. (A more complete discussion of these phenomena may be found in most introductory fluid mechanics textbooks; see, for example, Ref. 1, pp. 363 - 377.)

The analysis used to find the critical velocity, the behavior of hydraulic jumps, etc., is derived from a simple momentum balance and assumes only a hydrostatic pressure distribution and uniform velocities and densities. As it requires no knowledge of the material's internal structure or constitutive behavior, it applies nearly as well to granular flows as it does to fluids. Many of the phenomena discussed above have been observed in granular flows with particular attention applied to the study of granular jumps [2, 3]. This paper reports further observations of supercritical and subcritical flows in inclined channels.

The experiments were performed in an inclined open channel which is shown schematically in Fig. 1. The test section is 2.4 m long and has a movable wall that allows variation of the channel width b. The initial flow depth is controlled by a gate at the top of the test section. Another gate, further upstream, is used to turn the flow on and off. The variation in flow depth along the chute is measured by vernier-calibrated point probes stationed 75, 120, 165 and 210 cm from the gate (30, 75, 120 and 165 cm from the exit). The mass flow rate is determined by the delivery rate of material into a bucket thrust under the chute's end. When necessary, an average velocity $\bar{u}$ was calculated by $\bar{u} = m/\rho_c bh$, where ρ_c is the "critical density'. (Surface velocities measured from movies taken of colored beads dropped onto the free surface indicate that this is a good approximation to the actual flow velocity.) The experiments were performed with two sizes of glass beads manufactured by the Flexolite Division of General Steel. They will be referred to by Flexolite's designations, BT4 and BT6, for the larger and smaller beads respectively. Their properties are summarized in the table; all properties were determined using the same

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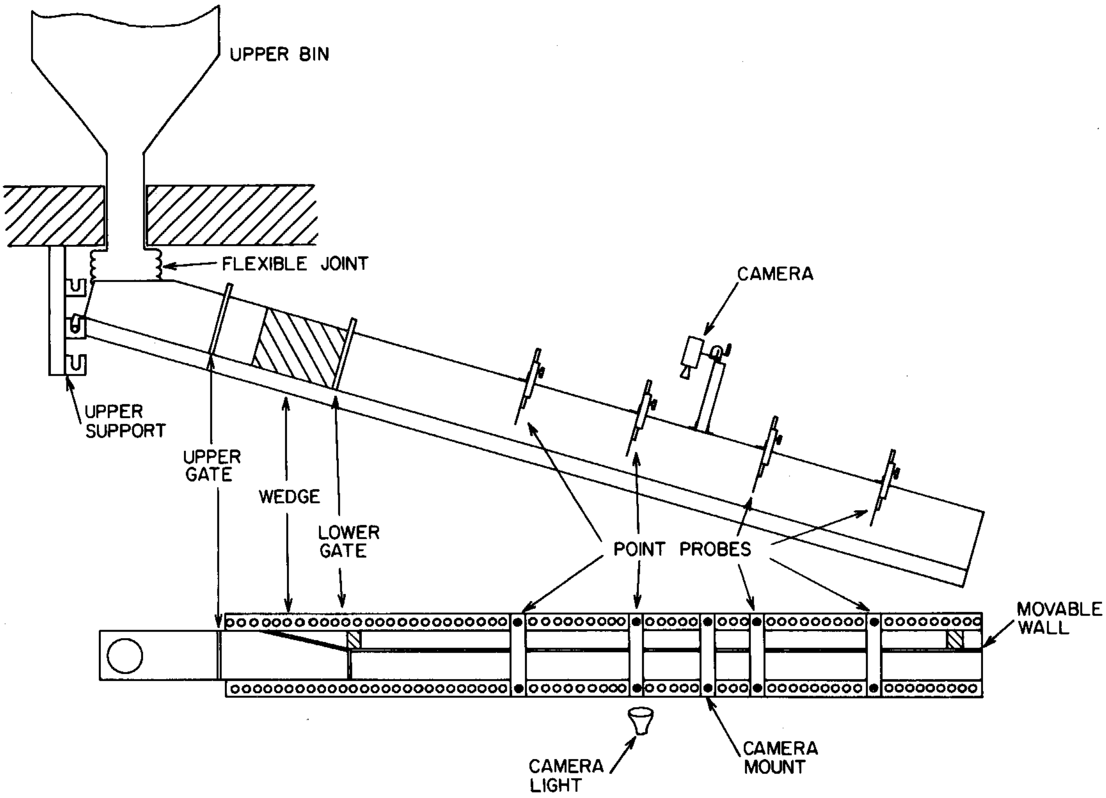

Fig. 1. Schematic of the experimental chute.

TABLE

Physical properties of glass beads used in the experiments

	BT4	BT6
Mean particle diameter (mm)	0.48	0.262
Standard deviation (mm)	0.101	0.037
Particle specific gravity	2.5	2.5
Critical specific gravity	1.44	1.43
Critical solid fraction	0.58	0.57
Internal friction angle	25.8°	16.0°

methods and apparatus as Pierce [4]. (For further details of the experimental technique, see Ref. 10.)

For shallow gate openings, the flow was nearly always supercritical (the sole exception being the $\alpha = 18°$ inclination, where nearly all the flows are subcritical), that is, the midspan Froude number is reasonably larger than one. Supercritical flows are shallow, fast and appear as a blur to the eye. As the gate opening is increased further, the flow may transition to subcritical. Immediately after material is admitted to the channel, the flow will almost always be supercritical and there will be some delay until downstream information can be felt upstream and the transition can begin. The flow will get progressively deeper and slower, until it reaches steady conditions. (Sometimes it will overshoot and slowly oscillate toward its eventual steady state.) The whole process may take up to 2 min. All subcritical data were measured during this terminal steady state. Subcritical flows are slow and deep and the particles on the free surface have a well-defined structure. Nowhere is the difference between the flow types more apparent than in the spanwise depth profiles shown in Fig. 2 for BT6 beads at a 22° inclination . Increasing the gate opening h_g from 1.8 to 2.5 cm causes a transition to subcritical flow and a threefold increase in the downstream depth. An interesting feature of the deep subcritical depth profiles is that they tend to approach a steady value until, between the third and fourth point probes, they undergo an abrupt acceleration with a corresponding decrease in depth. We refer to this phenomenon as ‘breaking’, but, as yet, cannot explain its occurrence.

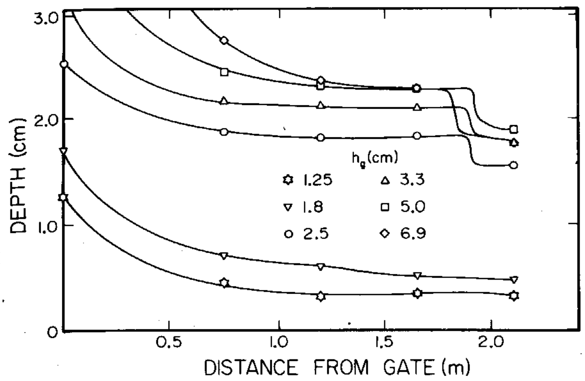

Fig. 2. Spanwise depth profiles, BT6 glass beads, α = 22°, b = 8.9 cm.

The described transitions are qualitatively similar to granular flow transitions observed by Choda and Willis [6] and Takahasi [7]. Choda and Willis' experiments were performed on fully enclosed curved ducts. As the duct inclination was decreased, the flow underwent a transition from 'fast flow' to 'slow flow'. Takahasi measured the flow out of 1 m long wooden chutes of various widths. The slope of the velocity curve, when plotted as a function of inclination angle, had a distinct discontinuity that accompanied a transition from 'laminar' to 'turbulent' flow; furthermore, he observed that the transition could be delayed by narrowing the chute. The Froude number was not determined in either investigation and consequently could not be linked to the observed transitions.

Figure 3 shows the mass flow rate of BT6 glass beads from the chute at an inclination angle of 22° as a function of gate opening h_g and chute width b. For small gate openings, the flow is supercritical and the mass flow increases with gate opening. At larger gate openings, the flow transitions to subcritical and the mass flow rate becomes independent of the gate opening. The gate opening at which transition occurs and the maximum mass flow rates increase with the chute width (presumably because the downstream influence is reduced). Figure 4 shows the mass flow rate of BT4 beads as a function of gate opening for b = 8.9 cm and various inclination angles. Subcritical transitions can be seen at inclinations up to 25°. (Takahasi [7] noted transitions as high as 45°.) The transition point appears to be a function of both inclination angle and chute geometry.

As long as the flow remains supercritical, the flow is completely controlled by the conditions at the gate and obeys the correlation

$$Fr_g = \frac{\bar{u}_g}{\sqrt{gH}} = \sqrt{2K}\sin\alpha$$

where $\bar{u}_g$ is the average velocity out the gate, H is the gate hydraulic diameter (as the gate is rectangular, $H = 2h_g b/(h_g + b)$) and K is a dimensionless material constant of order unity. For the BT6 glass beads, K = 1.07, and for the larger BT4 beads, K = 0.91. This predicts mass flow per unit width curves which, for comparison, are plotted in Figs. 3 and 4.

The maximum mass flows per unit width obtained from subcritical flows are plotted in

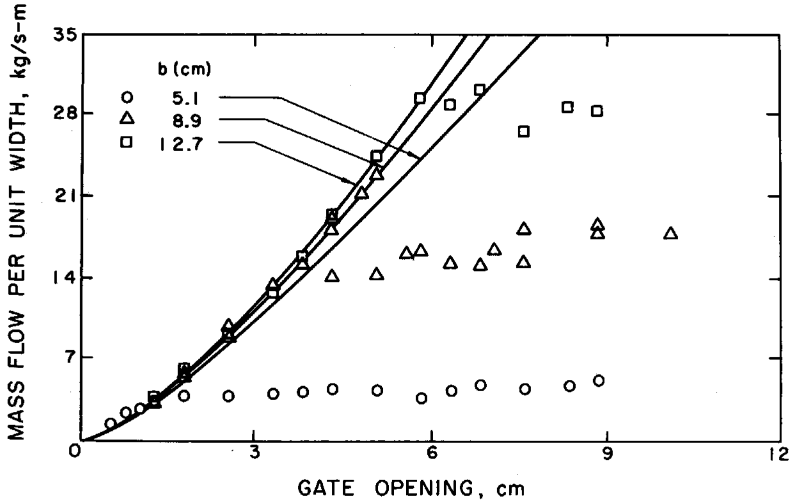

Fig. 3. Mass flow per unit width for BT6 glass beads, α = 22° and various chute breadths, compared with the supercritical mass flow correlation.

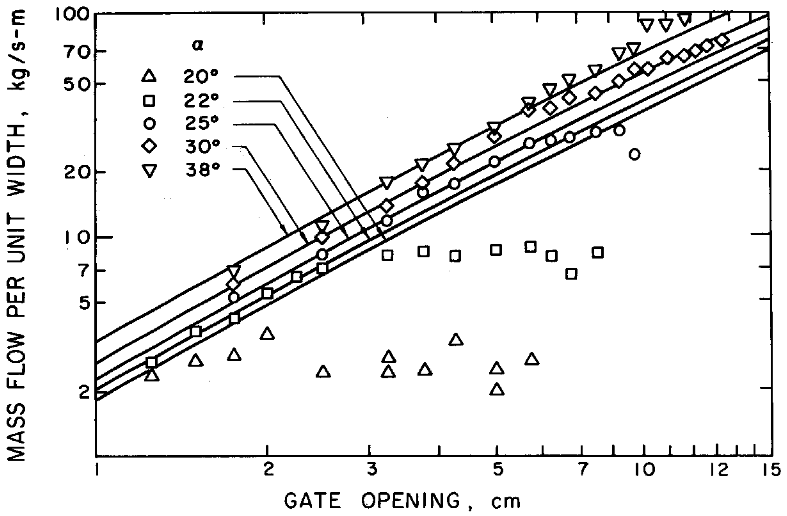

Fig. 4. Mass flow per unit width for BT4 glass beads, b = 8.9 cm, and various chute inclinations compared with the supercritical mass flow correlation.

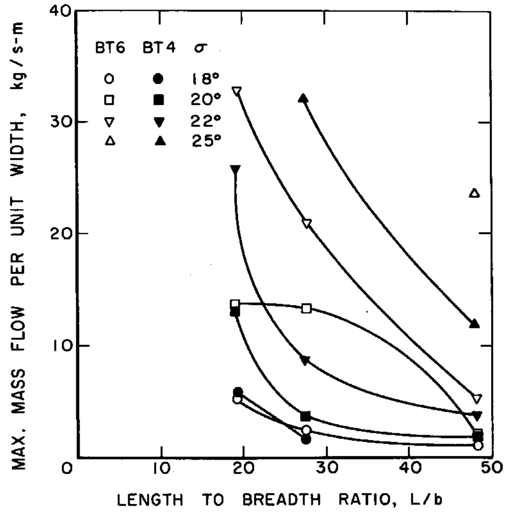

Fig. 5. Maximum subcritical mass flow rates as a function of chute geometry and inclination.

Fig. 5. as a function of inclination angle and chute geometry. The chute geometry is characterized here by the chute length to breadth ratio L/b. Only the breadth b was varied during these experiments, but the effect of length has been noted elsewhere: transition was observed at only extremely shallow angles in the short chute used by Brennen *et al.* [2], and Choda and Willis [6] observed that their fast-flow/slow-flow transition could be induced by adding a length of straight chute to their channel. The principle feature of Fig. 5 is that the maximum mass flow rate decreases with L/b and with particle size.

As mentioned earlier, a subcritical flow is subject to the constraint that the Froude number must assume its critical value somewhere before the chute exit. This imposes some downstream control on the flow but only requires that the mass flow rate vary as $(h_e)^{1/2}$, where h_e is the exit depth. In addition, some limit on the exit depth or velocity is required to explain the observed limit on the subcritical mass flow rate. In order to examine the conditions near the exit, the point probes were moved to locations 2, 27, 52 and 77 cm from the chute's end. The approach of the depth and Froude number towards their exit values are shown in Fig. 6 for BT6 glass beads, $\alpha = 22°$ and $b = 8.9$ cm. Consistent with the mass flow observations, both approach a single value at the exit for a wide variety of gate openings. Surprisingly, each curve approaches the exit value along a unique path. The complete set of measured values of the subcritical exit Froude number is shown in Fig. 7. Despite a wide range of entrance gate openings, the exit values show a spread of only about 10%. The exit Froude number is seen to vary with both chute geometry and inclination angle and does not assume the constant value $\sqrt{\cos\alpha}$ predicted for the critical Froude number. However, that prediction assumes uniform velocity and density across the flow depth. Many investigators [3, 5, 9, 10, 11, 12] have measured significant non-uniformities in the velocity and density profiles for granular chute flows. The shape of the profiles affects the value of

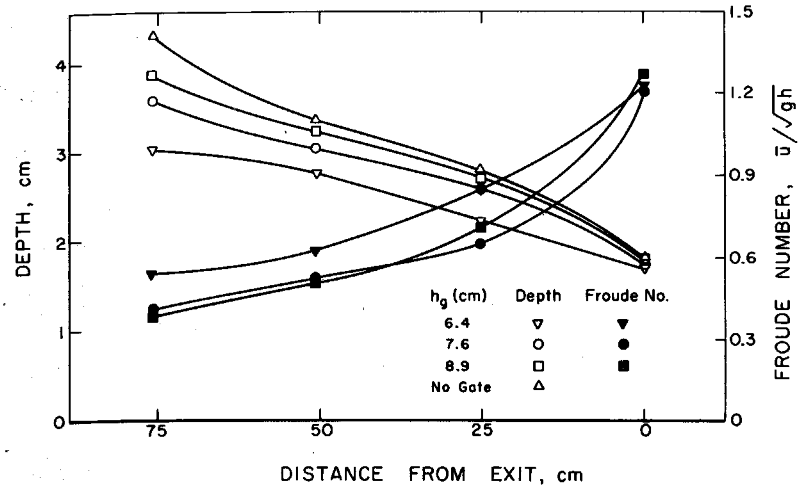

Fig. 6. Subcritical flow depths and Froude numbers near the chute exit, BT6 glass beads, $\alpha = 22°$.

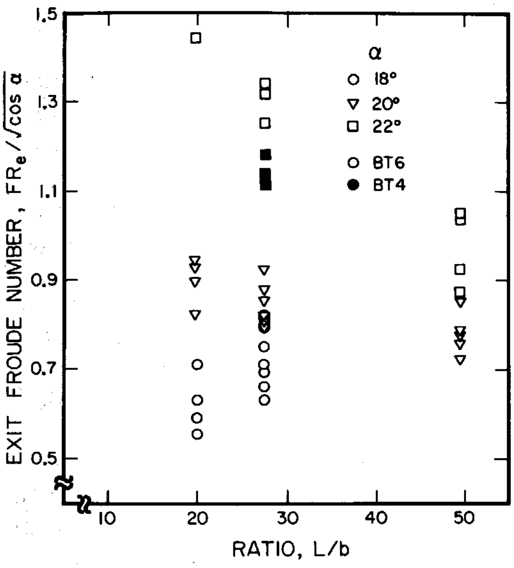

Fig. 7. Subcritical exit Froude numbers as a function of chute geometry and inclination.

the critical Froude number (for more information, see Appendix A of Ref. 10), and the evolution of the profiles could result in locally different values of the critical Froude number. It is possible that the flow could assume critical conditions at more than one position along the chute and such an occurrence could explain the phenomenon called 'breaking' that was observed in the spanwise depth profiles.

Conclusions

Granular chute flows may be classified as either supercritical or subcritical, depending on the relative magnitude of the local and critical Froude number. Supercritical flows ($Fr > Fr_c$) are swift and shallow, while subcritical flows $Fr < Fr_c$) are slow and deep. As long as the flow remains supercritical, the mass flow rate is completely controlled by the entrance gate, at which the flow obeys a simple correlation. However, when the flow transitions to subcritical, the mass flow rate becomes independent of the gate and depends only on the material properties, chute geometry and inclination angle. As such, the transition places an upper limit on the mass flow rate that can be obtained from a chute of fixed inclination and geometry.

The main difference between these subcritical flows and the 'laminar' flows observed by Takahasi [7] is that his flows were characterized by a thin layer of flowing material riding over a wedge of stagnant material. (Stagnant wedges were observed during the present investigations but only for the largest gate openings and only in the immediate region about the entrance gate.). However, Takahasi used various sands as experimental materials, and some preliminary experiments with sand performed during the course of the current investigations showed that wedge formation invariably accompanied the subcritical flow transition. Presumably, the transition slows the flow sufficiently for the highly angular and non-uniform sand particles to lock into the unshearable wedges. Wedging is then a byproduct rather than a salient feature of subcritical flows. It is interesting to note that it is common practice to flare the ends of industrial chutes to help prevent the formation of stagnant regions [13]. The effect is to expand the flow near the chute exit, which reduces the depth and increases the Froude number, preventing the subcritical flow transition.

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