Canyoneering Fluid Mechanics

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Extreme sports such as canyoneering have expanded greatly since the turn of the century yet little scientific attention has been paid to the analyses of the dangers of those activities. The author was much involved in promoting one such sport, namely canyoneering, and presents this paper as an example of the kind of fluids engineering analyses that are needed in order to objectively quantify those dangers and properly advise the participants. In canyoneering, the primary fluid-related sources of danger are the impact of falling water on the human body and the dangers a swimmer faces in a plunge pool. This paper presents rough evaluations of both dangers. [DOI: 10.1115/1.4034003]

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1 Introduction

Fluids engineering has made tremendous advances in the past 90 years. However, there are still many situations where fluid forces are encountered and simple analyses that could be performed are overlooked, yet are needed to make important sometimes critical decisions, as exemplified in this paper. During a long career in the academic fluid mechanics research, the author occasionally took time out to enjoy the marvelous outdoors of southwestern U.S. In particular, he was a pioneer in the sport of canyoneering (or canyoning as it is known outside of the U.S.), first in the San Gabriel mountains of California and later throughout the southwest and the world [1,2]. The sport of canyoneering involves entering the headwaters of a steep and narrow canyon and then traveling downstream over waterfalls and dryfalls, often using a rope to rappel (abseil) down the steeper drops. [The formation of such steep and narrow canyons is, in itself, an interesting fluid mechanical subject (see, for example, Ref. [3]) but is beyond the scope of this paper.] Sometimes, for example, in Death Valley National Park, the canyons are quite dry. At other times and in other places, a significant stream is flowing in the canyon, and part of the fun is the challenge of descending in or beside the waterfalls. Though accidents and injuries do occur [4], the dangers are slight for a canyoneer experienced in judging the forces involved. However, any bodily encounter with running water poses some risk, particularly when rappeling, and this paper attempts to quantify those dangers. While the most important readers are canyoneers, the analyses are sufficiently unusual that they may be of interest to other fluids engineers.

A few photographs (see Fig. 1) will help to set the scene. The left photograph shows a typical dry descent into a Grand Staircase/Escalante National Park canyon. The other two photographs show descents in moderate and heavy water flows, respectively.

2 Canyoneering Fluid Mechanics

Particularly in wet canyons or wet descents, the canyoneer must often contend with the forces imposed by the flow of water (see Fig. 1). In this section, we attempt to estimate the water velocities and forces that the canyoneer might experience and, where possible, provide a tool the canyoneer might use to anticipate the dangers posed by the flow. We focus on the two major phases of a wet descent, namely the exposure to the falling waterfall during the descent and the interaction with the plunge pool at the bottom at the end of the descent.

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The typical geometry of a waterfall and plunge pool is sketched in Fig. 2 and an example of the measured velocity magnitude distribution in the plunge pool is shown in Fig. 3.

3 Water Velocities and Flow Rates

We begin by evaluating the flows and forces experienced during the descent and this necessarily begins with the conditions that pertain at the lip of a waterfall. The flow at this point will be critical and if the velocity of the water were uniform over the cross section of the flow at that point, and the cross section of the flow was rectangular with a uniform depth, h, then, the velocity would be equal to $\sqrt{gh}$ (see Fig. 4), where g is the acceleration due to gravity. However, because the cross section is not rectangular and because the velocity is not uniform, it is traditional (see, for example, the USBR manual [5]) to introduce a coefficient, C^*, and express the average velocity, u, at the lip by

$$u = C^* \sqrt{gh} \tag{1}$$

The value of C^* varies with the cross-sectional geometry of the flow at the lip but is typically about 0.5. For example, if $h = 5\,\mathrm{cm}$ it means $u \approx 0.5\sqrt{9.8 \times 0.05} = 0.35\,\mathrm{m/s}$. Sometimes the canyoneer knows the water flow rate in the stream, commonly in the units of $\mathrm{ft^3/s}$ (*cfs*). Experienced canyoneers often consult the various internet water gauge flow rate data (see, for example, Ref. [6]) in order to determine the conditions within a canyon they plan to descend. Here, we will convert the *cfs* flow rate to a flow rate, Q, in $\mathrm{m^3/s}$ using $1cfs = 0.0283\ \mathrm{m^3/s}$. Then, if the cross-sectional area of the stream at the lip, B (in $\mathrm{m^2}$), can be estimated, the average velocity, u, can also be evaluated from

$$u \approx \frac{Q}{B} \tag{2}$$

For example, a velocity of $u = 0.35\,\mathrm{m/s}$ and a cross-sectional area $B = 0.05\,\mathrm{m^2}$ would correspond to a water flow rate of $Q \approx 0.175\,\mathrm{m^3/s}$ or $6.18cfs$.

Conversely, if we wish to estimate the flow rate in the stream we can do so by measuring the dimensions of the cross section of the flow at the lip, h (in m) and B (in $\mathrm{m^2}$). The flow rate, Q (in $\mathrm{m^3/s}$), then follows from

$$Q \approx B\, C^* \sqrt{gh} \approx 1.5B\sqrt{h} \tag{3}$$

If the cross section of the flow at the lip were rectangular with depth h and width b then $B = bh$; on the other hand if the cross section were triangular with surface width b and maximum depth h then $B = 0.5bh$. When the factors C^* and B/bh are combined, the expression 3 is conventionally written in the form

Fig. 1 The author on a dry descent in Grand Staircase/Escalante National Park (Egypt 2 Canyon) and on wet descents in the San Gabriel Mountains (Great Falls of the Fox) and the North Fork of the Kings River, Sierra Nevada Mountains, California. Photographs by Mark Duttweiler and Randi Poer.

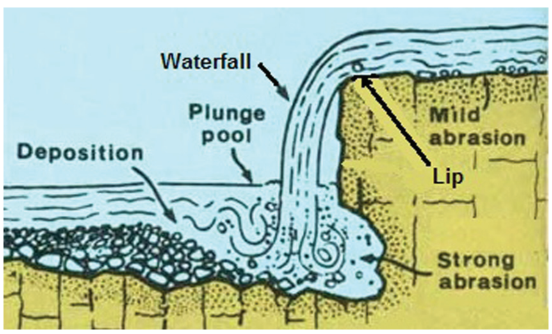

Fig. 2 Schematic of a waterfall and plunge pool. Adapted from USGS diagram.

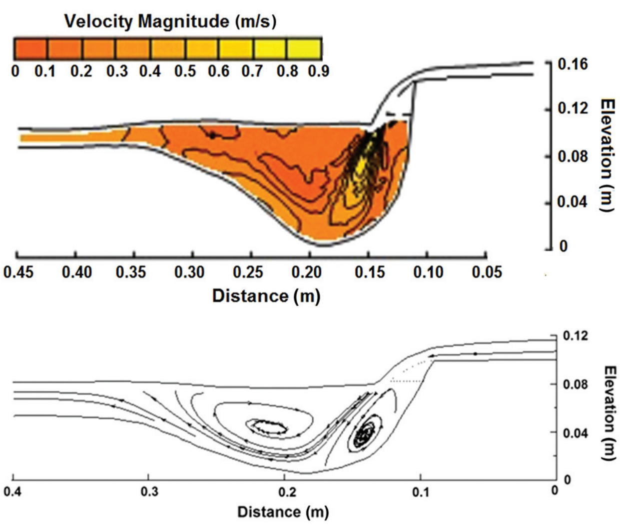

Fig. 3 Examples of measured velocity magnitudes and directions in a plunge pool. Adapted from Ref. [7].

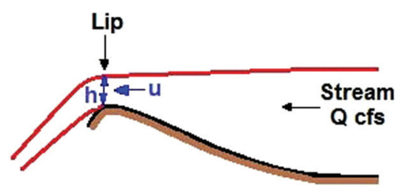

Fig. 4 Schematic of the flow over the lip of a waterfall

$$Q \approx Cbh\sqrt{h} \tag{4}$$

where the constant C is a discharge coefficient that is determined by experiments to lie somewhere in the range 1.5 (for a rectangular cross section) to about 0.7 for a triangular cross section. For the sake of simplicity, we will assume an intermediate value of $C = 1$ in what follows. We note that it may be useful to make such estimates before beginning a descent in the waterfall as part of the process of evaluating the potential danger.

Next we note that under the action of gravity a free-falling water stream will accelerate at the acceleration due to gravity. Of course, any interaction between the stream and nearby solid surfaces will tend to slow down this acceleration. But we can establish that the maximum acceleration will be $g = 9.8\,\text{m/s}^2$. This would lead to a water velocity, v (in m/s), at an elevation of y m below the lip given by

$$v^2 \approx u^2 + 2gy \tag{5}$$

For example, a water velocity of 0.7 m/s at the lip would increase to over 7.7 m/s after falling 3 m vertically if unimpeded by contacting nearby rock (this assumes no air drag on the water which would also reduce the result). Nevertheless, the increase in the stream velocity (and therefore, the forces it can produce) with decreasing elevation are clearly substantial.

In addition, as the falling stream accelerates, its cross-sectional area decreases; conservation of mass requires that

$$vB^{**} = \text{constant} = Q \tag{6}$$

where B^{**} (in m^2) is the cross-sectional area of the stream at the elevation where its velocity is v. Often, of course, the stream breaks up as it falls thus, decreasing the effective density of the stream and reducing its impact. Indeed, the fluid mechanics of falling waterfalls and the local updrafts they create can become quite complex as any observer of a high waterfall (such as the upper Yellowstone Falls) can attest.

One factor that we can and should incorporate in the present analysis is the fact that the aerodynamic drag causes a falling stream to cease accelerating once it reaches a terminal velocity. That terminal velocity is not easy to evaluate since it will depend on the extent to which the stream is broken up into droplets by the same aerodynamic forces. However, a not-unreasonable estimate would be to judge that the stream will have reached terminal velocity when it has free-fallen 10 m (this corresponds to a terminal velocity of 14 m/s). To apply such an observation, we should therefore use $H = 10\,m$ in all the subsequent analyses when the actual drop exceeds this value.

4 Water Impact Forces

We now evaluate the impact force, F, that a stream of water imposes on an object which is given by

$$F = \frac{\rho C^{**}}{2} v^2 B^* = \frac{\rho C}{2} vQ \tag{7}$$

where B^* (in m^2) is the cross-sectional area of the stream of velocity v impacting the object, ρ denotes the density of water ($\rho \approx 1000\,\mathrm{kg/m^3}$), and C^{**} is a coefficient whose value may lie between 0.5 and 1.0 depending on the shape of the object. For ease of interpretation, we will quote that force as that which a mass, F^*, experiences under the action of gravity, g, so that $F^* = F/g$. Using $C^{**} = 1$ and inputting the values of ρ and g, this yields $F^* \approx 50vQ\,\mathrm{kg}$. For example, if $v = 0.7\,\mathrm{m/s}$ and $Q = 0.35\,\mathrm{m^3/s}$, this yields a force that would be equivalent to the gravitational force on a mass of about 12 kg, a force that would be barely manageable if imposed on a human leg. (Anyone who has forded a fast moving stream deeper than about 0.3 m knows how difficult it is to move one's leg.) However, as the falling waterfall accelerates the force increases dramatically: at $v = 5\,\mathrm{m/s}$ and $Q = 0.35\,\mathrm{m^3/s}$ the force would be equivalent to that on a mass of about 88 kg, a force that a rappeling human could not realistically withstand.

A tool that would be most useful to the average canyoneer would be one (or more) criteria that would allow evaluation of the potential danger in a particular wet rappel. Of course, there are many variables in a waterfall, not just the quantifiable variables such as the flow rate and height of the waterfall, but the extent to which it is slowed by contact with nearby walls, the extent to which it is broken up by aerodynamic effects, and the descent route chosen by the canyoneer.

Let us assume that the canyoneer has no alternative but to follow the route of the falling water and to be wholly immersed within it (most experienced canyoneers know that once they begin to experience a substantial force in the waterfall there is usually little alternative but to continue straight down). Further, we assume that the falling water is not slowed by contact with the nearby solid surfaces and is not significantly broken up by the aerodynamic effects. These assumptions will lead to the *maximum* force the canyoneer might experience. Even if he/she is able to avoid or minimize this maximum by choice of anchor or route, the number calculated will still provide some, hopefully useful, guideline.

The two most basic parameters are the flow rate, Q (in m^3/s), and the height H of the waterfall (in m). The first can be obtained through Eqs. (3) or (4) by measurement of h and b at the lip and the second needs to be evaluated by the canyoneer from above the waterfall. The velocity, v, of the stream at the elevation H below the lip is then given approximately from Eq. (5) by $v = \sqrt{2gH}$ (since the term u^2 in Eq. (5) will be small) and therefore, the mass, F^* (in kg) experienced by the fully engaged canyoneer will, by Eq. (7), be given by

$$F^* \approx 50vQ\,\mathrm{kg} \approx 50\sqrt{2gH}\,bh\sqrt{h} \approx 220bh\sqrt{hH} \tag{8}$$

For example, if the flow at the lip is 0.1 m deep and 0.3 m wide and the unimpeded fall is 10 m then $F^* \approx 7\,\mathrm{kg}$, a significant but not unmanageable force. However, if the depth at the lip, h, is 0.2 m rather than 0.1 m then $F^* \approx 19\,\mathrm{kg}$ which would be difficult to manage. Thus, we propose a "danger" measure, namely, the "maximum force" mass, F_M^*, calculated from

$$F_M^*\,(\text{in}\,kg) \approx 220bh\sqrt{hH} \tag{9}$$

where b, h, and H are measured (or estimated) in m. Using the terminal velocity criterion discussed earlier, we should set H equal to 10 m even if the actual H is equal to or greater than 10 m. This simplifies the maximum force measure to

$$F_M^*\,(\text{in kg}) \approx 700bh\sqrt{h} \tag{10}$$

This formula is simple enough to present in graph form as shown in Fig. 5. Note how rapidly the mass, F_M^*, increases with increasing depth, h. For example, at a surface breadth of $b = 1\,\mathrm{m}$, F_M^* increases from about 8 kg at a lip depth of 0.05 m to about 22 kg at a lip depth of 0.1 m. Consequently, measurement of the lip depth, h is very important since this is required for an accurate evaluation of F_M^*.

We now turn to the second phase of a wet descent that we choose to evaluate, namely, the danger posed by the plunge pool at the bottom.

5 Waterfall Hydraulics

When a waterfall impacts a plunge pool at the bottom of its descent, it generates flow patterns in the pool that can pose significant danger for a canyoneer who descends into that pool, particularly if the pool is too deep to stand in. The plunging waterfall creates a vortex ahead of the plunging stream and, if there is room, a vortex behind the waterfall as shown in Fig. 6. The forward vortex can pose a substantial danger to the descending canyoneer if the water is more than about 1 m deep and if the velocity and breadth/width of the reverse surface velocity

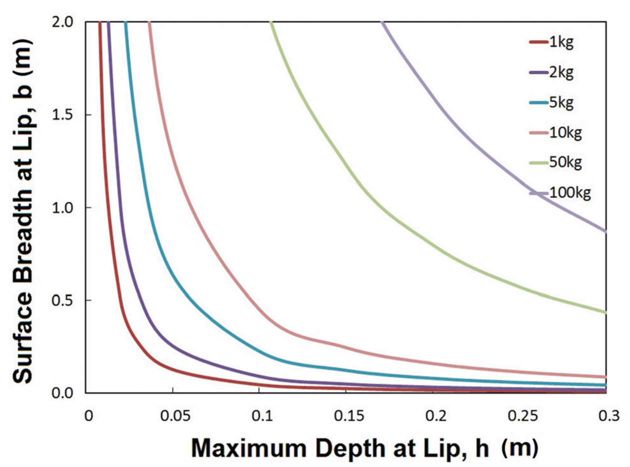

Fig. 5 The estimated "maximum mass", F_M^* (in kg), plotted as a function of the maximum depth of the stream at the lip, *h* (in *m*), and the breadth of the stream at the lip, *b* (in *m*)

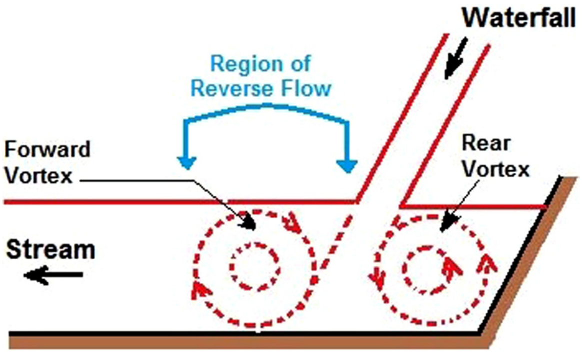

Fig. 6 Schematic of plunge pool vortices

generated by the vortex is too great for the canyoneer to swim against. A fully equipped canyoneer is unlikely to be able to swim any faster than about 2 kph or, say, 0.5 m/s. Consequently, if the velocity of the reverse flow in the pool is greater than about 0.5 m/s, the canyoneer will be unable to escape from the flow pattern in the pool unless he/she either dives deep to take advantage of the downstream current under the vortex (a very difficult strategy to follow in the maelstrom of the plunge pool) or swims sideways in a direction across the stream. We include Fig. 7 to convey an impression of the potential violence and turbulence in a plunge pool.

Typically, the vortices extend to the nearest solid surface, either the bottom of the pool or the rock wall behind the falls; they may also extend laterally toward the sides of the canyon. The danger is greatest when the plunge pool depth is too great to allow standing and there are lateral walls that restrict exit from the sides of the vortex. Typically, the maximum velocity of the water within the vortices will be comparable to though less than the velocity with which the waterfall impacts the plunge pool. Bennett and Alonso [7] have measured the distribution of velocities in a plunge pool and an example of their data is presented in Fig. 3 in which the magnitude of the velocity is color coded.

Given the velocities of the falling water described above and the fact that the swimmer has a maximum velocity of about 0.5 m/s, it is easy to understand the potential for entrapment within a vortex (and therefore for drowning). For this reason many canyoneers learn to setup and deploy guided rappels whenever the potential for a dangerous vortex exists. Of course, this requires the first descender to rappel down without such assistance in order to setup the guided rappel. Under such circumstances, the first descender should remain on rope until progress downstream

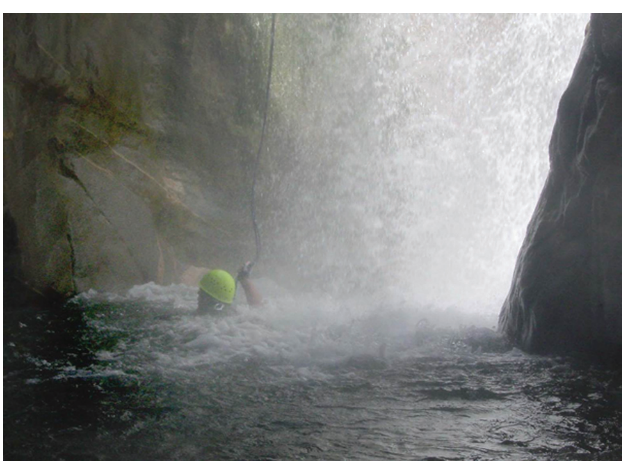

Fig. 7 Example of plunge pool violence during a white-water descent of the Great Falls of the Fox in the San Gabriel mountains of California. Photograph by Mark Duttweiler.

seems assured; he or she should also carry all possible buoyancy devices including an inflated backpack (one good strategy is to fill a dry bag in a backpack with air and then seal it). The force of buoyancy is one of the few available forces that may be able to overcome the drag forces of fast moving water—and is the reason why white-water rafters must wear life-jackets.

It is possible to add some quantification of the effectiveness of a life-jacket (or other buoyancy-adding device such as a wetsuit or backpack). A typical life-jacket provides buoyancy equivalent to a volume of about $V = 1\,\mathrm{L} = 0.001\ \mathrm{m}^3$ of air. The upward buoyancy force that this generates is therefore $1000 \times V \approx 1\,\mathrm{kg}$. Though this does not seem much, on a fully submerged human it would generate an upward velocity, U in m/s, relative to the water given approximately by

$$U \approx \left[\frac{2Vg}{C_D A}\right]^{\frac{1}{2}} = 0.6\,\mathrm{m/s} \qquad (11)$$

where we have assumed the swimmer has adopted a streamlined position so that the frontal projected area encountering the flow, A, is just $0.1\ \mathrm{m}^2$ and the drag coefficient, C_D, in that configuration is about 0.5. For any other configuration the velocity, U, is likely to be significantly smaller since both A and C_D will be larger. However, it seems clear that the swimmer's velocity relative to the water is considerably enhanced by the life-jacket (or other buoyancy device of a similarly effective air volume). Though it does not eliminate the potential for being drawn underwater by the downflow in a vortex or hydraulic, the life-jacket certainly enhances the possibility of escape in other parts of the flow.

One additional piece of information is relevant at this point. Nearly 30,000 people, men, women, and children, float down the Colorado through the Grand Canyon every year. In doing so, they descend numerous huge white-water rapids, over 150 in number [8] with 70+ major rapids including such notorious cascades as Lava Falls and Crystal Rapid. Many of these trips are made in small boats, inflatables and kayaks. It is extraordinary that there are so few casualties; a total of only 29 fatalities have occurred during the recorded history up to November of 1994 [9,10]. A major life-preserving factor is the insistence that everyone must wear a life-jacket (and those in small boats must wear helmets). So one must question whether or not there is any real danger for a canyoneer descending into plunge pool whose hydraulics are unlikely to be as violent or massive as those of the large Grand Canyon rapids. One conclusion must be that the wearing of a life-jacket hugely decreases the danger. The estimates in the preceding paragraph suggest why this is so, namely that a human wearing a life-jacket will typically generate an upward buoyancy-driven velocity that is greater than the velocity with which he/she could swim unaided—and that the life-jacket-driven velocity will always be upward, whereas the unaided swimmer can easily become disoriented. Thus, I would strongly recommend that all canyoneers descending canyons with substantial water flows wear life-jackets (even though wet-suits provide significant buoyancy).

But life-jackets may not be the whole story. The huge hydraulics of the Grand Canyon are not especially confined between narrow vertical walls whereas in some slot-canyons water often falls into laterally constricted plunge pools that prevent lateral escape from the generated hydraulics. Canyoneers need to be particularly careful in these circumstances, especially if the depth of the plunge pool cannot be discerned from above. If the plunge pool depth is less than about 1 m and one can stand and walk, the danger is clearly much less but plunge pool depths can vary greatly over time and are rarely discernible from above. If the waterfall is laterally constricted and the depth might be more than 1 m, it is clearly wise to send an experienced member down first attached to an additional rope so that he/she can be hauled up if necessary. As mentioned earlier, one option for the rest of the party is to setup a guided rappel that avoids the white-water. As a footnote, we should add that there are other complications such as entanglement with the rope that we do not address here; but even these are less dangerous if extra flotation is available.[1]

6 Concluding Comments

Extreme sports such as canyoneering have expanded enormously since the turn of the century yet, as far as the author is aware, little scientific attention has been paid to analyses of the dangers of those activities. The author was much involved in promoting one such sport, namely canyoneering, and presents this paper as an example of the kind of fluids engineering analyses that are needed in order to objectively quantify those dangers and to properly advise the participants. In this particular activity, two primary sources of danger are impact on the human body of falling water and the dangers a swimmer faces in a plunge pool. We have presented rough evaluations of both dangers and look forward to more accurate estimates in the future.

Acknowledgment

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[1]It is appropriate to ask for uncertainties in the estimates made in this paper. However, around the world, the great variety in the geometry of waterfalls and plunge pools, in the topography of the canyons and in the nature of the surfaces involved makes an estimate of such uncertainties impossible. Rather, the reader should regard the values used in this paper as illustrative examples.