Fluid Flow Equations For Rotordynamic Flows In Seals And Leakage Paths

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ABSTRACT

Fluid-induced rotordynamic forces produced by the fluid in an annular seal or in the leakage passage surrounding the shroud of a pump or turbine, are known to contribute substantially to the potential excitation forces acting on the rotor. In this paper we explore some of the important features of the equations governing bulk-flow models of these flows. This in turn suggests methods which might be used to solve these bulk-flow equations in circumstances where the linearized solutions (such as those of Childs 1987, 1989) will no longer be accurate. An example of a numerical solution is then presented.

Keywords: Rotordynamic forces, turbulent annular seals, bulk flow model.

1. BACKGROUND

Over the last few years a substantial body of experimental data has been gathered on fluid-induced rotordynamic forces (Brennen 1994) generated in narrow, fluid-filled annuli such as occur in turbulent annular seals (for example, Childs and Dressman 1982, Nordmann and Massmann 1984) or in the leakage flows surrounding the shrouded impellers of pumps or turbines (for example, Guinzburg *et al.* 1994). For example in the context of shrouded pump impellers, the effects of eccentricity, clearance, Reynolds number, leakage path geometry, inlet swirl and seals (both at the discharge and the suction) have been examined (Guinzburg *et al.* 1992, 1993, 1994, Sivo *et al.* 1995, Uy *et al.* 1997). Moreover, methods of changing these rotordynamic characteristics, for example by the installation of anti-swirl vanes, have also been explored (Sivo *et al.* 1995).

This now substantial body of data involves many different geometric and fluid flow parameters and it is not always easy for the potential user to find his or her particular choice of parameters within the database. To solve this problem and to allow for greater understanding of the underlying fluid mechanics it is clearly valuable to view this array of data in the context of an accurate analytical model and, if necessary, to tune the frictional and other parameters in the model to provide a reliable tool for the designer.

The problem with this strategy is that the available analytical models have not yet shown themselves capable of accurate and reliable predictions. Perhaps the most promising approach has been the bulk flow model developed by Childs (1987, 1989) and subsequently used by others (for example, by Guinzburg 1992). This model appears to give reasonable results in some cases and unreasonable, even bizarre results, in others. Nevertheless, it represents a coherent and rational starting point from which to begin. Before describing some of the inherent problems with this model we summarize it briefly.

2. BULKFLOW MODELS OF ROTORDYNAMIC FLOWS

As presented by Childs (1987, 1989), the bulk flow model depends on the fundamental assumption that the unsteady, three-dimensional, turbulent flow in the annulus can be accurately approximated in the following ways:

1. The dimensions of the flow can be reduced from three to two (a meridional coordinate and a circumferential coordinate) by assuming that the velocity profiles within the narrow passage are all self-similar so that the equations of flow can be averaged over the gap without undue error. This approximation is, of course, commonly employed in the Reynolds lubrication equations but also has its limitations, especially when frictional stresses are evaluated based on the gap-averaged velocities. For example, if the flow separates within the passage so that the velocity close to one wall is in a different direction than the gap-averaged velocity, then using the gap-averaged velocity to evaluate the shear stress on that wall will lead to stresses and forces which are quite incorrect. In many, uni-directional lubrication problems this is not a serious concern. But in unsteady, multi-directional flows (such as can occur in a pump leakage flow) this could represent a serious limitation. To examine the potential for such errors, Sivo *et al.* (1994) used a laser doppler velocimeter (LDV) to measure the velocity profile of a leakage flow experimentally. Flow reversal close to the rotor shroud was, in fact, observed and was in agreement with three-dimensional computations performed by Baskharone and Hensel (1993). The recirculation occurred at different locations in the leakage path for different conditions, and seemed to diminish at higher whirl ratios. In some cases, the recirculation regions were observed around the entire impeller. Such flow reversals could lead to a serious error in the bulk flow approach.

2. The Reynolds numbers of most of these flows are very high so that the flow is turbulent. This means that the bulk flow model requires expressions which relate the turbulent shear stresses to the gap- averaged velocities. In the current form of the bulk flow model, the shear stresses on the rotor and the stator are calculated using friction coefficients based on the work of Yamada (1962). These are defined by:
$$\frac{\tau}{\frac{1}{2}\rho u^2} = n\left(\frac{\rho u h}{\eta}\right)^m \tag{1}$$
   where u is the gap-averaged velocity and the m and n are denoted by ms and ns for the stator and mr and nr for the rotor. These expressions are taken from the work of Hirs (1973) who does, however, recommend that the coefficients m and n be "fitted to individual experiments." The frictional coefficients are dependent on six physical parameters, including the curvature of the surface, inertial effects, and roughness. Thus, the coefficients may not fully account for the curvature of the rotor in a particular annulus geometry or the inertial effects due to the curved path of the bulk flow. Nor will the roughness parameters be easy to gauge. Given the ease with which the frictional factors can be altered in the computational model, it is reasonable to consider fitting them so as to match the experimental data base.

   Of course, the use of the above expressions for the turbulent shear stresses is subject to an even more general criticism. They are correlations for steady turbulent flows based, primarily, on experimental observations of steady flows. In contrast, the rotordynamic flows of concern here are fundamentally unsteady. The problem is that we know very little about turbulent flows which are unsteady in the sense that the flow is being externally excited. Clearly, at present, this issue can only be resolved by careful comparison of the experimental and model results.

3. Childs treats the rotordynamic flow as a linear perturbation on the mean flow in the annulus. While this may be an accurate assumption for very small eccentricities, there is currently no way to know at what eccentricity this linearization begins to lose accuracy.

4. Though the basic equations appear to be accurate, there is much more doubt about the boundary conditions employed by Childs at the inlet to and discharge from the annulus. For example, Childs deploys a constant pressure condition as well as a uniform swirl velocity condition at inlet. Perhaps the first should be a constant total head condition? Moreover, it is assumed that the hydraulic loss through the orifices at inlet and/or discharge are related only to the meridional velocity (or flow rate) and are independent of the swirl velocity. This may not be accurate.

So, while the bulk flow model proposed by Childs model was a major step forward, there remain many questions which require resolution before a reliable predictive tool is available.

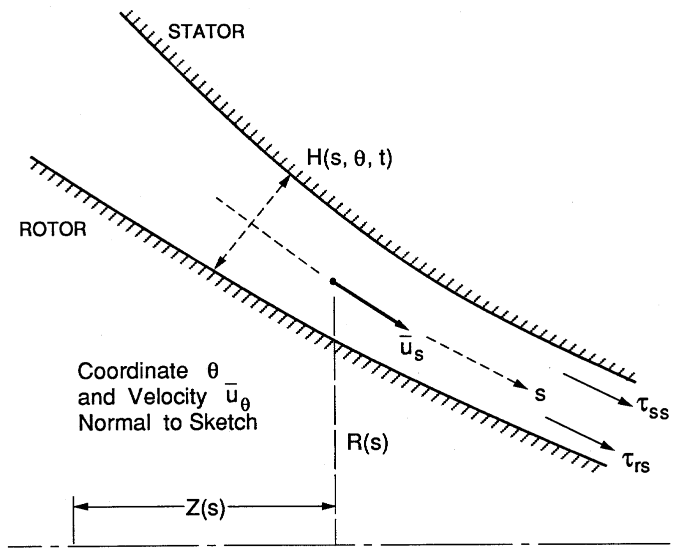

Figure 1: Sketch of fluid filled annulus between a rotor and a stator.

3. THE BULK FLOW MODEL EQUATIONS

Black and his co-workers (Black 1969, Black and Jensen 1970) were the first to attempt to identify and model the rotordynamics of turbulent annular seals. Bulk flow models (similar to those of Reynolds lubrication equations) were used. Several deficiencies in this early work caused Childs (1983a, 1983b) to publish a revised version of the bulk flow model for turbulent annular seals (see also Childs and Dressman 1982, 1985, Childs and Kim 1985, Childs and Scharrer 1986) and, later, to extend this model (Childs 1987, 1989) to examine the rotordynamic characteristics of discharge-to-suction leakage flows around shrouded centrifugal pump impellers. A general geometry is sketched in figure 1, and is described by coordinates of the meridian of the gap as given by $Z(s)$ and $R(s)$, $0 < s < S$, where the coordinate, s, is measured along that meridian. The clearance is denoted by $H(s, \Theta, t)$ where the mean, non-whirling clearance is given by $\bar{H}(s)$.

The equations governing the bulk flow are averaged over the gap. This leads to a continuity equation of the form

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial s}(H u_s) + \frac{1}{R}\frac{\partial}{\partial \Theta}(H u_\Theta) + \frac{H u_s}{R}\frac{\partial R}{\partial s} = 0 \tag{2}$$

where u_s and u_Θ are gap-averaged velocities in the s and Θ directions. The meridional and circumferential momentum equations are

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} = \frac{\tau_{Ss}}{\rho H} + \frac{\tau_{Rs}}{\rho H} - \frac{u_\Theta^2}{R}\frac{\partial R}{\partial s} + \frac{\partial u_s}{\partial t} + \frac{u_\Theta}{R}\frac{\partial u_s}{\partial \Theta} + u_s\frac{\partial u_s}{\partial s} \tag{3}$$

$$-\frac{1}{\rho R}\frac{\partial p}{\partial \Theta} = \frac{\tau_{S\Theta}}{\rho H} + \frac{\tau_{R\Theta}}{\rho H} + \frac{\partial u_\Theta}{\partial t} + \frac{u_\Theta}{R}\frac{\partial u_\Theta}{\partial \Theta} + u_s\frac{\partial u_\Theta}{\partial s} + \frac{u_\Theta u_s}{R}\frac{\partial R}{\partial s} \tag{4}$$

These are the equations used by Childs (1987, 1989). Note that they include not only the viscous terms commonly included in Reynolds lubrication equations (see for example Pinkus and Sternlicht 1961) but also the inertial terms (see Fritz 1970) which are necessary for the evaluation of the rotordynamic coefficients.

To determine the turbulent shear stresses, Childs employed the approach used by Hirs (1973). The turbulent shear stresses, τ_{Ss} and $\tau_{S\Theta}$, applied to the stator by the fluid in the s and Θ directions are given by:

$$\frac{\tau_{Ss}}{\rho u_s} = \frac{\tau_{S\Theta}}{\rho u_\Theta} = \frac{A_S u_s}{2}\left[u_s^2 + u_\Theta^2\right]^{\frac{m_S+1}{2}} (H/\nu)^{m_S} \tag{5}$$

and the stresses, τ_{Rs} and $\tau_{R\Theta}$, applied to the rotor by the fluid in the same directions:

$$\frac{\tau_{Rs}}{\rho u_s} = \frac{\tau_{R\Theta}}{\rho(u_\Theta - \Omega R)} = \frac{A_R u_s}{2}\left[u_s^2 + (u_\Theta - \Omega R)^2\right]^{\frac{m_R+1}{2}} (H/\nu)^{m_R} \tag{6}$$

where the constants A_S, A_R, m_S and m_R are chosen to fit the available data on turbulent shear stresses. Childs (1983a) uses typical values of these constants from simple pipe flow correlations:

$$A_S = A_R = 0.0664 \quad ; \quad m_S = m_R = -0.25 \tag{7}$$

Childs then proceeds to linearize the equations by dividing the clearance, pressure, and velocities into mean components (subscript 0) that would pertain in the absence of whirl, and small, linear perturbations (subscript 1) due to an eccentricity, ϵ, rotating at the whirl frequency, ω. He develops differential equations for the coefficients which are functions of r only.

The present paper will also focus on steady whirl with a constant eccentricity, ϵ, rotating at the whirl frequency, ω, which is superimposed on the shaft rotation whose speed is denoted by the radian frequency, Ω. Consequently, the fluid flow in a frame of reference rotating at ω is steady and it is clearly appropriate to rewrite the equations and to solve them in this rotating frame. Defining, therefore, a new angular variable, θ, and a new angular velocity, u_θ, in this rotating frame such that

$$\theta = \Theta - \omega t \quad ; \quad u_\theta = u_\Theta - \omega R \tag{8}$$

it follows that the continuity equation can be written as

$$\frac{\partial}{\partial \theta}\{H u_\theta\} + \frac{\partial}{\partial s}\{R H u_s\} = 0 \tag{9}$$

and this is most easily satisfied by defining a stream function, $\psi(s, \theta)$ such that

$$u_s = \frac{1}{RH}\frac{\partial \psi}{\partial \theta} \quad ; \quad u_\theta = -\frac{1}{H}\frac{\partial \psi}{\partial s} \tag{10}$$

It follows that the total volume flow rate, Q, at any meridional location, s, is given by

$$Q = \psi(s, 2\pi) - \psi(s, 0) \tag{11}$$

and this provides a periodic boundary condition on ψ in the θ direction.

In the rotating frame of reference, the equations of motion are usefully written using the total pressure, P, instead of the pressure, p, where

$$\frac{P}{\rho} = \frac{p}{\rho} + \frac{1}{2}(u_s^2 + u_\theta^2 - R^2\omega^2) \tag{12}$$

The total pressure, P, is sometimes called the rotalpy. The equations of motion, equations (3) and (4), then become

$$\frac{\partial}{\partial s}\left(\frac{P}{\rho}\right) = -H u_\theta \Gamma - u_s(g_S + g_R) \tag{13}$$

$$\frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{P}{\rho}\right) = H u_s \Gamma - (u_\theta + \omega R)(g_S + g_R) + \Omega R g_R \tag{14}$$

In these equations the functions, g_S and g_R, are the shear stress terms for the stator and rotor respectively and are given by

$$g_S = \frac{A_S}{2H}\left(\frac{H}{\nu}\right)^{m_S}\left\{u_s^2 + (u_\theta + \omega R)^2\right\}^{\frac{m_S+1}{2}} \tag{15}$$

$$g_R = \frac{A_R}{2H}\left(\frac{H}{\nu}\right)^{m_R}\left\{u_s^2 + (u_\theta + \omega R - \Omega R)^2\right\}^{\frac{m_R+1}{2}} \tag{16}$$

The important quantity, Γ, given by

$$\Gamma = \frac{1}{H}\left[-\frac{1}{R}\frac{\partial}{\partial s}(Ru_\theta + \omega R^2) + \frac{1}{R^2}\frac{\partial}{\partial \theta}(Ru_s)\right] \tag{17}$$

plays a crucial role both in understanding the fluid mechanics of these flows and in the solution methodology. It is the effective vorticity. A fundamental property of Γ can be discerned by eliminating P from equations (13) and (14) to obtain the following convection equation for Γ:

$$u_s\frac{\partial \Gamma}{\partial s} + u_\theta \frac{1}{R}\frac{\partial \Gamma}{\partial \theta} = \frac{1}{RH}\left[\frac{\partial}{\partial s}\left\{R(u_\theta + \omega R)(g_S + g_R) - \Omega R^2 g_R\right\} - \frac{\partial}{\partial \theta}\left\{u_s(g_S + g_R)\right\}\right] \tag{18}$$

This demonstrates that, in the absence of viscous effects ($g_S = g_R = 0$), the vorticity is invariant along any streamline. Conversely, the shear stresses are alone responsible for any change in Γ along a streamline. If ξ is a coordinate measured along a streamline, then equation (18) clearly implies that

$$\frac{\partial \Gamma}{\partial \xi} = \frac{1}{RH(u_s^2 + u_\theta^2)^{\frac{1}{2}}}\left[\frac{\partial}{\partial s}\left\{R(u_\theta + \omega R)(g_S + g_R) - \Omega R^2 g_R\right\} - \frac{\partial}{\partial \theta}\left\{u_s(g_S + g_R)\right\}\right] \tag{19}$$

Furthermore, when written in this way, the governing equations clearly indicate a physically reasonable approach to their numerical solution by iterative means.

4. BOUNDARY CONDITIONS AND NUMERICAL METHODS

It follows from the above that one method for the numerical solution of the equations for a rotordynamic flow would be to proceed as follows:

(1) First, for given or guessed values of the vorticity, $\Gamma(s, \theta)$, the Poisson-like equation (17), rewritten as

$$\frac{\partial}{\partial s}\left\{\frac{R}{H}\frac{\partial \psi}{\partial s} - \omega R^2\right\} + \frac{1}{R}\frac{\partial}{\partial \theta}\left\{\frac{1}{H}\frac{\partial \psi}{\partial \theta}\right\} = RH\Gamma \tag{20}$$

must be solved to obtain the stream function, $\psi(s, \theta)$. From this solution new values for $\psi(s, \theta)$, $u_s(s, \theta)$ and $u_\theta(s, \theta)$ can then be calculated. Appropriate boundary conditions on ψ for use in the solution of equation 20 are:

(i) Along $s = 0$, we specify an inlet swirl velocity, $u_\theta(0, \theta)$, which, in order to satisfy conservation of angular momentum, should normally be put equal to the swirl velocity in the reservoir upstream of the inlet.

(ii) An appropriate boundary condition at discharge, $s = S$, would be that the pressure in the flow exiting the annulus should be uniform for all θ or

$$\left(\frac{\partial p}{\partial \theta}\right)_{s=S} = 0 \tag{21}$$

For later discussion, we observe that the boundary condition

$$\left(\frac{\partial u_\theta}{\partial \theta}\right)_{s=S} = 0 \tag{22}$$

provides a convenient first approximation to the condition 21.

(iii) The periodic conditions on boundaries at $\theta = 0$ and $\theta = 2\pi$ such that

$$\psi(s, 2\pi) - \psi(s, 0) = Q \tag{23}$$

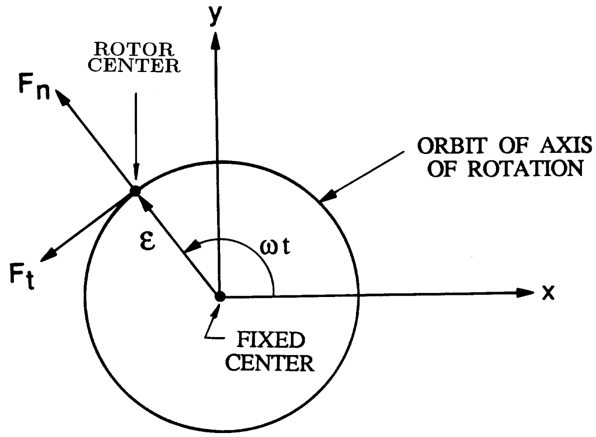

Figure 2: Schematic of the relation between the rotordynamic forces, F_n and F_t, the rotor center and the circular whirl orbit.

(2) Second, given the new values of $\psi(s,\theta)$, $u_s(s,\theta)$ and $u_\theta(s,\theta)$, we can proceed to integrate along streamlines to find new values for $\Gamma(s,\theta)$ using equation (19). This requires evaluation of the shear stress functions, g_R and g_S and values of Γ at the points where the streamlines enter the computational domain. Clearly this becomes more complicated when there is reverse flow either at inlet or at discharge. Here, we restrict out attention to the simpler circumstances in which there is no flow reversal and all the streamlines begin at the inlet. Then, assuming that the viscous stresses upstream of the inlet are negligible and that the inlet flow is circumferentially uniform, equation (14) requires that Γ at the inlet boundary, $s = 0$, must take a uniform value given by the definition (17).

These two steps are then repeated to convergence.

5. HEAD, PRESSURE AND ROTORDYNAMIC FORCES

Having obtained convergence, the total pressure, P, the pressure, p, and the rotordynamic forces can be calculated as follows. The total pressure is most readily obtained by an integration along the streamline similar to that for the vorticity, Γ. From equations (13) and (14) it follows that

$$\frac{\partial}{\partial \xi}\left(\frac{P}{\rho}\right) = \frac{1}{(u_s^2 + u_\theta^2)^{\frac{1}{2}}}\left[\Omega R u_\theta g_R - \left\{u_s^2 + u_\theta(u_\theta + \omega R)\right\}(g_R + g_S)\right] \tag{24}$$

which demonstrates that the total pressure (or energy in the flow) is constant along a streamline in the absence of viscous effects. Since the viscous terms are usually small in these calculations it is sensible to integrate equation (24) along a streamline in parallel with the Γ integration and so obtain the total pressure everywhere. If one chooses to neglect entrance losses between the upstream reservoir at the inlet plane ($s = 0$), and if the reservoir is circumferentially uniform, then this integration begins with the uniform value of $P(0,\theta)$ equal to the total pressure in the reservoir, P_{res}, and this can conveniently be chosen to be zero without loss of generality. On the other hand if entrance losses are to be included then $P(0,\theta)$ can be set to a value smaller than P_{res} by an amount equal to the entrance loss at that particular θ position. Other complications which could be incorporated include a non-uniform upstream reservoir (such as the volute of a pump operating off-design) which would imply a circumferentially varying $P(0,\theta)$.

Having obtained the pressure (and the viscous shear stresses), it only remains to integrate these to obtain the normal and tangential forces acting on the rotor. With the sign convention as defined

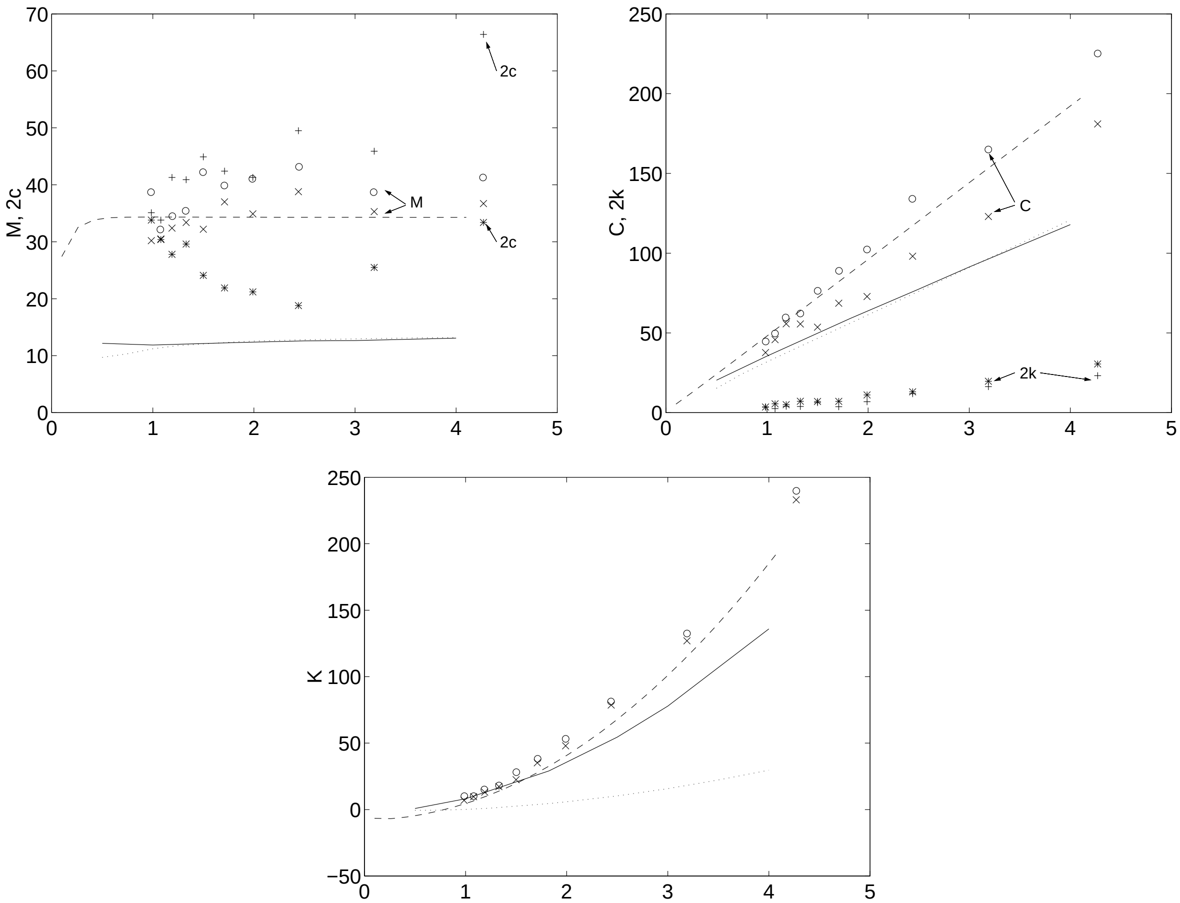

Figure 3: Comparison of the rotordynamic coefficients from the experiments of Nordmann and Massmann (1984) on a plain, untapered, smooth seal with various calculated results as follows. Dashed line: Childs' (1983a) analytical bulk flow model. Dotted line: results from numerical integration of Childs' (1983b, 1987, 1989) perturbation method for bulk flows. Solid line: present bulk flow calculation.

in figure 2, it follows that:

$$F_n = \int_0^S \left\{1 - \left(\frac{dR}{ds}\right)^2\right\}^{\frac{1}{2}} \int_0^{2\pi} \left(p\cos\theta + \tau_{R\theta}\sin\theta\right) Rd\theta \quad ds \tag{25}$$

$$F_t = \int_0^S \left\{1 - \left(\frac{dR}{ds}\right)^2\right\}^{\frac{1}{2}} \int_0^{2\pi} \left(p\sin\theta - \tau_{R\theta}\cos\theta\right) Rd\theta \quad ds \tag{26}$$

In the results quoted in this paper the contributions from the $\tau_{R\theta}$ parts of these integrals are very small and can often be neglected. The rotordynamic coefficients are then obtained by fitting quadratics to the functions, $F_n(\omega/\Omega)$ and $F_t(\omega/\Omega)$; the forces and coefficients are non-dimensionalized as described by Brennen (1994).

6. RESULTS

For purposes of illustration, we choose to examine the plain, untapered, smooth axial seal tested by Nordmann and Massmann (1984). This has an aspect ratio, $S/R = 1.67$, a clearance, $H/R = 0.0167$ and was tested at a Reynolds number, $Re = Q/2\pi R\nu = 5265$ (where Q is the volume flow rate through the seal). In Figure 3, the non-dimensional rotordynamic coefficients from the experiments of Nordmann and Massmann (see Brennen 1994) are plotted against a flow coefficient, $\phi = Q/2\pi H R^2 \Omega$. These are compared with three sets of calculated coefficients. The dashed lines are from the analytical expressions obtained by Childs' (1983a) as approximate "short seal" solutions to the bulk flow. Later Childs (1983b) published a more accurate "finite length" seal solution which involved the numerical integration of more accurate perturbation equations. The dotted lines represent the results of a similar perturbation analysis applied to the Nordmann and Massmann seal. The differences between the two Childs' methods are substantial but similar to the differences in the examples presented by Childs (1983b). They are presumably caused by different treatments of the circumferential velocity perturbations. Similar differences were noted by Guinzburg (1992), particularly in the direct added mass, M. Note that all three sets of calculated results use an inlet swirl velocity equal to a half of the rotor tip speed.

The solid lines represent results using the present numerical method. Apart from the direct stiffness, K, they are similar to Childs' (1983b) more accurate analysis. For reasons which are presently unclear our calculations yield a K which is closer to the short seal value and to the experimental data. The discrepancies between the more accurate analyses and the experimental data are disturbing and require further study.

7. CONCLUSIONS

This paper has explored some of the basic characteristics of the bulkflow model equations for the turbulent flow in a fluid-filled annulus generated by both rotational and whirling motions. The analysis unveils the definition of the appropriate vorticity for these flows and develops evolutionary equations both for the vorticity and for the total pressure, without resorting to linearization. Among other features demonstrated by these equations is the fact that the changes in vorticity and total pressure along a streamline are entirely due to the shear stresses imposed on the flow.

This equation structure naturally suggests a way in which numerical solutions to these equations might be sought, by iterating between a Poisson-like equation for the streamfunction using a preliminary vorticity distribution and an integration along the streamlines to revise that distribution. Several sample calculations are used to illustrate this technique.

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NOMENCLATURE

A_r, A_s	Empirical constants for rotor and stator respectively
C	Direct damping coefficient, normalized by $\rho\pi\Omega R^2 L\varepsilon$
c	Cross-coupled damping coefficient, normalized by $\rho\pi\Omega R^2 L\varepsilon$
F_n	Force normal to whirl orbit, normalized by $\rho\pi\Omega^2 R^2 L\varepsilon$

F_t	Force tangent to whirl orbit, normalized by $\rho\pi\Omega^2 R^2 L\varepsilon$
H	Clearance between impeller shroud and housing
K	Direct stiffness coefficient, normalized by $\rho\pi\Omega^2 R^2 L\varepsilon$
k	Cross-coupled stiffness coefficient, normalized by $\rho\pi\Omega^2 R^2 L\varepsilon$
L	Axial length of the impeller
M	Direct added mass coefficient, normalized by $\rho\pi R^2 L\varepsilon$
mr, ms	Empirical exponent for rotor and stator respectively
P	Total pressure
p	Static pressue
Q	Volumetric leakage flow rate
R	Radius of rotor
u_s	Mean leakage inlet path velocity of fluid
u_θ	Mean leakage inlet swirl velocity of fluid
Γ	Effective vorticity defined by Eq. 17
η	Fluid viscosity
ε	Eccentricity of whirl orbit
ρ	Fluid density
ϕ	Leakage flow coefficient, $\mathrm{u}_s/\Omega R$
ψ	Stream function, defined by Eq. 10
ω	Whirl radian frequency
Ω	Main shaft radian frequency
τ	Wall shear stress