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Vertical Oscillation of a Bed of Granular Material

A bed of granular material which is subjected to vertical vibration will exhibit at least one sudden expansion at a critical acceleration amplitude. This sudden expansion corresponds to a bifurcation similar to that exhibited by a single ball bouncing on a vibrating plate. Theoretical analysis based on this model yields results which are in accord with the experimental observations. Other bifurcations may occur at higher vibration levels.

1 Introduction

The vibration of granular materials is of interest for a number of reasons. First, vibration is sometimes used instead of an upward flow of gas to fluidize a particle bed reactor and in such devices it is clearly important to know the state of the bed. Secondly, vibration is often used to induce flow in recalcitrant bulk flow transport devices such as hoppers and chutes. It is also used to induce segregation of different density and different size particles. Clearly, knowledge of how vibration affects these granular materials provides important design information. As a third incentive we note that there has been a growing recognition of and interest in the granular state. In a recent review, Jaeger and Nagel (1992) have summarized some of the important issues, questions, and applications of knowledge of the granular state and highlight the need for understanding the response to vibration. The analogy to molecular dynamics is often drawn but an important difference is that the particles in a granular material are inelastic and therefore only sustain random motions when either (a) the material is flowing (more specifically, undergoing continuous deformation) in which case the random motions are produced by the collisions or (b) externally imposed vibrations generate particle motions. Consequently, research on the flow of granular materials and on the vibrational excitation of granular material would seem complementary and knowledge gained from one should provide insights to the other.

Several investigators have previously examined the response of a bed of particles subjected to vertical vibrations and identified a number of states and transitions between those states. Observations have been made for fine powders in which the interstitial fluid plays an important role in the response (see, for example Gutman, 1976a, 1976b) and for larger particles (typically > 0.1 mm diameter) in which the effects of the interstitial fluid are small. In this paper we shall focus on the latter case because, even in the absence of the interstitial fluid effects, phenomena occur which have yet to be adequately explained. The important variables are the radian frequency of vibration, Ω, vibration amplitude, a, particle diameter, d, and bed height at rest, h_0, as well as material properties such as the coefficient of restitution, ϵ_p, for collisions between the individual particles and the base plate. Clearly two appropriate dimensionless parameters which will influence the state of the material are the dimensionless acceleration amplitude, $\Gamma = a\Omega^2/g$, where g is the acceleration due to gravity, and the number of layers in the bed, h_0/d.

Most investigators agree that within the range of frequencies usually explored ($5 \rightarrow 100$ Hz) the phenomena are relatively independent of frequency but depend strongly on the acceleration level, Γ, and the bed thickness, h_0/d. We describe the phenomena which have been reported to occur as Γ is increased from zero. As long as $\Gamma < 1$, the visual appearance of the bed changes little; however Chlenov and Mikhailov (1965, 1972) report an increase in mobility and this manifests itself as a decrease in the angle of repose (Rajchenbach and Evesque, 1988). When Γ exceeds unity by a small amount, the bulk of the particles separate from the base plate each cycle of oscillation when the downward acceleration exceeds $1g$. We note, parenthetically, that one of the effects of the resistance to air flow in fine powders is to delay the inception of separation to values of Γ greater than unity (Thomas et al., 1989). For the larger particles (typically > 0.1 mm diameter), when Γ is just a little larger than unity the flight time of the particles, Δt (the time between separation and subsequent recontact), is short compared with the period, $T = 2\pi/\Omega$, of the oscillations. In these circumstances the material essentially comes to rest relative to the plate prior to the next flight.

A number of phenomena are observed to occur when the acceleration level, Γ, is increased to higher levels so that the flight time, Δt, approaches the period, T. It is clear that the events depend upon the layer thickness, h_0/d. Douady, Fauve, and Laroche (1989) examined fairly thick layers with h_0/d in the $10 \rightarrow 100$ range and observed that when the flight time becomes slightly greater than the period, a period doubling bifurcation occurred. This resulted in two different flights which alternated to produce a 2Ω component in the motion. The critical Γ at which this occurred increased from 4.5 for $h_0/d = 7$ to 5.3 for $h_0/d = 25$.

Thomas et al. (1989) examined much thinner layers including very dilute systems consisting of much less than a single layer of particles. They describe four identifiable states which can occur at large Γ (typically $2.5 \rightarrow 6.0$) and are primarily distinguished by different layer thicknesses, h_0/d. For very small h_0/d (of order 0.17) they describe a "Newtonian-I" state in which the particles are bouncing so randomly that the vertical concentration profile changes little during a cycle. At somewhat larger h_0/d (at 0.273 for example) there is a transition to a "Newtonian-II" state in which a dense layer of particles accumulates on the surface during one part of each cycle. Thicker layers of particles (for example $h_0/d = 1.7$) lead to a "coherent-expanded" state in which the particles all oscillate as a coherent mass. This mass does, however, expand and contract during each cycle. Bachmann (1940) had earlier observed the transi-

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Table 1 Bead and lid masses for the various experiments

Experiment No.	Bead Mass [gm.]	Lid Mass [gm.]
1	250	3.44
2	125	3.44
3	125	7.17
4	375	3.44
5	125	17.06
6	625	3.44
7	45	3.51
8	125	28.14

tion to coherent motion and reported that this occurred when $h_0/d = 6$.

Finally, Thomas et al. (1989) identify a "coherent-condensed" state at larger values of h_0/d on the order of 4. In this state the particles move as a mass but the mass remains compact throughout the cycle. They report that the transition from the "coherent-expanded" state to the "coherent-condensed" state is sudden and repeatable.

In the present paper we describe the phenomena which were observed to occur as the vertical acceleration of a bed of material is increased and identify a transition or bifurcation similar to that which occurs with a single bouncing ball on a vertically vibrating plate (Wood and Byrne, 1981; Holmes, 1982).

2 Experiments

Experiments were carried out to investigate the behavior of a bed of granular material subjected to vertical vibration. The materials used were A-285 glass beads with a mean diameter of 2.85 mm. Various quantities of these beads were placed in a rectangular box with cross-sectional dimensions of 11 cm by 13.2 cm which was in turn mounted on an electro-mechanical shaker and subjected to vertical vibration at frequencies between 4 and 10 Hz with amplitudes up to about $2.5g$. A Statham A73TC-4-350 accelerometer was used to measure the acceleration level accurately.

The box had a thick aluminum base and back but the other three sides were made of lucite so that the behavior of the beads could be observed. Paper lids of various thickness were placed on top of the beads leaving a clearance of about 1 mm between the edge of the lid and the walls of the box. When the box was vibrated vertically the bed of beads would expand and the lid would float on the beads. Fortunately, the lid proved to be quite stable and under all of the conditions used in the present experiments would remain horizontal and centralized with a roughly equal spacing all around the periphery. Because this spacing was smaller than the diameter of the beads, all of the beads would remain under the lid. A stroboscope was used to examine the motion of the lid and the beads during various parts of the oscillation cycle. By this means we were able to observe that the spacing, h, between the base and the lid did not vary greatly during the oscillations. The beads would bounce around below the lid but because of the resistance to the flow of air around the sides of the lid, the volume of beads and air would remain almost constant during a cycle of oscillation. Thus, using the strobe and a scale attached to the exterior of the box, it was possible to measure the height, h, for each operating condition.

Experiments were conducted by observing the evolution of the bed of beads as the vibration amplitude, a, was increased from zero to the maximum of which the shaker was capable. Such experiments were conducted over a range of frequencies ($4 \rightarrow 10$ Hz) for various quantities of beads and for lids with different weights as listed in Table 1.

It should be noted that a single packed layer of beads resting on the base of the box would weigh approximately 62 gm. Consequently the masses of beads range from less than a single layer to about ten layers. The 45 gm of experiment 7 was close to the minimum at which the lid would remain horizontal for the duration of the experiment.

3 Experimental Results

The results for the base-to-lid spacing, h, as a function of vibration amplitude will be presented in various ways but we focus here on the expansion of the bed, $h^* = h - h_0$, where h_0 is the spacing at rest. For reasons which will become clear, h^* will be presented both as a function of the non-dimensional acceleration amplitude $\Gamma = a\Omega^2/g$ where g is the acceleration due to gravity, and as a function of the vibration velocity, $a\Omega$. The typical behavior of the bed is best illustrated by the results from experiment 7 which are presented in Fig. 1.

The bed would begin to expand at an acceleration amplitude of about $1g$ and this expansion would gradually increase until one reached a certain critical value of the acceleration amplitude, Γ_c, which appeared to be independent of frequency but to vary with both the mass of beads and the mass of the lid. At this critical acceleration amplitude the lid would rise quite abruptly and then settle down at a substantially larger spacing, h. As illustrated in Fig. 1, further increase in the acceleration would result in further bed expansion but this was more gradual than the expansion encountered during transition. The top graph in Fig. 1 illustrates the fact that the critical conditions appear to occur at a given acceleration amplitude regardless of the frequency. On the other hand, the bottom graph in Fig. 1 illustrates the fact that the supercritical conditions correlate with the velocity amplitude, $a\Omega$, rather than the acceleration amplitude.

Using the strobe, one could observe that prior to the transition the motions of the particles were fairly uncoordinated. However, above the transition the beads began to move as a mass which collided once per cycle with the base and with the lid. The collision with the base seemed quite inelastic and it appeared

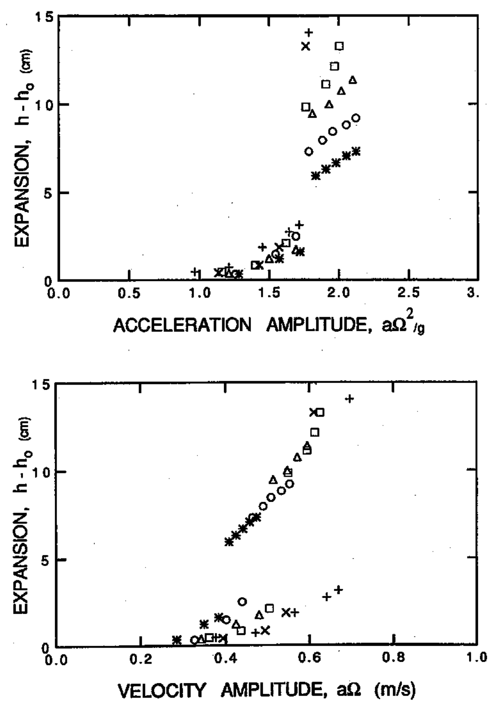

Fig. 1 The dependence of the bed expansion, $h - h_0$, on the acceleration amplitude, $\Gamma = a\Omega^2/g$, and the velocity amplitude, $a\Omega$ (in m/s), for experiment 7. Various frequencies as follows: + = 4 Hz, × = 4.5 Hz, □ = 5 Hz, △ = 5.5 Hz, ○ = 6 Hz, and * = 7 Hz.

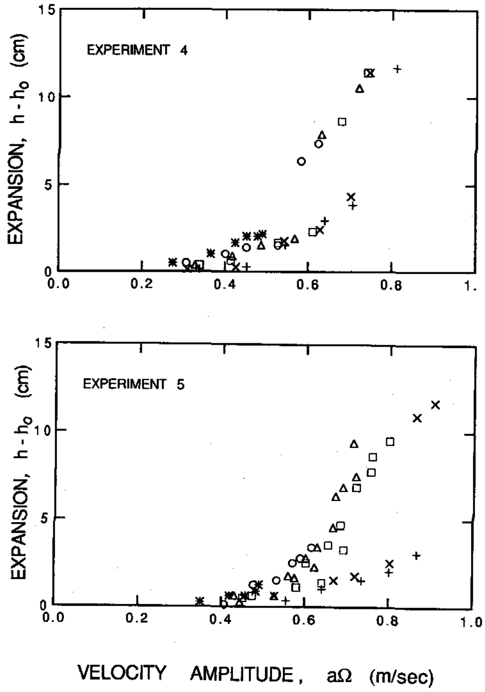

Fig. 2 Data from experiments 4 (top) and 5 (bottom) with frequency key as in Fig. 1

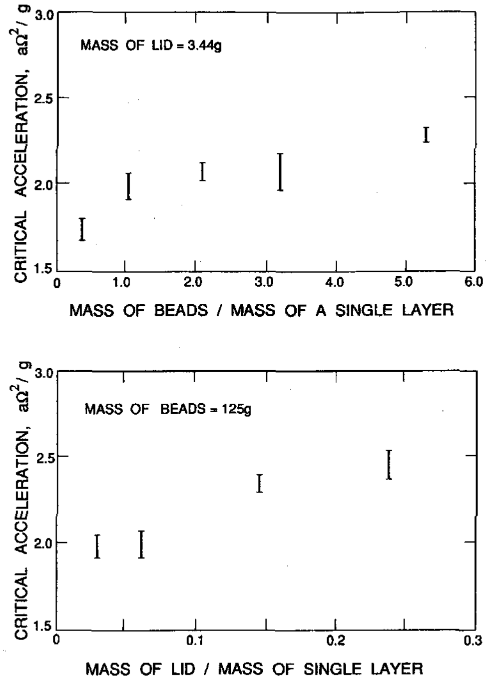

Fig. 3 The critical acceleration, $\Gamma_c = (a\Omega^2/g)_c$, as a function of the mass of beads (top graph for 3.44 gm lid) and as a function of the mass of the lid (bottom graph for 125 gm of beads)

that the mass only left the base again when the acceleration of the base exceeded some critical value. However, it is also important to emphasize that the mass expands and contracts substantially during each cycle being quite concentrated while it is in contact with the base but quite dilute while it is in flight.

Experiment 7 was chosen to illustrate the transition because it does so most clearly. This is because it used the smallest mass of beads. As the mass of beads was increased (for the same lid weight) the critical transition became less distinct in the sense that the expansion at the critical acceleration became somewhat less abrupt and somewhat smaller. The same trend was manifest as one increased the weight of the lid. Both effects are illustrated in Fig. 2 which presents data from experiments 4 and 5.

The critical acceleration, Γ_c, also increases with both the mass of the beads and the mass of the lid. These trends are shown in Fig. 3.

In order to understand the fundamental dynamics behind the above phenomena it is valuable to present the data non-dimensionally. This accomplished by nondimensionalizing the expansion as $(h - h_0)\Omega^2/g$ and plotting this versus the nondimensional acceleration amplitude, $\Gamma = a\Omega^2/g$. Examples from experiments 2 and 3 are shown in Fig. 4 in which the subcritical and supercritical data clearly form two distinct groups of points. Indeed the two groups of points both appear to lie close to quadratic curves which imply that each group of points correspond to a roughly constant value of the inverse Froude number,

$$\mathrm{Fr}^{-1} = \frac{[g(h - h_0)]^{1/2}}{a\Omega}. \quad (1)$$

To examine this further, the inverse Froude number is plotted versus the acceleration, Γ, in Fig. 5 for the typical data of experiments 2 and 3.

It seems particularly noteworthy that the subcritical data corresponds roughly to an inverse Froude number, Fr^{-1}, of between 0.5 and 1.0 and that the supercritical corresponds quite closely to $\mathrm{Fr}^{-1} = 1.5$ (recall that the values of $(h - h_0)$ and a for some of the subcritical data are quite small and this may account for the larger scatter in that group of points). The specific values for Fr^{-1} decrease significantly as the mass of beads increases and as the mass of the lid is increased. The subcritical data shows similar trends though they are less distinct due to greater scatter in the data.

4 Theoretical Analyses

The analytical solutions to the problem of a ball bouncing on a horizontal flat plate performing vertical oscillations (ampli-

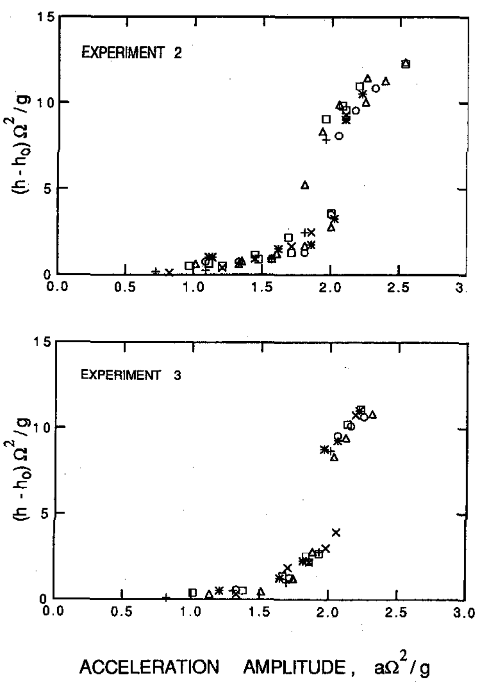

Fig. 4 Dimensionless expansion, $(h - h_0)\Omega^2/g$, plotted against dimensionless acceleration, $\Gamma = a\Omega^2/g$, for experiments 2 and 3. Frequency key as in Fig. 1.

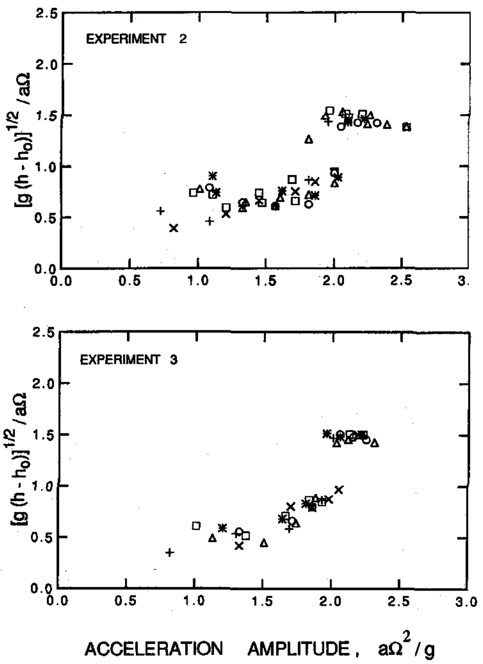

Fig. 5 Inverse Froude number, $[g(h-h_0)]^{1/2}/a\Omega$, plotted against dimensionless acceleration, $\Gamma = a\Omega^2/g$, for experiments 2 and 3. Frequency key as in Fig. 1.

tude, a, and radian frequency, Ω) are of interest for several reasons. First, the model could be considered appropriate for individual particles when particle/particle collisions are relatively rare, as, for example, in the case where less than a single layer of particles was used. Alternatively, in the case of a larger mass of particles, the solution might be considered applicable to the whole mass when it performs a coherent periodic motion. In either case, we shall consider that the particles bounce off a lid which, by some unspecified damping mechanism, is maintained at a constant height above the oscillating plate. The lid is, however, entirely supported in the mean by the impulses imparted by the particles; thus solutions will be sought for various ratios of the lid mass to the particle mass, f. The problem also requires specification of the coefficients of restitution, ϵ_p and ϵ_l for collisions with the plate and lid, respectively.

The dynamics of the ball bouncing problem without a lid have now become a classic example of the occurence of bifurcations (see, for example, Wood and Byrne, 1981; Holmes, 1982) and we shall see that this seems the probable explanation for the experimentally observed transition.

The first, simple solution which is useful is that for no lid and for $\epsilon_p = 0$. The ball remains in contact with the plate until the latter is accelerating downward at an acceleration equal to g. The maximum height, h_s, to which the ball rises above the plate can readily be identified parametrically as

$$\frac{h_s\Omega^2}{g} = \Gamma[(x_2 - x_1)\cos x_1 + \sin x_1 - \sin x_2] \qquad (2)$$

where

$$\sin x_1 = 1/\Gamma;\ x_2 - x_1 = \Gamma(\cos x_1 - \cos x_2). \qquad (3)$$

This relationship between the dimensionless "expansion," $h_s\Omega^2/g$ and the acceleration amplitude was obtained numerically and is identified in Fig. 6 as the "no bounce" solution. Note that it corresponds quite closely with the subcritical experimental data (in Fig. 6 we have used the data of experiment 2 as typical).

When one examines the specifics of this solution for the range of Γ values of interest here (less than about 2) one finds that after becoming airborne the particle (or particle mass) will return to impact the plate after less than about 0.6 of a cycle. Even if ϵ_p were nonzero and there were several small bounces following this impact there is more than sufficient time left in the cycle for the particle (or particle mass) to effectively come to rest on the plate before the next occurrence of a downward acceleration of $1g$. Thus the solution is valid for a range of ϵ_p.

The second benchmark which is of interest here is the periodic solution in which the particle (or particle mass) bounces off the plate and off the lid once per cycle of plate oscillation. In order for such a periodic solution to exist the relative velocity of departure from the lid collision, u_4, the relative velocity of incidence on the plate, u_1 (both u_4 and u_1 considered positive downward), the relative velocity of departure from the plate, u_2, and the relative velocity of incidence on the lid, u_3, (u_2 and u_3 considered positive upward) must be given by

$$\frac{u_1\Omega}{g} = \frac{2\pi(1+f)}{(1+\epsilon_p)};\quad \frac{u_2\Omega}{g} = \frac{2\pi\epsilon_p(1+f)}{(1+\epsilon_p)}$$

$$\frac{u_3\Omega}{g} = \frac{2\pi f}{(1+\epsilon_l)};\quad \frac{u_4\Omega}{g} = \frac{2\pi\epsilon_l f}{(1+\epsilon_l)}. \qquad (4)$$

The solution is most readily obtained parametrically by selecting the times t_1 and t_2 during a cycle when collision with the plate and the lid, respectively, occur. It then follows that

$$\Gamma = \frac{[\Omega(t_1 - t_2)]\left[\frac{u_2\Omega}{g} + \frac{u_3\Omega}{g}\right] + [\Omega(t_1 + t_2) + 2\pi]\left[\frac{u_1\Omega}{g} + \frac{u_4\Omega}{g}\right]}{2\pi(\cos\Omega t_1 + \cos\Omega t_2)} \qquad (5)$$

and that the expansion, h, defined as the increase in the spacing between the plate and the lid is given by

$$\frac{h}{a} = \sin\Omega t_1 - \sin\Omega t_2 + \frac{\Omega(t_2 - t_1)}{2}\left[\frac{u_2}{a\Omega} + \frac{u_3}{a\Omega} + \cos\Omega t_1 + \cos\Omega t_2\right]. \qquad (6)$$

Thus the choice of two arbitrary values of Ωt_1 and Ωt_2 corresponds to a solution for specific values of f and Γ and yields a specific value for h/a. In addition one must check to ensure that there are no unforeseen overlaps between the particle and the lid or plate during the oscillation cycle. Typical results for this analysis are included in Fig. 6 (identified as "with bounce" solution) for $\epsilon_p = 0.25$, $\epsilon_l = 0$, and $f = 0.01$, 0.1, and 0.2. Note that for a given lid and given coefficients of restitution there

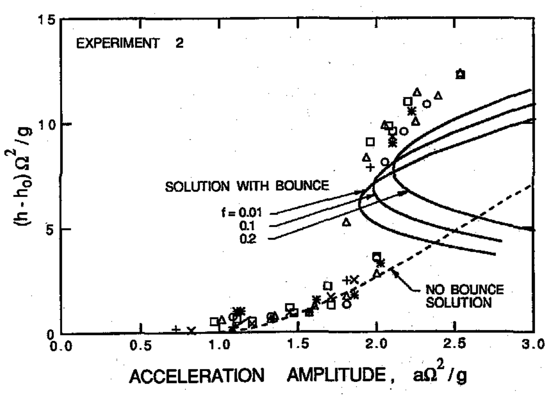

Fig. 6 Typical data (from experiment 2) compared with the analytical solutions described in the text

exist no periodic solutions of this type for accelerations below a certain critical level.

It should be noted in passing that there is a large variety of other possible periodic solutions. For example there exist the possibilities of one bounce for every two or more plate cycles and of two or more bounces in a single plate cycle. Alternatively the ball might cycle through two or more types of bounce before repeating itself. It is important to point out that studies of the dynamics of the much simpler system of a single particle on a vibrating plate (Wood and Byrne, 1981 and Holmes, 1982) have revealed a system of bifurcations at different critical values of the acceleration, Γ.

As the acceleration amplitude is increased the dynamics of the single ball exhibits the first bifurcation from the "no bounce" solution to the "with bounce" state at

$$\Gamma_c = \frac{\pi(1 - \epsilon_p)}{(1 + \epsilon_p)} . \quad (7)$$

The present experimental data clearly indicates that such a bifurcation also occurs with the granular mass. Though the analogy may only be of qualitative value, it is nevertheless of interest to observe that Eq. 7 yields $\Gamma_c = 1.88$ when $\epsilon_p = 0.25$, a value we have arbitrarily chosen to demonstrate the results of the analytical calculation. This analysis is qualitatively consistent with the current experimental data since the effective ϵ_p for the mass of particles may be as low as 0.25.

Thus the analysis is consistent with the following explanation of the observed experimental behavior. At small values of the acceleration just above $1g$, the data is consistent with the simple, no-bounce solution. However, when the acceleration approaches the critical or bifurcation value of Γ, a sudden expansion of the bed occurs as the particle mass begins to move as a fairly coherent whole, bouncing off the plate once each oscillation cycle.

A computer simulation was developed in order to determine if a column of inelastic particles vibrating on an oscillating plate and bouncing off one another would behave in a manner similar to a single particle. A hard sphere model was used to simulate a column of up to ten particles with zero radius, constrained to move vertically, supported by a sinusoidally vibrating plate. The separation height, h, between the top particle and the oscillating plate was averaged over many cycles for values of the parameter Γ between 1.0 and 5.0. It was found that a series of "jumps" in the bed expansion existed for combinations of N, the number of particles, and ϵ, the coefficient of restitution between particles and between the bottom particle and the plate. Figure 7 presents the results of a typical simulation where the dimensionless expansion $h\Omega^2/g$, is plotted against the acceleration amplitude $a\Omega^2/g$. For cases where N was small and ϵ was large, the effective coefficient of restitution of the column of particles is nonzero and sudden increases in the column expansion occurred. However, when N was large and ϵ was small, the effective restitution coefficient was zero and the column of particles remained grouped together. In this regime the characteristic sudden expansion was not observed for the range of acceleration amplitudes examined.

Clément et al. (1993) found similar results both experimentally and numerically for a column of spherical particles vibrating on a sinusoidally oscillating base. They plotted the separation height of the center of mass of a column of ten particles vibrating on a sinusoidally oscillating base as a function of acceleration amplitude and also found sudden jumps at particular values of Γ. They, however, did not discuss the cause of these jumps. Clément et al. also describe regimes where particles cluster together and move as a coherent mass. Here the column of particles behaves in a manner similar to a single particle with a coefficient of restitution equal to zero. Clearly the same phenomena are being observed in the present simulations.

As a last note, it is interesting to consider the possible role of the present bifurcations in the onset of the heaping phenomenon (and its convection pattern) observed in the experiments of Evesque and Rajchenbach (1989), Laroche et al. (1989), and others. Since the bifurcations observed here occur at nearly the same value of Γ as the onset value for heaping ($\Gamma_H = 1.2$) it is worth considering how the two phenomena might be related. The authors suggest that the two are in fact not related. The sudden bed expansion which occurs for the shallow beds examined here are due to a bifurcation in the dynamics of a bed that has an effective restitution coefficient which is greater than zero, $\epsilon_{eff} > 0$. Heaping, however, is observed for deeper beds where $\epsilon_{eff} = 0$ and the first bifurcation occurs when $\Gamma \approx 3.3$. Furthermore, when $\epsilon_{eff} = 0$, the bed does not exhibit the sudden expansion described in this paper but instead displays a period doubling bifurcation.

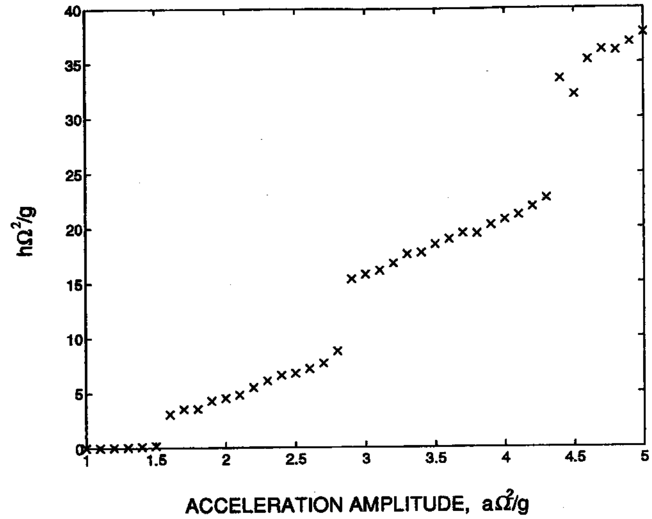

Fig. 7 Results from the computer simulation of a column of ten particles with zero radius. In the simulation, $\epsilon = 0.90$, and the separation height, $h\Omega^2/g$ was averaged over 400 oscillations.

5 Conclusions

A bed of granular material which is subjected to vertical vibration will exhibit at least one sudden expansion at a critical acceleration amplitude. This sudden expansion corresponds to a bifurcation similar to that exhibited by a single ball bouncing on a vibrating plate. Theoretical analysis based on this model yields results which are in accord with the experimental observations. Other bifurcations may occur at higher vibration levels.

Acknowledgments

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Undergraduate and graduate students are invited to participate in a student paper competition at the 33rd Annual Technical Meeting of the Society of Engineering Science to be held October 20–23, 1996 at Arizona State University in Tempe, AZ. Cash prizes of $1000 and $500 will be awarded for first and second place, respectively. A limited number of travel stipends of $350 will be available on a first come, first served basis. Student travel stipends also include waiver of the conference registration fee.

Presentations in any area of engineering science are appropriate. Topics may include, but are not limited to acoustics, atmospheric sciences, biological sciences, biomechanics, chemical sciences, compressible flow, computational sciences, computer sciences, composite materials, corrosion, dynamics, fracture mechanics, fluid/structure interactions geophysics, material sciences, mathematics, micromechanics, tribology and wear, and vibrations.

The competition is limited to 20 students. If more than 20 abstracts are received, the abstracts will be pre-screened. Students not selected for the competition will be invited to present their work in a poster session at the conference. The student paper competition is limited to those students who will not present papers in other sessions at the conference.

One-page abstracts must be submitted to Dr. Judy L. Cezeaux by *February 29, 1996*. Students must be first author on the abstract and the student authors and faculty advisors should be clearly indicated.

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