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Stability of Hydraulic Systems with Focus on Cavitating Pumps

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ABSTRACT

Increasing use is being made of transmission matrices to characterize unsteady flows in hydraulic system components and to analyze the stability of such systems. This paper presents some general characteristics which should be examined in any experimentally measured transmission matrices and a methodology for the analysis of the stability of transmission matrices in hydraulic systems of order 2. These characteristics are then examined for cavitating pumps and the predicted instabilities (known as auto-oscillation) compared with experimental observations in a particular experimental system

RÉSUMÉ

L'identification et l'analyse des critères de stabilité des écoulements non-stationnaires dans les composantes hydrauliques se font de plus en plus au moyen des matrices de fonctions de transfert.

Ce papier présente quelques caractéristiques générales qui se doivent d'être considérées en vue d'analyser une matrice de fonctions de transfert obtenue expérimentalement de même qu'une méthodologie pour mettre en évidence les critères de stabilité associés a une matrice de deuxième ordre.

Ce schème est part la suite appliqué aux pompes cavitantes et la prédiction des régimes instables (connue sous le nom d'auto-oscillation) est comparée aux observations expérimentales pour un système particulier.

1. INTRODUCTION

Hydraulic systems involving components in which phase changes occur often encounter instabilities which lead to large pressure and mass flow rate excursions. The prediction of such instabilities and the design of ameliorative hardware are usually hindered by a lack of knowledge of the dynamic response of the components in which the cavitation, boiling or other phase change process occurs.

In the present paper we present a general methodology for such problems and consider its application to the common instability problems which are experienced in systems involving cavitating pumps. Instabilities in such systems are often termed "auto-oscillation" and have been the subject of a number of studies (Refs. 1 to 15). They have been demonstrated to be system instabilities caused by the "active" nature of the dynamic characteristics of a cavitating inducer. In the next sections we developed a characterization for such activity and criteria for evaluating instability.

2. DYNAMIC ANALYSES OF HYDRAULIC SYSTEMS

The traditional procedures for the dynamic analyses of hydraulic systems involve the integration of the equations of motion in the time domain particularly by the method of characteristics (Refs. 16 and 17). These have the advantages that non-linear terms can be incorporated but the methods are not readily adaptable to complicated flows of the kind that occur in many hydraulic devices such as pumps and turbines. The alternative approach of solution in the frequency domain has been used less often (e.g. Ref. 18); it has the disadvantage that it is usually necessary to confine the analysis to small linear perturbations. However, more complex hydraulic devices can be readily incorporated in such an approach; furthermore, experiments to measure the dynamic characteristics of such devices are most readily performed by introducing perturbations over a range of frequencies and the results are then presented as functions of perturbation frequency (e.g., Refs. 12,15,19).

Within the context of the frequency domain analyses which have proliferated in recent years (e.g., Refs. 18,20,15) the vast majority have been guided by electric network theory (e.g., Ref. 21) and have been confined to systems in which the flow is completely described by two state variables, usually pressure, p, and flow rate, q, though total pressure, h, has advantages over the former as will be demonstrated later. This corresponds to so-called four terminal network theory in the electrical context and transmission matrices for any component of the system are 2 X 2 matrices which are functions of frequency, Ω, and the mean or time averaged flows in the component. For example, if the linear perturbations in total pressure and mass flow rate are described by $Re\{\tilde{h}e^{j\Omega t}\}$ and $Re\{\tilde{m}e^{j\Omega t}\}$ where t is time, j is the imaginary unit, Re denotes real part of and $\tilde{h}$ and $\tilde{m}$ are complex in general then the transmission function, $[T]$, can be defined as

$$\begin{Bmatrix} \tilde{h}_2 \\ \tilde{m}_2 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \tilde{h}_1 \\ \tilde{m}_1 \end{Bmatrix} \tag{1}$$

where subscripts 1 and 2 define values at inlet to and discharge from the component; $[T]$ is often referred to as the transfer function or matrix though it should strictly be termed the transmission function or matrix.

It is important to note that such a description is confined to the special class of hydraulic systems which can be broken down into components such that the flow at each dividing point (not necessarily all points) is characterizable by only two state variables. This confines the analysis to

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It is important to note that such a description is confined to the special class of hydraulic systems which can be broken down into components such that the flow at each dividing point (not necessarily all points) is characterizable by only two state variables. This confines the analysis to either (a) incompressible fluid flows (b) compressible fluid flows in which the perturbations are barotropic : the perturbation density, $\tilde{\rho}$, is directly related to the perturbation pressure, $\tilde{p}$ and therefore is not an independent state variable (c) components whose inlet and discharge are single phase flows of the type (a) or (b) though not necessarily the same phase (examples are a cavitating pump with single phase liquid flow in and out, or an ideal evaporator or condenser) (d) two-phase flows represented by homogeneous flow models since they are usually equivalent to (b).

General liquid/gas two phase systems do not however fall in this restricted class since they usually require at least four state-variables (e.g., pressure, gas flow rate, liquid flow rate and void fraction) for complete characterization though some reduction of the order of the system can be achieved with certain two-phase flow models (e.g., the drift-flux model in which the relative velocity is a function only of the void fraction). Very limited data is available on transmission matrices of order greater than two. Brown (22) presents a unified approach to such problems but the material is confined to uniform systems in which the coefficients of the governing differential equations are independent of position; this eliminates all but the simplest fluid systems or components.

The analysis in this paper is similarly confined to systems of order 2. In the next section we present some of characteristics of these systems and a methodology for stability analysis.

3. SOME PROPERTIES OF TRANSMISSION MATRICES

It is clear that provided one can construct transmission matrices for each of the components into which the hydraulic system is broken then one has available a complete dynamic model of the system to use for stability and transient analyses. The major difficulty is usually a lack of knowledge of the transmission matrices for complex hydraulic components. In this respect one must rely on experimental measurements of the transmission functions, though sometimes such measurements may suggest analytical approaches as in the case of cavitating pumps (Refs. 12,23,24). When faced with experimentally measured transmission matrices it is often desirable to evaluate certain properties of those matrices so that one can anticipate how that hydraulic component might affect the dynamics of a complete system incorporating that component.

One such property is the determinant, D, of $[T]$. The matrix $[T]$ is said to be reciprocal if $D = 1$ and the overall transmission matrix for any parallel or series combination of reciprocal components is also reciprocal. In the context of hydraulic systems it is readily shown that incompressible flows within rigid boundaries ($T_{21} = 0$, $T_{22} = 1$) and with total head losses which are functions only of flow rate ($T_{11} = 1$) are reciprocal. Furthermore, an accumulator or surge tank envisaged as acting at a point ($T_{11} = 1$, $T_{12} = 0$, $T_{21} = -j\Omega C$, $T_{22} = 1$) and having a compliance C is reciprocal. Systems comprised of the above elements are analogous to L,R,C networks and have the same properties.

As an addenda to this it is well-known and readily shown that any uniform system of any order, N, has a determinant, D, given by

$$D = \exp\left(j\ell \sum_{m=1}^{N} \gamma_m\right) \tag{2}$$

where ℓ is the distance between stations 1 and 2 and $\sum_{m=1}^{N} \gamma_m$ is the sum of of the complex wave numbers corresponding to the N wave propagation speeds in that system. Consequently, $|D|$ is unity. Though series combinations of such components retain the same property, general parallel combinations do not. For convenience we term such systems quasi-reciprocal since they tend toward reciprocity at low frequencies.

The classifications passive or active transmission matrices are more immediately relevant to the stability of the systems. A component is considered active if there is a possible state in which there is a net output of fluctuation energy from that component and passive if no such state exists. It is clear that if all elements of a system are passive then the system will be stable. Furthermore, most hydraulic system elements are passive; indeed L,R,C systems are always passive. In contrast, pumps or turbines may be active since they represent possible sources of fluctuation energy; hence the focus in the present paper.

We consider next the conditions for net gain or loss of fluctuation energy in a component considering only those cases of incompressible inlet and discharge flows it follows that the time-averaged flux of fluctuation energy into a hydraulic component $\Delta\tilde{E}$ is given by

$$\Delta\tilde{E}=\tilde{E}_1-\tilde{E}_2=\frac{1}{4\rho_L}\left[\bar{\tilde{m}}_1\tilde{h}_1+\tilde{m}_1\bar{\tilde{h}}_1-\bar{\tilde{m}}_2\tilde{h}_2-\tilde{m}_2\bar{\tilde{h}}_2\right] \tag{3}$$

where ρ_L is the fluid density and the overbar denotes the complex conjugate. Substitution for $\tilde{m}_2,\tilde{h}_2$ from the transmission matrix yields the alternative form

$$\Delta\tilde{E}=\frac{|h_1|^2}{4\rho_L}\left[-A-B\left(\frac{\tilde{m}_1}{\tilde{h}_1}\right)\left(\frac{\bar{\tilde{m}}_1}{\bar{\tilde{h}}_1}\right)+(1-C)\frac{\tilde{m}_1}{\tilde{h}_1}+(1-\bar{C})\frac{\bar{\tilde{m}}_1}{\bar{\tilde{h}}_1}\right] \tag{4}$$

where

$$A = T_{11}\bar{T}_{21}+T_{21}\bar{T}_{11} = \text{purely real} \tag{5}$$

$$B = T_{22}\bar{T}_{12}+T_{12}\bar{T}_{22} = \text{purely real} \tag{6}$$

$$C = \bar{T}_{11}T_{22}+\bar{T}_{21}T_{12} = \text{complex in general} \quad . \tag{7}$$

Note that C is somewhat suggestive of the determinant D; in fact

$$|C|^2 = |D|^2 + AB \quad . \tag{8}$$

From this it is readily shown that the component is

(A) Conservative (i.e. $\Delta\tilde{E} = 0$) for all modes of excitation if and only if $A = B = 0$ and $C = 1$. Therefore not only must it be quasi-reciprocal ($|D| = 1$) but also

$$\frac{T_{11}}{\bar{T}_{11}} = -\frac{T_{12}}{\bar{T}_{12}} = -\frac{T_{21}}{\bar{T}_{21}} = \frac{T_{22}}{\bar{T}_{22}} = D \quad . \tag{9}$$

(B) Completely Passive ($\Delta\tilde{E}>0$) for all modes of excitation if and only if

$$A < 0 \tag{10}$$

$$|D|^2 + 1 = 2RE(C) < 0 \quad . \tag{11}$$

Note that these imply $B < 0$.

(C) Completely Active ($\Delta\tilde{E} < 0$) for all modes of excitation if and only if

$$A > 0 \quad (12)$$

$$|D|^2+1-2Re(c) <0 \quad . \quad (13)$$

In contrast to the completely passive or active conditions (B) and (C) if

$$|D|^2+1-2Re(c) >0$$

the component can be either active or passive depending on the mode of excitation encountered by the component in the system.

4. TRANSMISSION MATRICES FOR CAVITATING INDUCERS

Since a pump may provide a source of fluctuation energy it provides a useful example and one for which transmission matrices have been measured experimentally for a range of frequencies. The values of A,B,C,D have been computed for transmission matrices obtained on two impellers (3 in. and 4 in. diameter models of the low pressure oxygen pump in the Space Shuttle Main Engine) in water over a range of operating conditions given by rpm, flow coefficient, φ, (mean inlet flow velocity/tip speed, U_T) and cavitation number σ (net positive suction pressure/ $\frac{1}{2}\rho_L U_T^2$) and frequency. These have been presented in Refs. 12 and 23.

It transpired that A was negative over the entire range of conditions though it tends to zero for low frequencies and at high cavitation numbers (i.e. in the virtual absence of cavitation). This is to be expected since $T_{11} \to 1$ and $T_{21} \to 0$ under such conditions. Since $A < 0$ the pump is never completely active. Thus the sign of the quantity $G = |D|^2 + 1 - 2Re(C)$ which we will call the "dynamic activity" of the pump (note that G, like D and C, is dimensionless) determines whether the pump is completely passive (negative values) or whether it can be active (positive values). Numerical values for the two pumps are presented in Fig. 1 and 2 as functions of both the actual frequency and a non-dimensional frequency based on tip-speed and blad tip spacing.

In Fig. 1 data is presented for the 3 in. impeller operating at 9000 rpm with a flow coefficient of 0.07 and five different cavitation numbers. The case $\sigma = 0.114$ has very limited cavitation and 0.040 represent a fair degree of cavitation, $\sigma = 0.024$ is on the brink of breakdown and $\sigma = 0.023$ is into breakdown. Note that the dynamic activity is slight until close to breakdown when it rises dramatically. Some subsequent decline in activity following breakdown is also suggested. The data for the larger 4 in. impeller in Fig. 2 (6000 rpm, $\varphi = 0,07$) demonstrates the same trend toward higher activity with greater cavitating though the smallest cavitation number for which data was obtained was 0.044. In both cases there is also a trend toward higher activity at the higher end of the frequency range.

5. SYSTEM STABILITY

Stability of a complete hydraulic system is most readily assessed using the same expression (3) for the net of gain of fluctuation energy; [T] is now the overall transmission matrix for the system.

If the problem involves an open system then it is only necessary to apply one appropriate boundary condition on the fluctuating quantities to the expression (3) or (4). A simple example would be a system originating from a reservoir of constant total head so that $\tilde{h}_1 = 0$. Then it is clear from (3) that the sign of ΔE is identical to the sign of $Re(-\tilde{h}_2/\tilde{m}_2)$ and hence to the sign of $Re(-T_{12}/T_{22})$ which is the real part of the output impedance, $-T_{12}/T_{22}$. Thus the system is stable if $Re(-T_{12}/T_{22}) >0$ and unstable in the reverse circumstances. Similarly a system discharging with a constant total head ($\tilde{h}_2=0$)

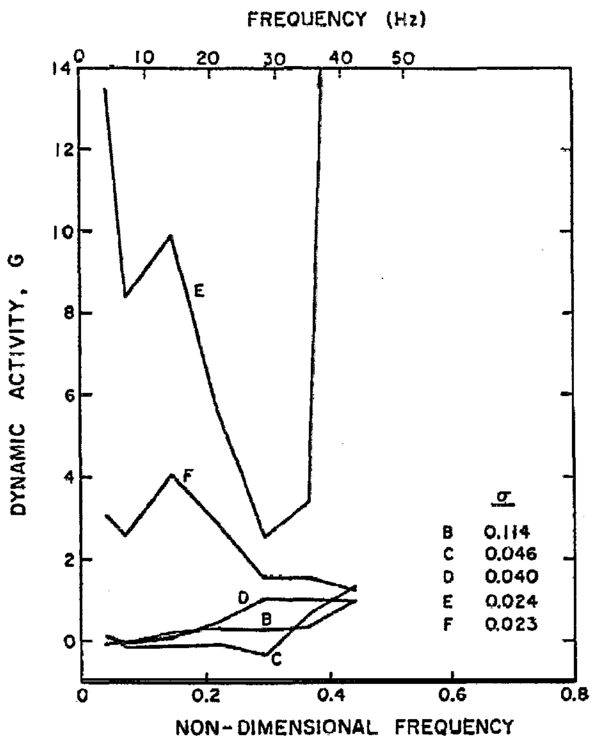

Figure 1. The dynamic activity, G, of the 3 in. impeller operation at 9000 rpm with a flow coefficient, φ, of 0.07 and various cavitation numbers, σ, as shown.

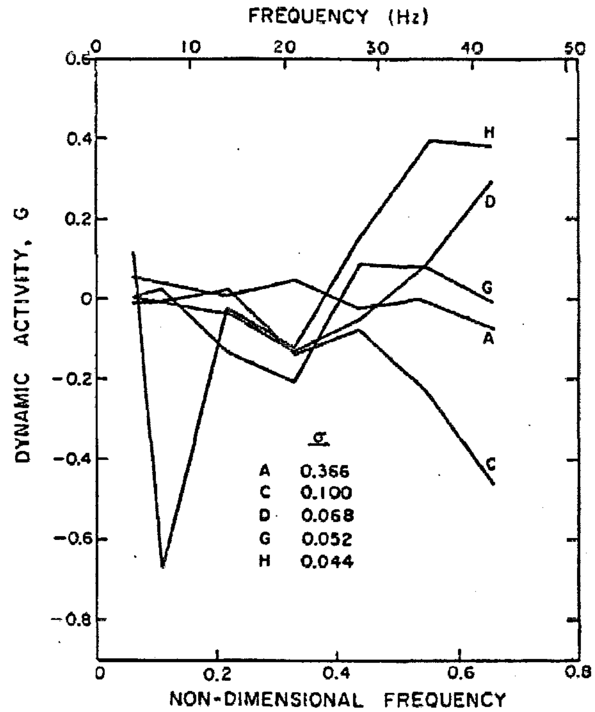

Figure 2. The dynamic activity, G, of the 4 in. impeller operating at 6000 rpm with a flow coefficient, φ, of 0.07 and various cavitation numbers, σ, as shown.

is stable if the real part of the input impedance, $Re(-T_{12}/T_{11})$ is greater than zero.

When the system is closed it should be broken at any arbitrary point so that subscripts 1 and 2 refer respectively to the conditions downstream and upstream of the breakpoint. Then if the fluctuating flow rates across the breakpoint are equated stability is determined by the sign of

$$\Delta\tilde{E} = \frac{|\tilde{h}_1|^2}{2\rho_L} Re \left\{ \frac{\bar{T}_{21}[(1-T_{11})(1-\bar{T}_{11})-T_{21}T_{12}]}{(1-T_{22})(1-\bar{T}_{22})} \right\} \tag{14}$$

which is readily computed from the overall transmission matrix [T]. The crucial term is therefore the numerator in the curly brackets; note that the value of the determinant, d, of the matrix [T]-[I] plays a central role here and that the condition for stability can be reduced to

$$Re(d\bar{T}_{21}) < 0 \quad . \tag{15}$$

6. APPLICATION TO AUTO-OSCILLATION ANALYSIS

The transmission matrices for cavitating inducer pumps used in Section 4 were measured in a closed loop system designed for that purpose and called the Dynamic Pump Test Facility (DPTF) (see Refs. 11,12,15 and 23). A schematic is included here as Fig. 3. In addition to the transmission matrices for the cavitating inducers (denoted by [Y]) the dynamic characteristics of the remainder of the loop have also been measured (Ref. 15). For the purpose of assessment of the stability of this closed loop system it can be

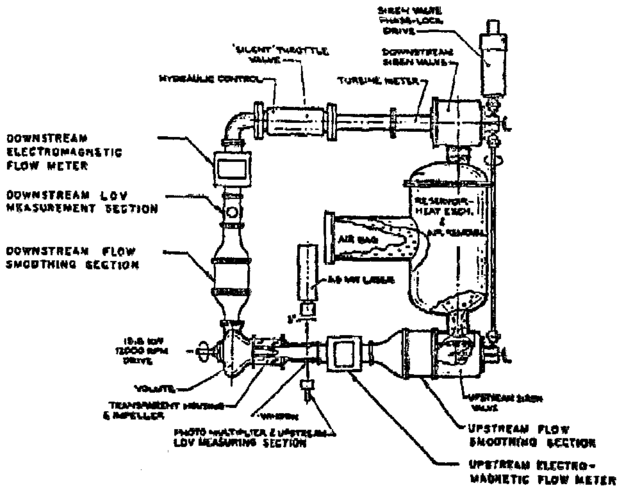

Figure 3. Schematic plan view of the Dynamic Pump Test Facility used for measuring the transmission matrices for cavitating inducer pumps (Refs. 11,12 and 23) and for auto-oscillation studies (Ref. 15)

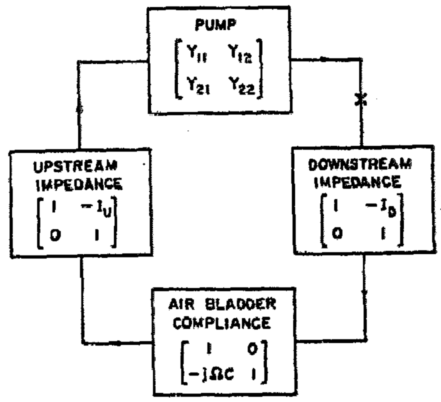

Figure 4. Schematic of the dynamic model used for auto-oscillation prediction and assessment of the stability methodology.

divided into the four elements depicted in Fig. 4, namely (i) the pump (ii) inlet and discharge lines which are dominated by their impedances I_U and I_D respectively which were measured as functions of frequency and various valve settings (Ref. 15) and (iii) the large air bladder used for pressure control which is dominated by its compliance, c(real).

With the arbitrary choice of the breakpoint, X(Fig. 4), just downstream of the pump the transmission matrix [T] immediately follows and substitution into the relation (14) yields

$$\frac{2\rho_L \Delta\tilde{E}}{|\tilde{h}_1|^2} = Re(\bar{F}G/H\bar{H}) \tag{16}$$

$$F = Y_{21}+j\Omega c(I_U Y_{21}-Y_{22}) \tag{17}$$

$$G = (1-Y_{11})(1-Y_{22})+Y_{21}(I_U+I_D-Y_{12})+j\Omega c[Y_{12}+I_U I_D Y_{21}-I_U Y_{11}-I_D Y_{22}] \tag{18}$$

$$H = (I_U+I_D)Y_{21}-Y_{11}+1+j\Omega c(I_U Y_{21}-Y_{22}) \quad . \tag{19}$$

Considerable simplification is effected by the observation that the air bladder compliance, c, is very large so that F,G,H are all cominated by the terms involving $j\Omega c$. Consequently

$$\Delta e \cong \frac{2\rho_L \Delta\tilde{E}}{|h_1|^2} Re \left\{ \frac{I_U Y_{11}+I_D Y_{22}-Y_{12}-I_U I_D Y_{21}}{(Y_{22}-I_U Y_{21})} \right\} . \tag{20}$$

Note that in the absence of cavitation since $Y_{11}\to 1$, $Y_{22}\to 1$ and $Y_{21}\to 0$ the sign of Δe is simply determined by the sign of $Re(I_U+I_D-Y_{12})$, that is to say by the sign of the sum of the resistances of the lines and the pump.

Then if the pump resistance, $Re(-Y_{12})$, which is given at low frequencies by the negative slope of the HQ characteristic of the pump, becomes sufficiently negative (i.e. positive HQ slope) to cause the total resistance to become negative the system becomes unstable. Such cases are known and lead to the surge phenomena observed for example in compressors (Ref. 25) and centrifugal pumps (Ref. 26).

However, it is clear from the form of (20) that in the presence of cavitation a negative Δe can occur even when the resistance is positive (negative HQ slope) depending on the other elements of the pump transmission matrix.

Values of Δe were computed using each of the experimentally measured cavitating pump transmission matrices described in Section 4 and presented in Refs. 11,12 and 23 plus appropriate experimentally measured inlet and discharge line impedances. Due to variable valve settings in both the inlet and discharge lines various combinations could be generated which produced the same total mean flow resistance (matching the mean flow head rise of the pump operating under the conditions at which the transmission matrix was measured) but different impedance functions $I_U(\Omega)$ and $I_D(\Omega)$. In the present system it was found that increasing the contribution of the total in the inlet line tended to stabilize the system; such a trend was also observed during experimental observations of auto-oscillation.

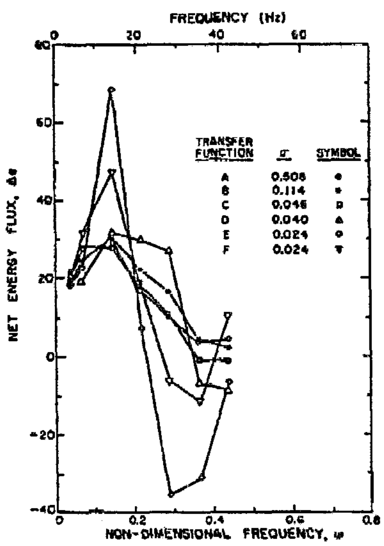

Figure 5. The net energy flux, Δe, as a function of frequency for six cavitation numbers at which transmission functions (A to F) were obtained for the 3 in. impeller at $\varphi = 0.07$ and 9000 rpm.

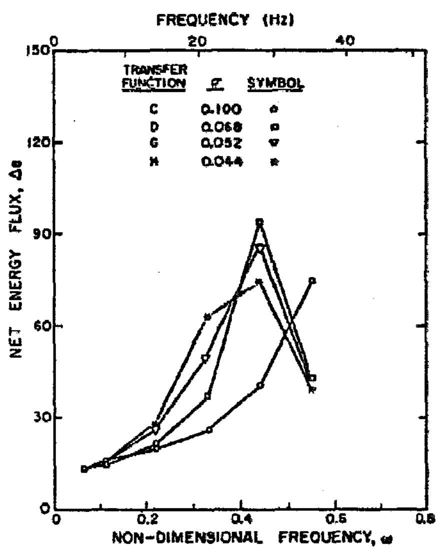

Figure 6. The net energy flux, Δe, as a function of frequency for four cavitation numbers at which transmission functions (C,D,G,H) were obtained for the 4 in. impeller at $\varphi = 0.07$ and 6000 rpm.

For a given distribution of impedances (namely that used during most of the experimental observations of instability) the net energy flux, Δe, varied with cavitation number as shown in Figs. 5,6 and 7. Figures 5 and 7, show the behavior of the system with the 3 in. impeller installed and operating at 9000 rpm, φ , 0.07 and 12,000 rpm, $\varphi = 0.07$; Fig. 6 presents Δe for the 4 in. impeller operating at $\varphi = 0.07$ and 6000 rpm. All of these figures show that the system is stable (positive Δe) when the cavitation in the pump is minimal (large σ). They also demonstrate that the system tends to become

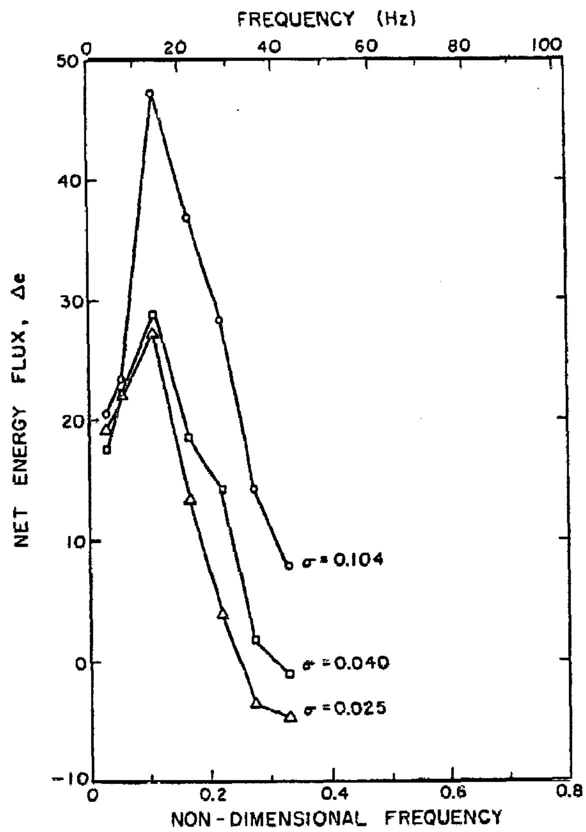

Figure 7. The net energy flux, Δe, as a function of frequency for three cavitation numbers at which transmission functions (G,H and I)were obtained for the 3 in. impeller at $\varphi = 0.07$ and 12000 rpm.

unstable in the higher range of frequencies as the cavitation number is decreased; however it remains stable at the lower frequencies.

7. COMPARISON WITH OBSERVATIONS OF AUTO-OSCILLATION

Experimental observations of the onset of auto-oscillation were also performed for the conditions corresponding to Figs. 5,6 and 7 and are reported in detail elsewhere (Refs. 11 and 15). Though the values of the onset cavitation number, σ_A, from repeated runs performed by slowly reducing of the cavitation number were rather scattered the frequency of auto-oscillation was quite repeatable. Observed results for the 3 in. impeller at $\varphi = 0.07$ were onset conditions in the range $\sigma_A = 0.025$ to 0.035 and at a reduced frequency of 0.3 to 0.35 for both 9000 and 12000 rpm. For the 4 in. impeller onset occurred in the range $\sigma_A = 0.05$ to 0.06 and at a reduced frequency of about 0.55. The predictions from Figs. 5,6 and 7 are qualitatively consistent with these actual observations and thus provide a fair degree of substantiation of the stability methodology. More accurate predictions would have required more transmission matrices giving wider coverage of the cavitation number spectrum. Finally, we note that the range of frequencies and cavitation numbers at which auto-oscillation occurs could be anticipated from the calculated "dynamic activities" presented in Figs. 1 and 2.

8. CONCLUDING REMARKS

This paper has provided some background on the analysis of the dynamic response and stability of hydraulic systems of order two. It has been shown that the potential trouble which a particular component might cause when incorporated in a hydraulic system can be characterized by a quantity called its "dynamic activity". Experimentally measured transmission matrices for cavitating inducer pumps are used as an example to demonstrate that cavitation in a pump can cause such a device to become dynamically active.

Finally a methodology for the analysis of stability of open or closed hydraulic systems is presented. As an example this is applied to a system with a cavitating pump and the predictions compare fairly well with the observed onset of instability (called auto-oscillation in the context of cavitating pumps).

9. ACKNOWLEDGEMENTS

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