On the Unsteady, Dynamic Response of Phase Changes in Hydraulic Systems

CHRISTOPHER BRENNEN
California Institute of Technology
Pasadena, California, USA

1. ABSTRACT

This paper is concerned with the unsteady, dynamic behavior of hydraulic systems and, in particular, with the dynamic characteristics of internal flows involving phase-change and two-phase flows. This emphasis is motivated by the large number of different flows of this kind which exhibit "active" dynamic characteristics (see Section 3) and therefore have the potential to cause instability in the whole hydraulic system of which they are a part (see Section 4). We begin, first, with a discussion of the form and properties of dynamic transfer functions for hydraulic systems. Then, following the discussion of a number of examples we present an analysis leading to the transfer function for a simple phase-change and demonstrate its "active" dynamic character.

2. HYDRAULIC SYSTEM TRANSFER FUNCTIONS

Traditionally, unsteady flow problems in hydraulic systems have been tackled in the time-domain utilizing the method of characteristics [1]. Such methods are often convenient for relatively simple flows for which the differential equation can be constructed with some degree of certainty. On the other hand with a few notable exceptions [2, 3] little use seems to have been made of the other classical approach namely construction of the problem in the frequency domain. One of the underlying themes of this paper is that such an approach can have significant advantages in the analysis of complex, unsteady flows. However, unlike the time domain methods they are limited in practice to small amplitude pertubations on some mean flow.

The basic approach is analogous to that of electrical network analysis. The pressure, p, and mass flow rate, m, at every point in the hydraulic system are subdivided into mean flow components, $\overline{p}$ and $\overline{m}$ which are independent of time, t, and small linear fluctuating components, $\tilde{p}$ and $\tilde{m}$, for each frequency, Ω:

$$p = \overline{p} + \mathrm{Re}\{\tilde{p}e^{j\Omega t}\} \quad ; \quad \overline{m} = \overline{m} + \mathrm{Re}\{\tilde{m}e^{j\Omega t}\} \tag{1}$$

where j is the imaginary unit, Re denotes the real part and $\tilde{p}$, $\tilde{m}$ are complex in general. Alternatively, it is often convenient to use the total pressure, h, and to similarly subdivide it into $\overline{h}$ and $\tilde{h}$.

The next step is to identify the transfer functions, [Y] or [X], for each element of the hydraulic system; this relates the fluctuating pressures at inlet to the element, $\tilde{p}_1$ (or $\tilde{h}_1$) and $\tilde{m}_1$, to the fluctuating quantities at discharge, $\tilde{p}_2$ (or $\tilde{h}_2$) and $\tilde{m}_2$:

$$\begin{Bmatrix} \tilde{p}_2 \\ \tilde{m}_2 \end{Bmatrix} = [Y] \begin{Bmatrix} \tilde{p}_1 \\ \tilde{m}_1 \end{Bmatrix} \text{ or } \begin{Bmatrix} \tilde{h}_2 \\ \tilde{m}_2 \end{Bmatrix} = [X] \begin{Bmatrix} \tilde{h}_1 \\ \tilde{m}_1 \end{Bmatrix} \tag{2}$$

Clearly once [X] or [Y] is known the other can readily be constructed; in some cases the [Y] formulation is more convenient, in others the [X] formulation is preferable. In either case the transfer matrix consists of four complex elements which will, in general, be functions not only of frequency but also of the mean flow conditions within the hydraulic element. If these transfer functions are known for each element in a hydraulic system then one could (i) perform stability analyses (ii) investigate transient response by inverse Fourier transforms and (iii) design appropriate corrective hardware to mitigate problems in stability or transient response.

The major task is therefore to identify the transfer functions for devices such as pumps, evaporators, etc. Before proceeding to discuss some examples it is instructive to examine some basic properties of transfer functions.

3. SOME TRANSFER FUNCTION PROPERTIES

Some basic properties of transfer functions known from four-terminal network analysis for electrical systems [4]:

(i) If the hydraulic element under consideration is entirely conservative with no internal dissipation or production of flow energy then it can readily be demonstrated that the elements of [X] must satisfy the relations

$$X_{11} = d\bar{X}_{11} \quad ; \quad X_{12} = -d\bar{X}_{12} \quad ; \quad X_{21} = -d\bar{X}_{21} \quad ; \quad X_{22} = d\bar{X}_{22} \tag{3}$$

where d is the determinant of [X] and is complex in general and the overbar denotes the complex conjugate. It follows that

$$|d| = 1 \quad . \tag{4}$$

For example the transfer function for frictionless compressible fluid flow in a uniform pipe of length, ℓ, [5] has $d = \exp(2j\Omega\ell M^2/\bar{u}(1-M^2))$ where $\bar{u}$ is the mean fluid velocity and M is the Mach number.

(ii) It is important to note that if $|d| = 1$ (i.e. the system is reciprocal) then it does not necessarily follow that the system is conservative. For example, any composite system comprised of discrete passive elements which are either resistances, R, inertances, L, or compliances, C has $d = 1$ yet energy is dissipated. Furthermore, for many simple single phase flows with conventional fluid mechanical dissipation (such as compressible pipe flow with friction) it transpires that $|d| = 1$, though this may not be true in general.

(iii) It follows from (i) and (ii) that a hydraulic component which has the potential for being dynamically active will exhibit the property

$$|d| \neq 1 \quad . \tag{5}$$

In this paper we shall examine the characteristic transfer functions for some potentially active hydraulic components involving two-phase flow.

4. DYNAMICS OF FLOW WITH PHASE CHANGE

There are many practical and experimental observations of hydraulic system instabilities connected with flows involving phase change or two-phase flows. We will mention here just a few examples for illustrative purposes. Instabilities associated with evaporators and boilers are well known; though they have been the subject of dynamic testing [6,7] complete transfer functions have not as yet been obtained. Instabilities associated with condensing flows have also been observed, most recently in the context of nuclear reactor safety systems. "Condensation oscillations" have been reported in the testing of emergency core cooling systems, pressure relief systems and pressure suppression pool operation. For example in the last case [8] steam escaping in drywell of a B. W. R. will expand down through vent pipes into a pool of water (Fig. 1). Following the initial drywell air-venting period, the steam condenses at some interface in the vicinity of the submerged vent pipe exit. It has, however, been observed that this flow can be quite unstable leading to large amplitude motion of the interface in and out of the vent pipe exit; large amplitude pressure oscillations accompany this so-called "chugging" phenomena [9,10]. Somewhat similar oscillations were observed many years ago in the context of underwater jet propulsion using condensable gases [11].

Furthermore, instabilities are frequently encountered in hydraulic systems involving pumps or turbines which are cavitating. One example of this kind is the "auto-oscillation" phenomenon associated with the operation of cavitating inducer pumps [12 → 17]. Briefly when the overall mean pressure level in the system is reduced until the cavitation in the pump is sufficiently extensive, the system of which the pump is a part can become unstable, resulting in large pressure and mass flow rate fluctuations within the entire system. This behavior is the result of changes in the character of the dynamic transfer function for the pump when cavitation becomes sufficiently extensive [16,17,18].

Recently we have conducted a series of experiments designed to measure dynamic transfer functions for cavitating pumps [16,17,18,19]. The purpose was to provide some knowledge of the dynamic characteristics of such flows so that instabilities, such as the POGO instability [20,21] endemic to all liquid propelled rockets, might be more readily understood and analyzed. Some typical results are presented in Fig. 2 (see [16] for further details). The four elements of the matrix, $[ZP] = [X] - [I]$, (defined for non-dimensionalized fluctuating total pressures and mass flow rates; $[I]$ is the unit matrix) are plotted against frequency (also non-dimensionalized; actual frequencies ranged from 4 → 42 Hz.) for a three inch diameter model of the impeller of the low pressure oxidizer pump in the Space Shuttle Main

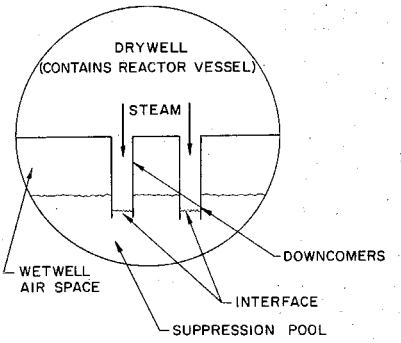

Figure 1. Schematic of BWR pressure suppression system.

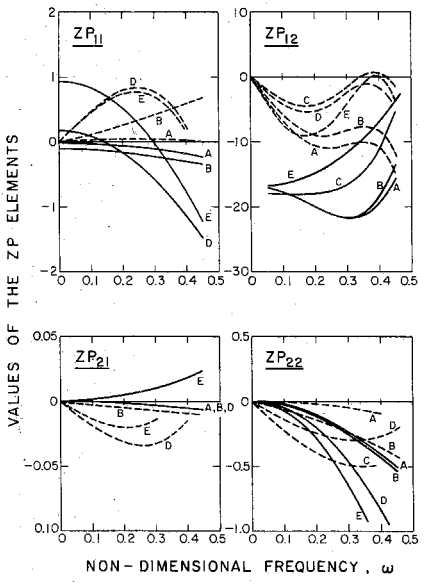

Figure 2. Dynamic transfer functions for a cavitating axial inducer pump. The real and imaginary parts of each of the matrix elements are shown by solid and dashed lines respectively and are plotted against a reduced frequency. The lines are polynomial curve fits to experimental data of reference [16]. The data was obtained at a flow coefficient of 0.07 and a pump speed of 9000 rpm. In the curves A → E the extent of the cavitation is progressively increased (see [16,18,19]).

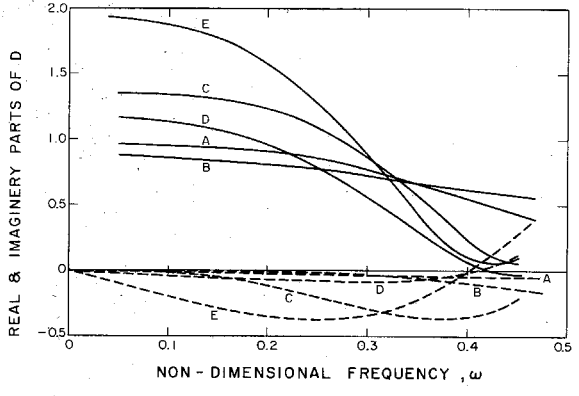

Figure 3. The real and imaginary parts of the determinant, d, for the results of Fig. 2.

Engine. The real and imaginary parts are displayed by solid and dashed lines respectively; for clarity the data is replaced by polynomial curve fits for

different degrees of cavitation in the pump; the extent of cavitation increases through the curves labelled A to E. Though the dynamics are quite complex it can be observed that in the absence of cavitation the data suggests a single non-zero element ZP_{12} which represents the non-cavitating impedance of the pump. This non-cavitating impedance is roughly comprised of a real resistance and an imaginary inertive component increasing with frequency; similar non-cavitating results were obtained by Anderson, Blade and Stevens [22] for a centrifugal pump.

One of the most significant results from these experiments involved the determinant, d, for these transfer functions (Fig. 3). In the absence of cavitation d was close to unity. However, even a modest amount of cavitation was sufficient to cause substantial departure from unity as indicated in Fig. 5. This property of cavitating pump flows clearly indicates why auto-oscillation occurs. Furthermore, these experimental results also demonstrate that some of the older dynamic models for cavitating pumps [21,23] which were based on combinations of discrete resistive, inertive and compliant components (and would therefore have $d = 1$) were inadequate.

More recently we have proposed a simple bubbly flow model [19] which does remarkably well in reproducing most of the basic trends in the transfer functions of Figs. 2 and 3. This model strongly suggests that the active nature of the dynamics arises from the fluctuating production of cavitating bubbles in the vicinity of inlet to the inducer. Though the situation there is even more complex than in condensers or evaporators it seems evident that the dynamic response of this partial phase change is a most significant part of the overall dynamics. One tentative conclusion which can be drawn from all of these examples is that many two-phase flows are capable of exhibiting "active" dynamic characteristics. If we could synthesize the transfer functions for such flows and couple them with the transfer functions for the rest of the hydraulic system then we have available the necessary analytical tools for dealing with those instabilities.

Of course, even under stable conditions many of the nominally steady flows described above are very complicated and not readily amenable to theoretical synthesis (e.g. evaporator, cavitating pump). Consequently, detailed unsteady analysis is often prohibitively complicated. Nevertheless, one of the purposes of this paper is to show that very crude and approximate models of these flows with phase change still indicate qualitative mechanisms through which they may exhibit active dynamic behavior. In the next section we briefly derive the transfer function for a very simplistic phase change. This could be used as a first approximation in synthesizing the dynamics for condensing or evaporating flows.

5. PHASE CHANGE TRANSFER FUNCTION

Consider the characteristic flow with phase change represented in Fig. 4; it is assumed that any steady velocity of the phase change interface has been removed by a suitable Galilean transformation. Then the origin of the coordinate, x, in the direction of flow is the mean position of the interface and the analysis which follows is spatially one-dimensional.

The purpose here is to isolate the transfer function of the interface alone. In order to eliminate incorporation of the dynamic effects of the single phase flows on either side of interface (since these will be manifest in the transfer functions for those single phase flows) we shall consider the transfer function between two fixed boundaries, 1 and 2, in the two different phases which are considered to be infinitesimally close to the origin, $x = 0$. Since the analysis is purely linear the amplitude of the fluctuation in the position of the interface can be considered infinitesimally small in order to realize this configuration.

It is convenient to define phase 1 ($x < 0$) as that in which the mean flow

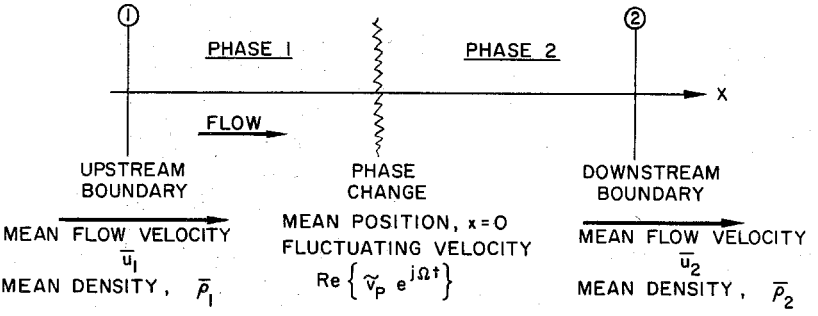

Figure 4. Schematic of the non-inertial x-coordinate system used in the dynamic analysis of a phase change.

is toward the interface (the "upstream" phase) and phase 2 ($x>0$) as the "downstream" phase; the subscript, $i = 1$ or 2, will designate characteristics of the two phases at the boundaries 1 and 2.

The fluid velocities, u_i, mass flow rates per unit cross-sectional area, m_i, pressures, p_i, and densities ρ_i, are divided into mean and fluctuating components in the manner of equations (1). The interface velocity v_p, and temperature, T_p are similarly represented as

$$v_p = Re\{\tilde{v}_p e^{j\Omega t}\} \quad ; \quad T_p = \bar{T}_p + Re\{\tilde{T}_p e^{j\Omega t}\} \quad . \tag{6}$$

Clearly it follows that

$$\bar{p}_1 = \bar{p}_2 = \bar{p}_p \text{ and } \tilde{p}_1 = \tilde{p}_2 = \tilde{p}_p \tag{7}$$

where p_p is the interface pressure. Furthermore, if the interface condition follows the saturated liquid/vapor line then by the Clausius-Clapeyron relation

$$\tilde{p}_p = \left(\frac{\bar{\rho}_v \mathcal{L}}{\bar{T}_p}\right)\tilde{T}_p \tag{8}$$

where $\bar{\rho}_v$ is the mean saturated vapor density and $\mathcal{L}$ is the latent heat at the mean interface temperature, $\bar{T}_p$, or pressure, $\bar{p}_p$.

It follows from conservation of mass that $\bar{m}_1 = \bar{\rho}_1\bar{u}_1 = \bar{m}_2 = \bar{\rho}_2\bar{u}_2$ and, moreover, if $\tilde{m}_p$ represents the fluctuating mass flow rate relative to the interface then

$$\begin{aligned} \tilde{m}_p &= \bar{\rho}_1(\tilde{u}_1 - \tilde{v}_p) + \tilde{u}_1\tilde{\rho}_1 = \bar{\rho}_2(\tilde{u}_2 - \tilde{v}_p) + \tilde{u}_2\tilde{\rho}_2 \\ \tilde{m}_1 &= \tilde{m}_p + \bar{\rho}_1\tilde{v}_p \\ \tilde{m}_2 &= \tilde{m}_p + \bar{\rho}_2\tilde{v}_p \end{aligned} \quad . \tag{9}$$

Now the unsteady heat transport in the vicinity of the phase change must be considered in order to obtain one final relation for $\tilde{T}_p$ in terms of $\tilde{m}_p$. For this purpose we assume that the frequency of the oscillations is sufficiently large so that the thermal convection terms of the equation governing heat transport in the neighborhood of the interface are negligible and the unsteady thermal transport is governed by

$$j\Omega\tilde{T} = D_i \frac{\partial^2 \tilde{T}}{\partial y^2} \quad \begin{cases} y>0 \ , \ i = 2 \\ y<0 \ , \ i = 1 \end{cases} \tag{10}$$

where D_1, D_2 are the thermal diffusivities of the two phases and y is a

modified coordinate measured relative to the instantaneous position of the interface (and is thus not the same as x). A much more complete analysis for lower frequencies at which the convective heat transport must be included will be presented elsewhere [5,24]. However, the present assumptions cover many problems of practical interest. It follows that the solution of equation (10) in the two phases is

$$\tilde{T}\begin{cases} = \tilde{T}_p \exp\left((1+j)\left(\frac{\Omega}{2D_1}\right)^{\frac{1}{2}} y\right), & y<0 \\ = \tilde{T}_p \exp\left(-(1+j)\left(\frac{\Omega}{2D_2}\right)^{\frac{1}{2}} y\right), & y>0 \end{cases} \quad . \tag{11}$$

By equating the fluctuating enthalpy fluxes into and out of the interface the desired relation is obtained:

$$\tilde{m}_p = S\tilde{T}_p \tag{12}$$

where

$$S = \frac{(1+j)}{\mathcal{L}}\left(\frac{\Omega}{2}\right)^{\frac{1}{2}}\{\bar{\rho}_1 c_{p1} D_1^{\frac{1}{2}} + \bar{\rho}_2 c_{p2} D_2^{\frac{1}{2}}\} \tag{13}$$

and c_{pi}, $i = 1,2$ are the specific heats of the two phases. It is of value to point out that since the saturated vapor density, $\bar{\rho}_v$, is many orders of magnitude less than the saturated liquid density, $\bar{\rho}_L$ (unless the temperature, $\bar{T}_p$, is close to the critical temperature for the fluid) then

$$S \approx (1+j)\left(\frac{\Omega}{2}\right)^{\frac{1}{2}} \frac{\bar{\rho}_L c_{pL} D_L^{\frac{1}{2}}}{\mathcal{L}} \quad . \tag{14}$$

The transfer matrix, [Y], for the interface follows directly from (7),(8),(9) and (12):

$$Y_{11} = 1 \quad ; \quad Y_{12} = 0$$

$$Y_{21} = -\frac{(\bar{\rho}_2 - \bar{\rho}_1)\bar{T}_p S}{\bar{\rho}_1 \bar{\rho}_v \mathcal{L}} \quad ; \quad Y_{22} = \frac{\bar{\rho}_2}{\bar{\rho}_1} \quad . \tag{15}$$

6. DISCUSSION OF THE PHASE-CHANGE TRANSFER FUNCTION

The dynamic characteristics of a phase change are thus functions only of the frequency, Ω, and the mean interface temperature $\bar{T}_p$ since, given the latter, the saturated liquid and vapor properties $\bar{\rho}_L, \bar{\rho}_v, c_{pL}, c_{pv}, D_L, D_v$ and $\mathcal{L}$ may be determined. One other physical consideration may however cause exception to this statement. If the thermal diffusivities are governed not by molecular diffusivity, but by turbulent mixing on the scale of the unsteady thermal boundary layer thickness, $(D_L/\Omega)^{1/2}$, then the effective turbulent diffusivity will be determined by factors other than Ω and $\bar{T}_p$. This problem will not be addressed here, but we shall examine the implications of a turbulent mixing length, $\ell_L = D_L/\bar{u}_L$.

Another simple way of representing the dynamics manifest in the transfer function (15) is by means of the model presented in Fig.5. It is readily demonstrated that whether phase 1 is liquid and phase 2 is vapor or vice-versa (i.e. whether the mean flow is condensing or evaporating) the interface dynamics can be represented by a mass flow rate amplifier of amplification $\bar{\rho}_L/\bar{\rho}_v$ and an impedance I^{**} to ground, whose admittance per unit area of the interface, G^*, is given by

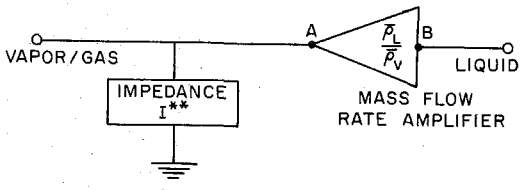

Figure 5. A model of the dynamics of a phase-change at high frequencies. The mass flow rate amplification is such that $\tilde{m}_B = \bar{\rho}_L \tilde{m}_A / \bar{\rho}_v$.

$$G^* = \left(\frac{\bar{\rho}_L}{\bar{\rho}_v} - 1\right) c_{PL} \bar{T}_P D_L^{\frac{1}{2}} (1+j) \left(\frac{\Omega}{2}\right)^{\frac{1}{2}} \mathcal{L}^2 \tag{16}$$

where the simplification (14) has been employed. Notice that this impedance is neither resistive nor compliant but varies with frequency like $(j\Omega)^{-1/2}$; such an impedance is characteristic of unsteady diffusive processes and implies a "receding memory".

If the molecular thermal diffusivity of the liquid is used then $G^* = M^*(1+j)(\Omega/2)^{1/2}$ where

$$M^* = \frac{c_{PL} \bar{T}_P}{\mathcal{L}^2} \left(\frac{\bar{\rho}_L}{\bar{\rho}_v} - 1\right) \left(\frac{k_L}{\bar{\rho}_L c_{PL}}\right)^{\frac{1}{2}} \tag{17}$$

and k_L is the thermal conductivity of the liquid. If on the other hand turbulent mixing dominates the thermal diffusion then the admittance per unit total mass flow rate, G^{**}, is given by

$$G^{**} = G_L(1+j) \left(\frac{\Omega \ell_L}{\bar{u}_L}\right)^{\frac{1}{2}} \quad ; \quad G_L = \frac{c_{PL} \bar{T}_P}{\rho_L \mathcal{L}^2} \left(\frac{\bar{\rho}_L}{\bar{\rho}_v} - 1\right) . \tag{18}$$

For the purposes of demonstrating the large effect on the interface dynamics of changes in the mean interface temperature, the crucial quantities $\bar{\rho}_L/\bar{\rho}_v$, M^* and G_L have been presented in Figs. 5,6 and 7 as functions of $\bar{T}_P$ for various liquids. The non-dimensional temperature used, namely $\theta = (\bar{T}_P - T_T)/(T_C - T_T)$, is one which roughly spans the interval between the triple point temperature, T_T, and the critical point temperature, T_C, for each fluid. For water, sodium, ethane, oxygen and hydrogen these end points to the nearest degree Kelvin are (273,647), (371,2733), (90,305), (54,155) and (14,33) respectively. For Freon-12 and mercury arbitrary values of (115,385) and (234,1823) were used for the purposes of presentation. The primary purpose of Figs. 5,6 and 7 is to show that if the mean interface temperature increases both the amplification factor and the admittance decrease dramatically except close to the critical point temperature.

Returning to the original purpose, we observe that the transfer function for the interface has a determinant whose magnitude is not unity. Consequently the presence of a phase-change in a hydraulic system could cause that system to be unstable over a range of flow conditions and frequencies. It is important to note that, though the value of d for the phase-change transfer function is dominated by $\bar{\rho}_L/\bar{\rho}_v$, the potential "active" character of the entire system of which the phase change is only one part will depend on both the amplification, $\bar{\rho}_L/\bar{\rho}_v$, and the impedance, I^{**}.

It should also be pointed out that a more complete analysis of the transfer function for a phase change has been completed [5]. Among the

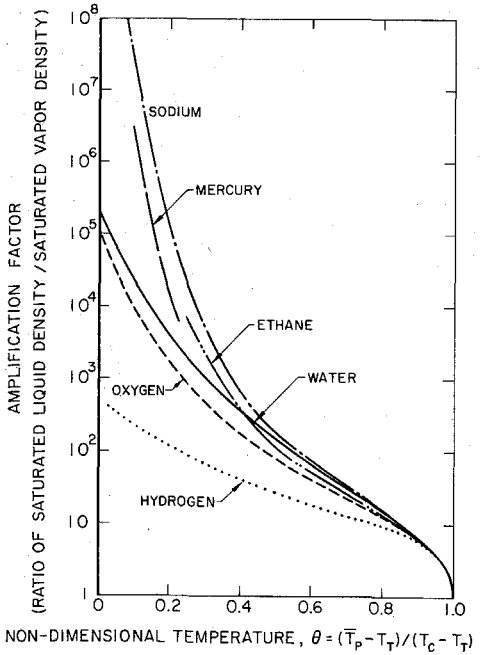

Figure 6. Ratio of saturated liquid to vapor densities, $\bar{\rho}_L/\bar{\rho}_v$, for various fluids as a function of the interface temperature.

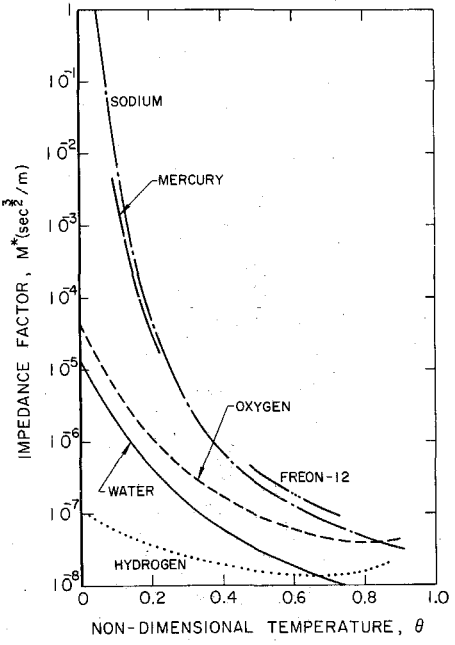

Figure 7. Values of M^* in units of $sec^{3/2}/m$ for various saturated liquids as functions of the interface temperature.

features which are delineated in this more complete study is the lower limit of frequency for which the above analysis is relevant (particularly the assumptions implicit in equation (10)). A typical measure of this lower limit is

$$\Omega \gg \frac{H}{\bar{\rho}_2 c_{p2}} \times \frac{(\text{Perimeter of flow})}{(\text{Area of flow})}$$

where H is the typical heat transfer coefficient in the downstream phase to the walls containing the flow.

Finally, we also observe that the dynamics of a condensing phase change may be radically affected by the presence in the vapor phase of a second "contaminant" gas which is relatively insoluble in the liquid. Such a contaminant gas will tend to accumulate just upstream of the phase-change and inhibit the rate of condensation of the vapor. This will clearly affect the dynamic transfer function for the phase-change. Indeed an analysis of the effect of such a contaminant on the dynamics of a condensation interface has been completed [24]. This shows that the major effect of a small mass concentration, α, of contaminant gas is to supplement the impedance, I^{**}, by a second "contaminant" impedance I_c^{**}. If β denotes the ratio of the molecular weight of the contaminant to that of the vapor then it transpires that for small α:

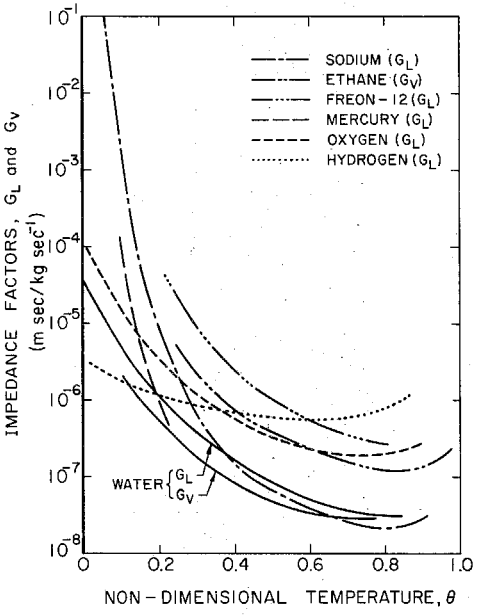

Figure 8. Values of G_L in units of m sec^2/kg for various saturated liquids as functions of the interface temperature. (Some data for a comparable quantity, G_V, in which c_{PV} replaces c_{PL} is also shown).

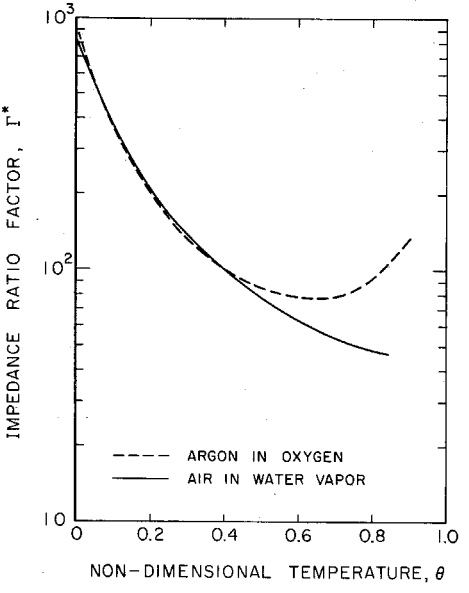

Figure 9. The factor, Γ^*, which determines the effect of a contaminant gas on the impedance of a condensation interface.

$$\frac{I_c^{**}}{I^{**}} = \alpha\beta\Gamma^*$$

where Γ^* is a quantity which is a function of the interface temperature, $\bar{T}_p$, and the thermal and mass diffusivities. Values of Γ^* based on molecular diffusivities are presented in Fig. 9 as functions of $\bar{T}_p$ for two particular fluid systems namely air contaminating a water vapor condensation and argon contaminating an oxygen condensation. The results based on turbulent diffusivities are similar [24]. Thus if β is of the order of unity it follows that the impedance due to the contaminant in these two fluid systems is generally negligible if the mass concentration is less than about 0.1%. At low temperatures mass concentrations of the order of 1% cause the dominant impedance to be I_c^{**}; at higher temperatures relatively larger concentrations of contaminant are required to have the same effect.

7. CONCLUDING REMARKS

In the present paper we have attempted to outline some of the basic features of the unsteady dynamics of flows with phase-change. The analysis in the preceding two sections leading to the transfer function for a phase-change can only be considered a first attempt to understand the hydraulic

system instabilities so often associated with such flows. Clearly the actual form and configuration of most real phase-changes are usually much more complex than the simple form assumed here. In such cases the present analysis can only be considered to provide guidelines as to the form which the transfer function might take. What is really needed are some basic experiments designed to measure the transfer functions in these more complicated cases.

Despite these qualifications, the present analysis clearly indicates why systems with a single phase change (such as the B. W. R. suppression pool system) can become unstable. Indeed if the rest of the elements in such a system are reciprocal the determinant of the composite system made up only of the series combination of the elements (but not parallel combination) will have a spectral radius equal to that of the phase change transfer function.

There are two other points which require consideration in dealing with more complex flows such as those in a cavitating pump. Firstly, since the blade passage flows represent parallel combination of transfer functions it should be noted that though a series combination of reciprocal elements ($|d| = 1$) leads to a composite with the same property the same is not necessarily true for parallel combinations (though it will, of course, be true if the elements are conservative as well as reciprocal). The second point concerns the determinant of a system in which there is phase change to vapor followed later by the reverse (as in a cavitating flow); for a pure series combination the composite determinant should have a spectral radius of unity if all the single phase flows are reciprocal. It is clear from these remarks that the basic reasons underlying the active nature of cavitating pump flows are less readily understood and that further analytical studies are needed.

This work was supported by the National Science Foundation under Grant Eng. 76-11225. Support on the cavitating pump dynamics by NASA under contract NAS 8-28046 is also gratefully acknowledged. The author wishes to thank Professor Allan Acosta for his valuable advice.

REFERENCES

1. Streeter, V. L. and Wylie, E. B. 1967. Hydraulic Transients. McGraw-Hill, New York.

2. Fanelli, M. 1972. Further considerations on the dynamic behavior of hydraulic turbo-machinery. Water Power, June 1972, pp. 208-222.

3. Brennen, C. and Acosta, A. J. 1976. The dynamic transfer function for a cavitating inducer. J. Fluids Eng., Vol. 98, pp. 182-191.

4. Hennyey, Z. and Izsak, N. 1962. Linear electric circuits. Addison-Wesley Publ. Co., Inc.

5. Brennen, C. 1978. The thermo-hydraulic transfer function for a phase change in a flowing fluid system. Submitted for publication.

6. Krejsa, E. A. 1968. Model for frequency response of a forced flow, hollow, single tube boiler. NASA TM-X-1528.

7. Krejsa, E. A., Goodykoontz, J. H., and Stevens, G. H. 1967. Frequency response of forced-flow single tube boiler. NASA TN D-4039.

8. Wade, G. E. 1974. Evolution and current status of the BWR containment system. Nuclear Safety, Vol. 15, No. 2.

9. Class, G. and Kadlec, J. 1976. Survey of the behavior of BWR pressure suppression systems during the condensation phase of a LOCA. Paper contributed to the American Nuclear Society, 1976 International Conference, Nov. 15-19, 1976, Washington, D. C.

10. Koch, E. and Karwat, H. 1976. Research efforts in the area of BWR pressure suppression containment systems. Paper presented at the 4th Water Reactor Safety Research Meeting, Gaithersburg, Maryland, Sept. 1976.

11. Kiceniuk, T. 1952. A preliminary investigation of the behavior of condensable jets discharged into water. Report No. E-24.6, Hydrodynamics Lab., Calif. Inst. of Tech., Pasadena, Calif.

12. Sack, L.E. and Nottage, H.B. 1965. System oscillations associated with cavitating inducers. J. Basic Engr., Vol. 87, Series D, No. 4, pp. 917-925.

13. Young, W.E., Murphy, R. and Reddecliff, J.M. 1972. Study of cavitating inducer instabilities. Pratt and Whitney Aircraft, Florida Research and Development Center, Rept. PWA FR-5131, August 1972.

14. Natanzon, M.S., Bl'tsev, N.E., Bazhanov, V.V. and Leydervarger, M.R. 1974. Experimental investigation of cavitation-induced oscillations of helical inducers. Fluid Mechanics, Soviet Res., Vol. 3, No. 1, pp. 38-45.

15. Kamijyo, K., Suzuki, A., Shimura, T., Hashimoto, R., Watanabe, M., Watanabe, Y., Iwabuch, T., and Mori, Y. 1975. Experimental investigation of small, high-speed, high-head liquid oxygen pump. Nat. Aero. Lab. (Japan), N.A.L. Rept. TR-415.

16. Ng, S.L. and Brennen, C. 1978. Experiments on the dynamic behavior of cavitating pumps. To appear in the J. Fluids Eng.

17. Braisted, D.B. and Brennen, C. 1978. Observations on instabilities of cavitating inducers. ASME Cavitation and Polyphase Flow Forum, June 1978.

18. Brennen, C. 1978. The unsteady, dynamic characterization of hydraulic systems with emphasis on cavitation and turbomachines. Vol. 1, Proc. of Joint ASCE/IAHR/ASME Symp. on Design and Operation of Fluid Machinery, Fort Collins, Colo., June 1978, pp. 97-107.

19. Brennen, C. 1977. Bubbly flow model for the dynamic characteristics of cavitating pumps. Submitted to J. Fluid Mech.

20. NASA, 1970. Prevention of coupled structure-propulsion instability. NASA Rept. SP-8055.

21. Rubin, S. 1966. Longitudinal instability of liquid rockets due to propulsion feedback (POGO). J. Spacecraft and Rockets, Vol. 3, No. 8, pp. 1188-1195.

22. Anderson, D.A., Blade, R.J. and Stevens, W. 1971. Response of a radial-bladed centrifugal pump to sinusoidal disturbances for non-cavitating flow. NASA, TN D-6556.

23. Vaage, R.D., Fidler, L.E. and Zehnle, R.A. 1972. Investigation of characteristics of feed system instabilities. Final Rept. MCR-72-107, Martin Marietta Corp., Denver, Colo.

24. Brennen, C. 1978. The effect of a contaminant gas of the unsteady dynamics of condensation. Submitted for publication.