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The unsteady, dynamic characterization of hydraulic systems with emphasis on cavitation and turbomachines.

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Abstract

The ability to analyze and model the unsteady, dynamic response of hydraulic systems has arisen from the need for operational stability, flexibility and controlled transient behavior. Following a general discussion, this paper discusses experiments and analysis directed toward identification of the dynamic response of hydraulic pumps, both cavitating and non-cavitating. It is shown that rather modest amounts of cavitation cause the pump to become dynamically active and therefore capable of exciting instabilities and resonance with the hydraulic system.

La capacité pour analyser et modeler la réponse dynamique, en régime irrégulier, de systèmes hydrauliques, provient de la nécéssité d'obtenir leur stabilité operationelle, leur flexibilité et de controler leur comportement en régime transitoire. Après une discussion générale, des expériences et analyses effectuées en vue de l'identification de la réponse dynamique de pompes hydrauliques (à la fois cavitantes et non cavitantes) sont discutées dans cet article. Il est montré qu'une pompe peut devenir active de façon dynamique même pour de petites cavitations et est donc capable de susciter des instabilités et d'entrer en résonance avec le système hydraulique.

## I. Introduction

This paper is concerned with the unsteady, dynamic response of hydraulic systems and, in particular, with the rôle played by turbomachines in determining the stability or transient response of such internal flow networks. The need for such information has arisen because of (i) an increasing sensitivity to hydraulic system instabilities for reasons of safety, operational stability and flexibility (ii) requirements for the analyses of the response of hydraulic systems to externally imposed transients and (iii) a need for analytical models for use in the design and positioning of corrective hardware.

The basic approach adopted here is analogous to that of electric circuit analysis. The pressure, $p$, and mass flow rate, $m$, at every point in the hydraulic system are subdivided into mean flow components, $\bar{p}$ and $\bar{m}$, which are independent of time, $t$, and fluctuating components, $\tilde{p}$ and $\tilde{m}$, for each frequency, $\Omega$:

$$p = \bar{p} + Re\left\{\tilde{p}\, e^{j\Omega t}\right\} \quad ; \quad m = \bar{m} + Re\left\{\tilde{m}\, e^{j\Omega t}\right\} \tag{1}$$

where $j$ is the imaginary unit, $Re$ denotes the real part, and $\tilde{p}$, $\tilde{m}$ are complex in general. As the above implies, the analysis will be confined to small, linear perturbations about the mean flow. Furthermore, any general transient response will be synthesized from knowledge of the system response for individual frequencies, $\Omega$. For this purpose, we require the dynamic transfer function for each element in the hydraulic system. This transfer function or matrix, $[Z]$, relates the fluctuating pressure and mass flow rate at discharge from the element, $\tilde{p}_2$ and $\tilde{m}_2$, to those quantities at inlet, $\tilde{p}_1$ and $\tilde{m}_1$; in hydraulic systems $[Z]$ is most conveniently chosen to be defined as

$$\begin{Bmatrix} \tilde{p}_2 - \tilde{p}_1 \\ \tilde{m}_2 - \tilde{m}_1 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{Bmatrix} \tilde{p}_1 \\ \tilde{m}_1 \end{Bmatrix} \tag{2}$$

The transfer matrix, $[Z]$, will be a function not only of frequency, $\Omega$, but also of the mean flow conditions in the hydraulic element. A few simple examples will help our interpretation later of more complex transfer functions. The flow of an incompressible fluid in a rigid, uniform pipe will have only one non-zero matrix element, namely $Z_{12}$; $-Z_{12}$ is the impedance of the pipe. At low frequencies it will be comprised of $R + j\Omega L$ where $R$ is the frictional resistance obtained from the mean flow friction factor and $L$ is the inertance due to the acceleration of the mass of fluid in the pipe. At higher frequencies it will become a complex function of frequency which will reflect the non-trivial behavior of the boundary layers under fluctuating conditions. On the other hand a simple compliance consisting of a volume of gas communicating pressure-wise with the flow in a duct will yield a non-zero value of $Z_{21}$ of the form $-j\Omega C$ where $C$ is known as the compliance. The electrical analogy is a capacitor to ground.

There are several general properties of the transfer matrix, $[Z]$, which are particularly important to discuss at this point and they involve the value of the determinant, $D$, of $[Z] + [I]$ where $[I]$ is the unit matrix:

(a) Any simple dynamic system which is described by L, R and C elements will, by the theorem of reciprocity, have a determinant, D, equal to unity. Note that such a description implies an infinite wave propagation speed. All such systems are dynamically passive.

(b) In so far as the author has been able to determine the value of D for any passive system with a <u>finite</u> wave propagation speed is complex in general, but its spectral radius, $|D|$, is still equal to unity. This is true, for example for the flow of a compressible fluid in a uniform duct (which, incidentally, has a transfer function in which all four elements are non-zero).

Several conclusions can be drawn from the above observations. First, it is clearly important to concentrate attention on those elements in a hydraulic system which are dynamically "active" and therefore capable of exciting instabilities and resonances within the system. Such elements can therefore be identified either experimentally or analytically as those in which the spectral radius of the determinant, D, is <u>not</u> equal to unity. Pumps and turbines could clearly be active dynamic elements and the remainder of this paper will concentrate on the dynamic transfer function for pumps (Fanelli (1972) was among the first to consider transfer functions for pumps). It is also worth noting that two-phase flows involving phase change also exhibit active transfer matrices (e.g. Brennen (1978a)). Hence particular attention will be paid transfer matrices for cavitating pumps.

## 2. Dynamic Transfer Functions for Pumps

Though the need for dynamic information on pumps extends to many hydraulic systems, it has been most intensively studied in the context of liquid-propelled rockets because of the need for analysis of the POGO instability endemic to all such rockets (e.g. Rubin (1966), Vaage, Fidler and Zehnle (1972), Rocketdyne Report (1969)). Most of the early attempts to synthesize the dynamics of the fuel and oxidizer turbopumps were essentially quasi-static. The pump resistance was obtained from the slope of the steady state head rise versus flow rate curve at the operating point; to this was added some estimated inertance to obtain the quasi-static pump impedance. In one of the few previous experimental investigations, Anderson, Blade and Stevens (1971) measured the dynamic impedance of a non-cavitating centrifugal pump for frequencies up to 50 Hz. Their results depart from the quasi-static values at quite low frequencies, the resistance increasing and the inertance decreasing with frequency.

Under cavitating conditions two other non-zero elements were generally added; the slope of the steady state head rise versus inlet pressure yielded some value for $Z_{11}$ (the "pressure gain" term) and some estimated volume of cavitation in the pump was used to obtain a "cavitation compliance", $C$, for the pump so that $Z_{21} = -j\Omega C$ (see for example Rubin (1966), Vaage, Fidler and Zehnle (1972)). Later Brennen and Acosta (1976) pointed out that the quasi-static approach applied to the cavitating case would lead not only to a a non-zero $Z_{21}$ but also to a value for $Z_{22}$. The mean operating state within the pump is described by the flow coefficient, $\varphi$ ($=U_A/U_T$ where $U_A$ is the mass average axial velocity of the flow at inlet and $U_T$ is the impeller tip speed) and the cavitation number $\sigma = (\bar{p}_1 - p_v)/\frac{1}{2}\rho U_T^2$ where $p_v$ is the vapor pressure and $\rho$ the liquid density. The head coefficient, $\psi$ (defined at the total pressure rise across the pump divided by $\rho U_T^2$) is then a function of $\varphi$ and $\sigma$. Also the volume of cavitation is a function primarily of $\varphi$ and $\sigma$. Then under quasi-static fluctuations $(\tilde{m}_2 - \tilde{m}_1)$ is simply related to the rate of change of this volume with time. Brennen and Acosta (1976) utilized a cavitating cascade model with fully developed blade cavities to evaluate (i) the change in volume with inlet pressure (or $\sigma$) which leads to a value of the compliance, $C$ ($Z_{21} = -j\Omega C$) and (ii) the change in volume with angle of attack (or $\varphi$) which leads to a value of the "mass flow gain factor", $M$ ($Z_{22} = -j\Omega M$).

The above analysis has two limitations. First it is purely quasi-static and departures from this might be expected at fairly low frequencies. Secondly, cavitation in a pump occurs in a number of forms (see, for example, Brennen (1973)); bubbly cavitation is the most common type and blade cavities, if they occur at all, are usually accompanied by considerable bubbly cavitation. Relatively little analytical research has been done on the dynamics of bubbly or backflow cavitation though in some preliminary studies (Brennen (1973)) it was shown that streams of cavitating bubbles do have an identifiable compressibility or compliance. However, it is clear that the most valuable information at the present time will come from experimental measurements of the dynamic transfer functions of turbomachines; in the next section we describe the results of such an experiment.

## 3. Experimental measurement of pump transfer functions

A facility fabricated for the purpose of measuring the transfer function of cavitating and non-cavitating axial flow pumps was built at Caltech and is shown diagrammatically in Fig. 1. Details of this facility can be found elsewhere (Ng (1976), Ng and Brennen (1978)). The fluctuating pressures and mass flow rates were measured at inlet and discharge for a range of fluctuation frequencies from 4→42 Hz. Fluctuating pressures were measured with pressure transducers. The most difficult experimental problem was the measurement of the fluctuating mass flow rates and laser doppler velocimeters (LDV) were employed for this purpose (see Ng (1976)).

The fluctuations were imposed on the mean flow by means of two fluctuators (siren valve devices) driven at a prescribed frequency by a reference signal generated electronically. Two fluctuators are necessary in order to excite the system in a number of linearly independent modes at the same frequency and mean operating state, $(\sigma, \varphi)$; by inspection of the form of the transfer function it is clear that measurements of the fluctuating pressures and mass flow rates for at least two linearly independent modes are necessary before one can solve for the matrix elements, $[Z]$. By adjusting the relative amplitudes and phase of the two fluctuators, three or more linearly independent mode measurements were actually made and the transfer functions were obtained by a least-squares fitting procedure.

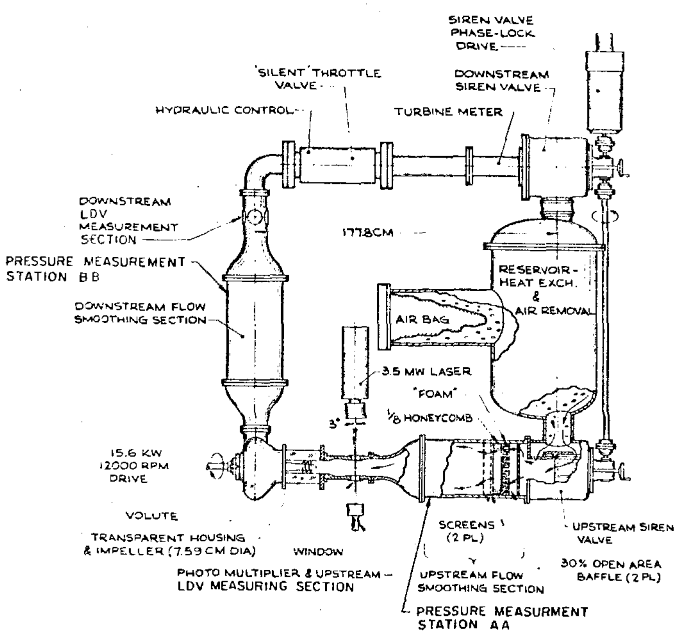


Figure 1. Schematic plan view of the dynamic pump test facility.

The typical levels of fluctuation in the mass flow rate were about 2% of the mean mass flow rate; the corresponding pressure fluctuations were a few percent of the mean total pressure rise across the pump. Tests showed that these levels were small enough to be considered linear perturbations.

Considerable effort was expended to ensure the accuracy of the LDV measurements of fluctuating flow rate. Prior to these measurements the flow was smoothed by means of screens and honeycombs followed by 9:1 converging nozzles in order to assure uniform velocity profiles at both measuring stations and to reduce the turbulence there to a level of about 1/2% (this is particularly difficult at discharge and accounts for the large downstream flow smoothing section). A signal to broad-band noise ratio of about 4:1 was obtained. However, by cross-correlating the returning fluctuating signals with the basic reference signal driving the system the effect of the noise was substantially reduced. For details of this, many other ancillary measurements (accelerations, shaft axial motion and fluctuating rotating speed) and the processing of the data see Ng (1976) and Ng and Brennen (1978).

Though a number of impellers have been tested, we concentrate here on the results for a 1/4 scale model of the low-pressure oxidizer pump in the Space Shuttle Main Engine (Impeller IV). The transfer functions for other impellers are qualitatively similar suggesting that the results are rather insensitive to the particular geometry (indeed it is expected that centrifugal pumps would have similar transfer functions). The steady state performance of this model is shown in Figs. 2 and 3 and compares favorably with the full scale impeller performance (Rocketdyne Division, Rockwell International, personal communication) obtained later. The relevant data in Fig. 2 is that with the stator.

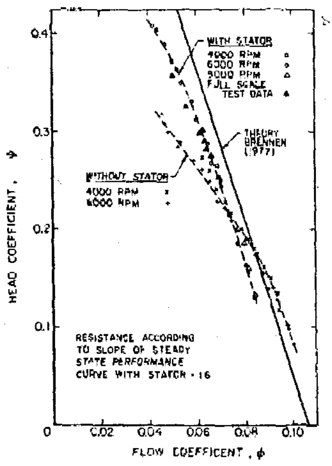


Figure 2. Non-cavitating performance of Impeller IV with and without the stator for various rotative speeds. Also shown are full-scale test data for this impeller (Rocketdyne Division, North American Rockwell, personal communication) and the theoretical, loss-less performance curve (Brennen (1977)).

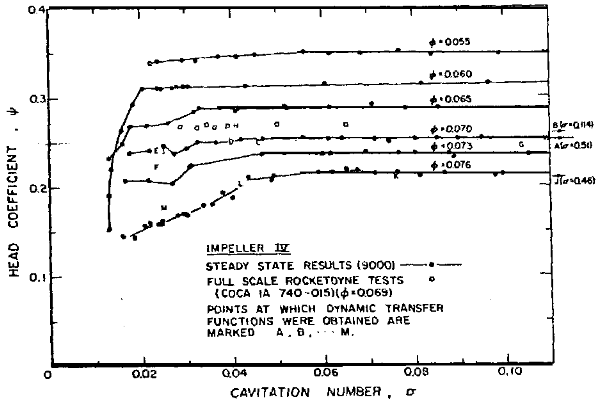


Figure 3. Measured steady state cavitation performance for Impeller IV at 9000 rpm and various flow coefficients, φ, as indicated (———). Also shown are the results of some full scale tests for φ = 0.069 performed by Rocketdyne (COCA 1A 740-15). The points at which dynamic transfer functions were obtained are shown by the letters; test A⇒F were performed at 9000 rpm and φ≈0.070; test G⇒I at 12,000 rpm and φ≈0.070; tests J⇒M at 9000 rpm and φ≈0.076.

Transfer functions were obtained over a range of mean operating states at points shown by letters in Fig. 3. It is interesting to note that the present hydraulic system became unstable without external fluctuation within a range of operating states near breakdown. This auto-oscillation behavior will be the subject of future study (Braisted and Brennen (1978)) but clearly reflects the active nature of the pump dynamics in this range of operation. Only one attempt was made to measure a transfer function in this range (point E).

## 4. Results

The results are presented non-dimensionally by generating a transfer function relating non-dimensional pressures and mass flow rates, defined by dividing by $\frac{1}{2}\rho U_T^2$ and $\rho U_T A_1$ respectively where $A_1$ is inlet area of the pump. The reduced frequency, $\omega$, is defined as $\Omega H/U_T$ where H is the blade tip spacing. Furthermore, the following results are for transfer functions, [ZP], in which instantaneous total head is used instead of static pressure.

Since transfer matrices for individual mean operating points appear elsewhere (Ng (1976), Ng and Brennen (1978)), a summary of the results for Impeller IV is presented in Fig. 4. The four elements of the matrices are each plotted against frequency, the real and imaginary parts of each element being given by the solid and dashed lines respectively. Simple polynomials in the Laplace variable $j\omega$ have been fitted through the actual data for clarity of presentation in Fig.4. Results are presented which show the progressive changes occurring as the cavitation number is reduced, beginning with the non-cavitating results for the operating point A (Fig.3). The corresponding values for the determinants, D, are shown in Fig. 5.

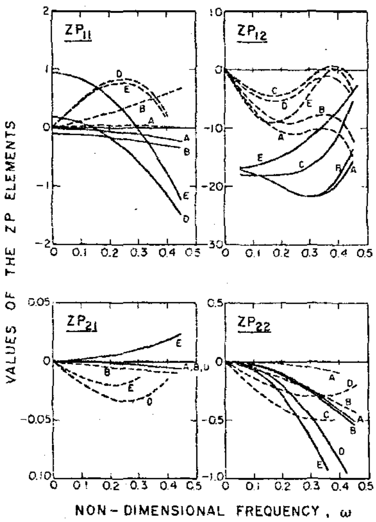


Figure 4. Polynomial curve fitting to experimental pump transfer matrices, [ZP], obtained for Impeller IV at $\varphi = 0.070$ and a rotational speed of 9000 rpm by Ng and Brennen (1978). The real and imaginary parts of the matrix elements are presented as functions of frequency by solid and dashed lines respectively. The letters A to E are the designations used in Fig.3 and denote matrices taken at five, progressively diminishing cavitation numbers, $\sigma$, as follows: (A) 0.508 (B) 0.114 (C) 0.046 (D) 0.040 (E) 0.023.

Two general conclusions can be drawn from this data. First the departure from quasi-static behavior occurs at rather low reduced frequencies of the order of 0.05. For example, under conditions of little or no cavitation (B or A) the resistance (real part of $-ZP_{12}$) first increases and then decreases with frequency and the inertance (imaginary part of $-ZP_{12}/\omega$) also tends to decrease with frequency (these effects are similar to those observed by Anderson, Blade and Stevens (1971)). Under non-cavitating conditions (A) the other elements should be zero.

However, even a modest amount of cavitation (C, D) clearly causes significant changes in all of the matrix elements. Of particular interest is the reduction in both the resistance and inertance at higher frequencies. Though both compliance and mass flow gain factor effects can be discerned (imaginary parts of $ZP_{21}$ and $ZP_{22}$) the changes are clearly more complex than these simple models.

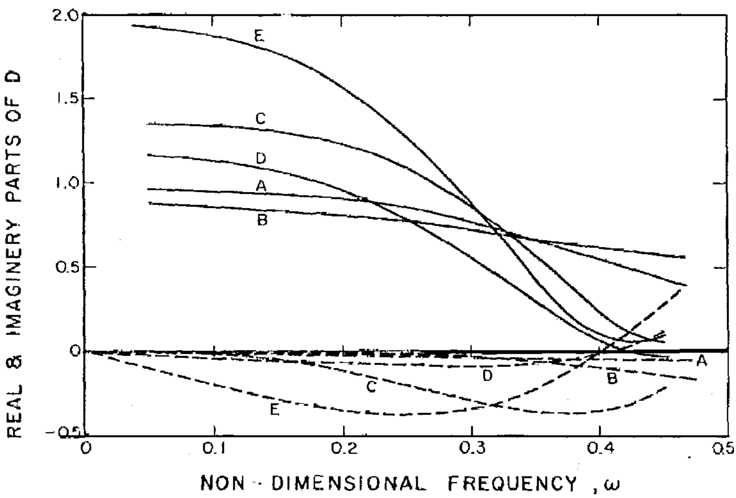


Figure 5. The determinants, D, for the results of Fig. 4; solid and dashed lines again represent real and imaginary parts respectively.

The results of Fig. 5, clearly demonstrate one of the most important conclusions. Though the pump acts like a dynamically passive device (with $D \approx 1$) under non-cavitating conditions even a modest amount of cavitation is sufficient to cause the pump to become dynamically active. It should be noted from Fig. 3 that this occurs prior to any significant change in the steady state performance of the pump. Such a conclusion has important consequences for the stability of hydraulic systems involving pumps and clearly demonstrates the need for dynamic pump models which incorporate this active dynamic characteristic.

## 5. Theoretical Bubbly Flow Model

It has been established earlier (Brennen (1973)) that a stream of cavitating bubbles exhibits a compressibility or compliance when subject to global pressure fluctuations. This compliance remains real provided the fluctuation frequency is well below the natural resonant frequency of the bubble nuclei entering the cavitating region. Since these latter frequencies are usually much larger than the fluctuation frequencies of concern here we may regard the cavitating bubbly flow as similar to a two-phase mixture with some finite compressibility and sonic velocity. Thus the imposed fluctuations give rise to dynamic waves which propagate through the bubbly mixture in the blade passages of the impeller at sonic velocity. Since this sonic velocity is difficult to determine analytically with any degree of accuracy it is represented in the theory by a compressibility parameter, K (see Brennen (1978b)).

But the two-phase bubbly mixture in the blade passages is also subject to kinematic (or continuity) waves. These are generated by a fluctuating rate of production of cavitation bubbles at the inlet to the passages caused by the fluctuations in the angle of attack. The factor of proportionality is difficult to determine analytically and is therefore represented by a parameter, M, in the theory.

Consequently a simple one-dimensional model of the bubbly flow in the blade passages was constructed and its dynamic response analyzed. Only a fraction, $\epsilon$, of the total length of the blade passage contains this bubbly flow. This fraction increases as the cavitation number, $\sigma$, is decreased and is therefore used in place of the mean flow variable, $\sigma$, for the purpose of presentation of the results in this paper. Thus, the flow through the impeller is divided into four parts and dynamic relations for each part are used to synthesize the dynamics of the pump: (i) the relations between the upstream inlet fluctuations and those at entrance to a blade passage (ii) the bubbly flow region within a blade passage (iii) the single phase liquid flow in the remainder of the blade passage following collapse of the cavities and (iv) the relations between the fluctuations at the end of a blade passage and the downstream conditions. Despite these complications it transpires that the overall transfer function for the pump is predominantly determined by the response of the bubbly region and the interactions between the upstream and downstream fluctuations caused by the dynamic and kinematic waves propagating through this region.

Some typical analytical transfer functions based on the model are presented in Fig. 6 for increasing values of $\epsilon$ (decreasing $\sigma$ ). The qualitative trends in the data with increasing frequency, $\omega$, are relatively independent of the choice of the parameters K and M, though of course the quantitative magnitudes do depend on the particular values of these parameters.

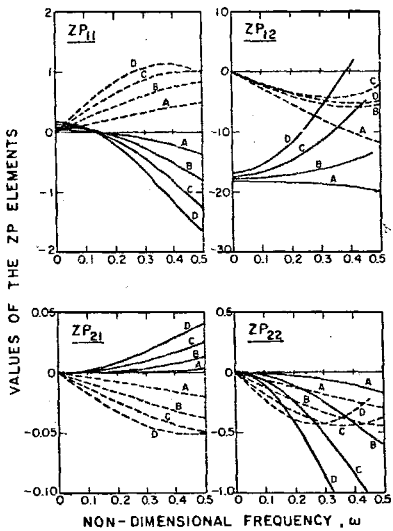


Figure 6. Theoretical pump transfer matrices ,[ZP], for Impeller IV at $\varphi$ = 0. 07 as functions of reduced frequency, $\omega$. The lettered curves are for different fractional lengths, $\epsilon$, of the bubbly region and correspond to decreasing cavitation numbers, $\sigma$: (A) $\epsilon$ = 0. 2 (B) $\epsilon$ = 0. 4 (C) $\epsilon$ = 0. 6 (D) $\epsilon$ = 0. 8. The curves are for one specific choice of the parameters, K and M (see Brennen (1978b)).

Comparison with Fig. 4 suggests that this simple model has reproduced most of the important trends in the transfer function as both the frequency and cavitation number are changed. Since the individual elements of the transfer functions show satisfactory agreement, the analytical determinants are in similar agreement with the experimental results of Fig. 5.

At present the merit of this analytical model is that it suggests qualitative reasons for the mechanics underlying the experimental results. For example, the "active" nature of the dynamics arises from the formation of kinematic waves as the angle of attack fluctuates. Without this mechanism the pump dynamics would be passive even under cavitating conditions. More detailed numerical comparison between theory and experiment requires further theoretical work on the values of K and M and further experimental exploration of the detailed processes occurring in the blade passages.

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