CHAPTER 7

CHANNEL FLOWS OF GRANULAR MATERIALS AND THEIR RHEOLOGICAL IMPLICATIONS

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7.1 BASIC FEATURES OF CHANNEL FLOWS OF DRY GRANULAR MATERIAL

7.1.1 Introduction

While the flow of a dry granular material down an inclined channel may seem at first sight to be a relatively simple flow, the experiments which have been conducted up to now suggest sufficient complexity to warrant further analysis and investigation. They also suggest complexities which may be present in all but the very simplest granular material flows; consequently it is important to our general understanding of granular material rheology that these experimental observations be fully understood. This review of the current knowledge of channel flows will focus on the basic mechanics of these flows and the contributions the observations have made to an understanding of the rheology. In order to make progress in this objective, it is necessary to avoid some of the complications which can occur in practice. Thus we shall focus only on those flows in which the interstitial fluid plays very little role in determining the rheology. In his classic paper, Bagnold (1954) was able to show that the regime in which the rheology was dominated by particle/particle or particle/wall interactions and in which the viscous stresses in the interstitial fluid played a negligible role could be defined by a single, Reynolds-number-like parameter. It transpires that the important component in this parameter is a number which we shall call the Bagnold number, Ba, defined by

$$Ba = \rho_s d^2 \delta / \mu_F$$

where ρ_s, μ_F are the particle density and interstitial fluid viscosity, d is the particle diameter and δ is the principal velocity gradient in the flow. In the shear flows explored by Bagnold δ is the shear rate. Bagnold (1954) found that when Ba was greater than about 450 the rheology was dominated by particle/particle and particle/wall collisions. On the other hand, for $Ba < 40$, the viscosity of the interstitial fluid played the dominant role. More recently Zeininger and Brennen (1985) showed that the same criteria were applicable to the extensional flows in hoppers provided the extensional velocity gradient was used for δ. This review will focus on the simpler flows at large Ba where the intestitial fluid effects are small.

Other important ancilliary effects can be caused by electrical charge separation between the particles or between the particles and the boundary walls. Such effects can be essential in some flows such as those in electrostatic copying machines. Most experimenters have observed electrical effects in granular material flows, particularly when metal components of the structure are not properly grounded. The effect of such electrical forces on the rheology of the flow is a largely unexplored area of research. The lack of discussion of these effects in this review should not be interpreted as a dismissal of their importance.

Apart from electrical and interstitial fluid effects, this review will also neglect the effects caused by non-uniformities in the size and shape of the particles. Thus, for the most part, we focus on flows of particles of spherical shape and uniform size. It is clear that while an understanding of all of these effects will be necessary in the long term, there remain some important issues which need to be resolved for even the simplest granular material flows.

7.1.2 Mass and Momentum Equations for Open Channel Flow

Perhaps the best starting point is the existing body of knowledge of open-channel flows for incompressible liquids. Thus Savage (1979) and Brennen *et al.*(1983) identify different types of granular material chute flow distinguished by a Froude number, Fr, defined in the present paper as

$$Fr = U/(gh)^{\frac{1}{2}} \tag{1}$$

where $U = u_m$ is the cross-sectionally-averaged velocity, h is the depth of flow and g is the acceleration due to gravity. The basic equations which govern the open-channel flow of any substance whether liquid or granular material are those of conservation of mass and a momentum equation. If the depth of the flow, h, varies slowly with position, z, along the channel then the flow can be represented by a velocity, u, parallel to the base and a solid fraction, α, which may be functions of the coordinate y measured perpendicular to the base (the origin of y is taken at the free surface). It is convenient to define profile shape parameters a, b and c as follows.

$$a = \int_0^1 \frac{\alpha}{\alpha_m} \frac{u}{U} d\left(\frac{y}{h}\right) \tag{2}$$

$$b = \int_0^1 \frac{\alpha}{\alpha_m} \left(\frac{u}{U}\right)^2 d\left(\frac{y}{h}\right) \tag{3}$$

$$c = \int_0^1 \frac{2p}{\rho_s \alpha_m g h \cos\theta} d\left(\frac{y}{h}\right) \tag{4}$$

where α_m is the cross-sectionally average solid fraction, ρ_s is the particle density and θ is the inclination of the channel to the horizontal. Note that if the solid fraction, α, and velocity, u, were uniform over the depth then $a = b = 1$. Moreover we define c so that if the pressure, p (normal stress in direction parallel to the base), varied hydrostatically over the depth then the third shape parameter, c, would also be unity.

If we assume for simplicity that the profiles of α, u and p are sufficiently self-similar so that a, b and c are simple constants then we can proceed with a conventional open-channel flow analysis in which, for steady flow, the conservation of mass requires that

$$d\left(\alpha_m h U\right)/dz = 0 \tag{5}$$

and, utilizing this, the equation of momentum can be written as

$$f = \frac{\tau P}{\rho_s \alpha_m g w h \cos\theta} = \tan\theta + \frac{h}{\alpha_m}\frac{d\alpha_m}{dz}\left(\frac{bFr^2}{\cos\theta} - \frac{c}{2}\right) + \frac{dh}{dz}\left(\frac{bFr^2}{\cos\theta} - c\right) \tag{6}$$

where τP is the shear force per unit channel length between the material and the walls and w is the width of the flow. Note that since $\rho_s \alpha_m g h$ is a typical normal stress at the channel base, the factor f is similar to a Coulomb friction factor. Note also that the momentum equation reduces to a more familiar form if the solid fraction is assumed constant ($d\alpha_m/dz = 0$) and the shape factors b and c are assumed to be unity (Patton *et al.*(1987)). The conventional behavior of an open-channel flow can be readily extracted from the above equation (6). In the simpler case of $d\alpha_m/dz = 0$, $b = c = 1$, it is clear that the evolution of the flow as it proceeds down the channel depends on whether the flow is subcritical ($Fr < 1$) or supercritical ($Fr > 1$). For example, if the angle θ is greater than a particular value given by f, if f remains constant as a Coulombic friction law might suggest and, if the flow is supercritical ($Fr > 1$), then the depth, h, will continuously decrease (since $dh/dz < 1$). Thus the flow will continuously accelerate and Fr will get larger still. In flows of liquid this process is terminated because f increases with velocity and hence the flow reaches an equilibrium in the sense that the depth and velocity no longer change with z. In the present paper the term "fully developed" is used to refer to such circumstances. In granular material flow the processes leading to fully developed flow are not well understood. We shall delay further discussion on this important issue until the basic concepts have been introduced.

Another consequence which is familiar in open channel flows or compressible flows is that, if one postulates a fully developed flow, then there exist two flows with the same flow rate (two "conjugate" states, one subcritical and the other supercritical) which satisfy the governing equations. This was confirmed analytically by Richman and Marciniec (1988) who searched for steady, fully developed solutions for the flow of granular material down "bumpy inclines." For a fixed flow rate, the range of inclinations for which fully-developed flow was possible was obtained. For certain combinations of inclination and flow rate, two conjugate solutions were found, one dilute and fast, the other dense and slow.

When an open channel flow is supercritical, kinematic waves cannot propagate upstream and so the evolution of the flow does not depend on downstream conditions or obstacles. What frequently happens under these circumstances is that a kinematic shock or "hydraulic jump" occurs in which the flow undergoes a transformation from a supercritical state ($Fr > 1$) to a subcritical state ($Fr < 1$). Hydraulic jumps have been observed in granular material flows and have been studied by Savage (1979), Morrison and Richmond (1976) and Brennen *et al.*(1983). If the upstream supercritical state and the downstream subcritical state are respectively denoted by subscripts 1 and 2 then application of the same mass and momentum equations leads to the following relation across the shock (Brennen *et al.*(1983)):

$$2Fr_1^2 \left\{ \left(\frac{b_2}{a_2^2}\right)\left(\frac{h_1}{h_2}\right) - \frac{b_1}{a_1^2} \right\} = c_1 - c_2 \frac{h_2^2}{h_1^2}. \tag{7}$$

The equations used by Savage (1979) and Morrison and Richmond (1976) are examples of this relation for special values of the shape parameters a, b and c; for example, Morrison and Richmond assume $a = b = 1$, $c_1 = (1 - \sin\phi)/(1 + \sin\phi)$, $c_2 = (1 + \sin\phi)/(1 - \sin\phi)$ where ϕ is the internal friction angle. Brennen *et al.*(1983) show that the experimental observations of hydraulic jumps in granular material agree with the form of equation (7).

Granular jumps are most commonly manifest in the following way. Most of the experimental facilities (Savage (1979), Ishida and Shirai (1979), Brennen *et al.* (1983), Johnson *et al.* (1990)) use a sluice-gate-like control valve to regulate the flow from a supply hopper to the open channel. At larger channel inclinations this generates a supercritical flow which continues to accelerate if unobstructed. However, when a sufficiently large obstruction is placed in the channel, it will create a granular jump which will propagate upstream like a hydraulic bore. If the obstruction is large enough the jump will propagate all the way to the sluice-gate and result in a substantial decrease in the flow rate. Lesser obstructions may cause the jump to find an equilibrium position some distance upstream of the obstruction (Brennen *et al.* (1983)). In either case, if the obstruction is removed, the flow will revert to its unobstructed state through an expansion wave which propagates upstream from the location of the obstruction and cancels out the hydraulic jump.

Before progressing further with a discussion of the flow of a granular material down an inclined chute it is necessary to examine two other relations which are needed to complement the mass and momentum equations. First we must discuss the concept of granular temperature (section 7.1.3) for it is through that concept that one can follow the constitutive laws (section 7.1.4) which relate the solid fraction, α, to the stresses and the velocity field. Secondly some discussion of the boundary conditions both at a solid wall and at the free surface (section 7.1.5) is necessary for these are clearly more complex than in conventional fluid mechanics.

7.1.3 Concept of Granular Temperature and its Conduction

One of the most important characteristics of rapid granular flow is the "granular temperature" which is a measure of the energy contained in the random or incoherent motions of the particles in the material. Clearly, the velocity of an individual particle can be decomposed into the mean or ensemble-averaged velocity and the fluctuation velocity associated with the departures from this mean. The mean-square value of these fluctuation velocities is commonly referred to as the granular temperature, and is analogous to the thermal motion of molecules in the kinetic theory of gases. As discussed in Campbell (1990), two mechanisms are responsible for the generation of granular temperature. The first mechanism is the collision between particles. The rapid flow of granular materials is characterized by high deformation rates and this rapid shearing motion of the flow causes collisions between particles. Since the directions of their resultant velocities after collision are dependent on the impact angle, their rotational velocity and particle properties, the collisions produce random velocity components. The other mechanism is a by-product of the random motions when these occur in a non-uniform flow field. A particle which moves in the direction of a velocity gradient yields departures from the local mean velocity. This "streaming" mechanism can only generate random velocity in the direction of the mean flow while the collisional mechanism yields components in all directions. Consequently, one would expect some anisotropy in the granular temperature especially at low densities where the streaming mechanism is more effective than the collisional mechanism as observed by Walton and Braun (1986a, b), Campbell and Gong (1986) and Campbell (1989).

Though many studies of granular materials draw on the analogy with the kinetic theory of gases, there are important differences between granular materials and gas molecules. One of the major differences is that the collisions between granular particles are inelastic. Thus fluctuation energy associated with the granular temperature is always dissipated by interparticle collisions. On the other hand, one of the similarities between granular materials and gas molecules is that granular temperature may be conducted by a temperature gradient. That is, fluctuation energy can be transferred from a region of high fluctuation energy to a region of low fluctuation energy through "granular conduction." Later we will discuss two mechanisms of granular conduction. For the present it is sufficient to observe that a local fluctuation energy equation is required to represent the balance between the generation of fluctuation energy due to shear motions, the dissipation due to inelastic collisions and the transport due to granular conduction. For example, when the fluctuation energy generated by shear motion is greater than the energy dissipation, the granular temperature of the system increases and/or excessive energy is conducted out of the system. In the opposite case, the granular temperature decreases and/or energy is conducted into the system from the outside.

For simple shear flow, no granular conduction is expected. In this case, the dissipation rate is uniform and equal to the production rate. Furthermore, the local magnitude of the granular temperature is influenced only by the local production of fluctuation energy, which is a direct result of the velocity gradient. Therefore, the granular temperature can be characterized by the velocity gradient alone. In channel flow, however, there are large gradients of granular temperature (Campbell and Brennen (1985) and Ahn *et al.* (1989b)). Therefore, the fluctuation energy is conducted along the gradient of granular temperature and the local granular temperature cannot be characterized by the local velocity gradient alone. In channel flow the direction of granular conduction plays a significant role in determining the profile of solid fraction and other characteristics of the flow (Ahn *et al.* (1989b)). If the fluctuation energy is being conducted from the bulk of flow to the channel base (that is, if granular temperature in the bulk is higher than near the channel base) then the solid fraction decreases monotonically with a distance from the base. On the other hand, if the energy is being conducted from the channel base to the bulk, the solid fraction is low at the channel base, reaches a maximum toward the center, and then decreases near the free surface. Ahn *et al.* (1989b) show that the direction of granular conduction in fully developed flow is determined by the channel inclination and the coefficient of restitution between particles.

In spite of its importance, the granular temperature had not been investigated experimentally until Ahn *et al.* (1988, 1989a) employed fiber optic probes to measure one of its components, namely the velocity fluctuation in the direction of the mean flow. Future developments will hopefully allow a more complete study of granular temperature in many different granular material flows.

7.1.4 Rheological Behavior of Granular Materials

Bagnold (1954), in his simple analyses and experiments of a cylindrical shear cell, distinguished three different regimes of flow behavior, namely, macroviscous, transitional, and grain inertia regimes, according to the importance of the role of the interstitial fluid in determining the dynamics of granular materials. In the grain inertia regime, where the interstitial fluid plays a minor role, the behavior of a granular flow is governed largely by direct interactions between particles. In this case, following Bagnold's argument, both the particle collisional rate and momentum transfer during the collision are proportional to the shear rate and, as a result, the stresses depend on the square of the shear rate.

Like Bagnold's experiments, most of stress measurements have been made in annular shear cells because the measurement technique in the shear cell is relatively simple compared to other experimental devices. The reader is referred to Savage (1978), Savage and McKeown (1983), Savage and Sayed (1984), Hanes and Inman (1985) and Craig *et al.* (1986). Savage and Sayed (1984) found that at the lower solid fractions and high shear rates both normal and shear stresses were proportional to the particle density and to the square of the shear rate and particle diameter. At higher solid fractions and lower shear rates, the stresses were found to be proportional to the shear rate raised to a power less than two. One of characteristics of the friction coefficient (the ratio of shear stress to normal stress) was that the friction coefficient was a weak function of the shear rate and that it increased with decreasing solid fraction. Campbell (1989) presented the results of computer simulations of an imposed simple shear flow with inelastic spheres. The stresses were normalized by the particle density and the square of the particle diameter and the shear rate, and were characterized by a "U" shape when presented as functions of solid fraction, with asymptotes toward large values as the solid fraction approached both zero and the shearable limit. The relative importance of the streaming (or kinetic) and collisional modes of momentum transfer was examined. The collisional mode dominated at high concentrations where collisions are frequent and the streaming mode dominated at low concentration where particles travel long distances between collisions. As a result, anisotropic behavior of the normal stresses was observed with decreasing concentration. The friction coefficient was also found to increase with decreasing solid fraction, mainly due to the streaming contribution.

For simple shear flow, all the theoretical analyses predict the same behavior which is a natural extension of that originally proposed by Bagnold, namely,

$$\tau_{ij} = \rho_s f_{ij}(\alpha) d^2 \left(\frac{du}{dy}\right)^2$$

where τ_{ij} is the stress tensor, ρ_s the particle density, f_{ij} a tensor function of the solid fraction α, d the particle diameter and du/dy the local mean shear rate. That the stress should be proportional to the square of the shear rate can be simply demonstrated by dimensional analysis. In the grain inertia regime where the interstitial fluid effect is negligible, the only available dimensional quantities are ρ_s, d, du/dy, the velocity fluctuations (or the square root of granular temperature) and the material properties of the particles. For simple shear flows in which the granular temperature is uniform, fluctuation energy is generated by the shearing motion and dissipated by the inelastic collisions. Thus, the magnitude of granular temperature is linearly proportional to the square of the shear rate. Savage and Jeffrey (1981) introduced the parameter S defined as

$$S = \frac{d\frac{du}{dy}}{T^{\frac{1}{2}}}$$

where T is the granular temperature. Lun *et al.* (1984) have evaluated S for a simple shear flow as a function of the solid fraction and coefficient of restitution between particles.

Channel flows are more complicated because the granular temperature may vary not only over the depth of the flow but also with position along the channel. The local granular temperature can not be described only by the local mean shear rate, the solid fraction, and the coefficient of restitution, but the fluxes of fluctuation energy (or granular temperature) in all directions have to be considered. Thus, the rheological behavior of the channel flow is not as simple as that of the simple shear flow and the above rheological model for simple shear flows does not hold for channel flows. Therefore, by studying the rheological behavior of the channel flow, more fundamental rheological models (not limited only to a simple case) can be examined.

7.1.5 Boundary Conditions at the Free Surface and the Wall

The conventional treatment of a solid boundary condition in fluid mechanics is the no-slip condition in which the fluid "particles" in contact with the wall cannot move relative to the wall. However in granular flow, there may be slip at a solid boundary. Furthermore, the boundary of the system generates granular temperature through shear work at a rate equal to the product of the slip velocity and the shear stress at the wall. At the same time, collisions of particles with the wall boundary dissipate fluctuation energy. Therefore, the boundary of the system may serve as either a source or a sink of fluctuation energy, depending on whether the generation or the dissipation dominates. These boundary conditions for granular flow can not be obtained independently of the flow field and studies of the slip velocity and the fluctuation energy flux at the boundary face many difficulties, some of which remain to be resolved (Hui *et al.* (1984), Jenkins and Richman (1986), Johnson and Jackson (1987), Richman and Chou (1988), Richman (1988) and Gutt and Haff (1988)).

The other type of boundary is a free surface and it is clear from the experimental observations that there are two types of free surface (Ahn *et al.* (1989a and b)). For dense and slow flows (perhaps all subcritical flows), the free surface is clearly defined. The material near the surface is being sheared either very slowly or not at all and particles are not thrown up out of the bulk. On the other hand for dilute and fast flows (perhaps all supercritical flows) particle saltation causes a substantial density gradient at the surface so that the precise location of the free surface is difficult to define. In a time-averaged view the solid fraction tends asymptotically approaches zero.

When the velocities at the base and free surface of a channel flow are compared, different base surface conditions yield quite different results. Various surface conditions have been explored by Bailard (1978) who used a surface onto which grains were glued, by Savage (1979) who attached roughened rubber sheets to the surface and by Ishida and Shirai (1979) who attached very rough sandpaper. In all these cases of rough base surface, the ratio of the velocity at the base, u_w, to the velocity at the free surface, u_s, was close to zero. Augenstein and Hogg (1978) obtained a spectrum of values for u_w/u_s

ranging from zero to values near one as the size of the particles glued to the base ranged from larger than the particles in the flow to much smaller than the particles in the flow. In the computer simulations of Campbell and Brennen (1985b) in which a smooth surface with high friction coefficient was employed, the ratio of u_w to u_s was about $0.4 \sim 0.5$. The high friction coefficient ensured a no-slip condition at the contact surfaces; however the particles at the wall still had non-zero slip velocities at their centers. Ahn *et al.* (1989a) have presented experimental measurements of the velocity ratio, u_w/u_s, as a function of the solid fraction for different surface conditions (see Figure 7-1).

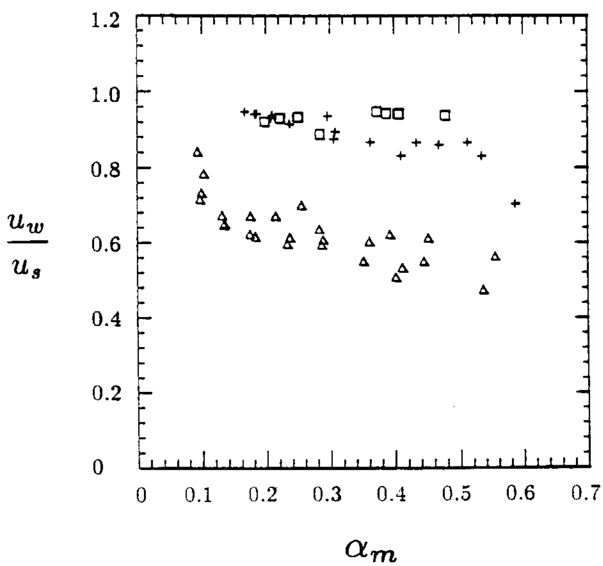

FIGURE 7-1. The ratio of velocity at the chute base to velocity at the free surface, u_w/u_s, presented as a function of the mean solid fraction, α_m. $\square$, the smooth surface; $+$, the moderately smooth surface; $\triangle$, the rubberized surface.

For a smooth aluminum surface, the ratio remains fairly constant and is independent of the solid fraction. For a surface coated with a thin rubber layer (the rubberized surface), the ratio gradually increases as the solid fraction decreases; an abrupt increase at a solid fraction of about 0.1 also occurred. Despite these data, the present state of knowledge does not allow prediction of the slip at the wall. Indeed the features of the surface or of the flow which determine the slip are not well understood.

The surface conditions also influence velocity fluctuations at the wall. If the friction between the particle surface and the wall surface is high, there will be no slip at the contact point during a collision and the particle will acquire substantial rotational velocity as a result of the collision with the wall. Subsequent collisions of this particle with other particles will cause some of this rotational energy to be converted to translational energy or granular temperature. On the other hand, if the friction at the collisional contact between the particle and the wall is small, slip will occur and relatively little rotational velocity will be imparted to the particle, resulting in lower granular temperatures. Therefore, the higher the friction, the larger the granular temperature near the wall. This is observed in the experiments of Ahn *et al.*(1989a), in which the measured velocity fluctuations were larger for the rubberized surface with a high Coulombic friction coefficient than for the smooth aluminum surface with a low Coulombic friction coefficient. Therefore, for rapid flows rough surfaces can result in flows of lower density and higher granular temperature than would be the case for smooth surfaces.

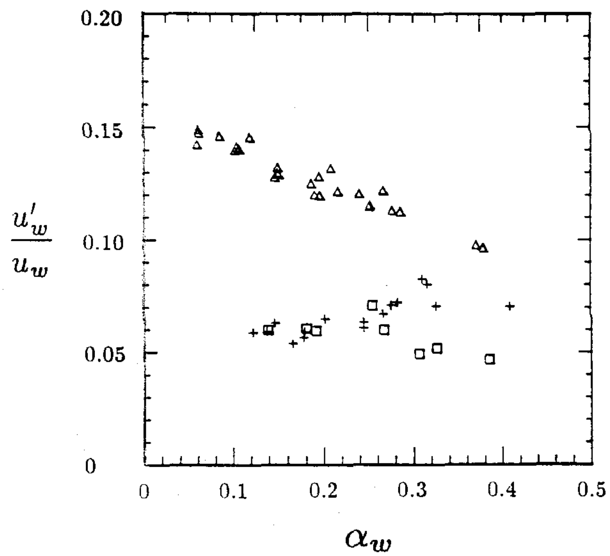

FIGURE 7-2. The longitudinal velocity fluctuation at the chute base normalized by the mean velocity, u'_w/u_w, against wall solid fraction, α_w. □, the smooth surface; +, the moderately smooth surface; △, the rubberized surface.

We will now turn to discussion of the fluctuation velocities themselves. Ahn *et al.*(1989a) present measurements of the velocity fluctuation near the base, u'_w, for various channel base surface conditions. This data is plotted against the solid fraction in Figure 7-2. For the smooth aluminum surface, the ratio of u'_w to the slip velocity, u_w is small and fairly constant. Note that u'_w itself increases with the decreasing solid fraction but the ratio, u'_w/u_w, appears to be constant. On the other hand u'_w/u_w for the rubberized surface is large and increases with decreasing solid fraction. These observations suggest the following explanation of why u_w/u_s for the rubberized surface abruptly increases at low solid fraction (Figure 7-1). The rubberized surface is characterized by large velocity fluctuations particularly at lower solid fractions as shown in Figure 7-2. The large fluctuations and the low solid fraction allow particles to move more freely from one location to another. One of the consequences is a decrease in the velocity gradient because when the particles move from a layer with low mean velocity to an adjacent layer with high velocity, the mean velocity of the adjacent layer is reduced. On the other hand when particles move from the upper layer with the higher mean velocity to the layer underneath they increase the mean velocity of the lower layer. This may help explain the large increase in u_w/u_s at low solid fractions exhibited in Figure 7-1.

7.1.6 Evolution of an Open Channel Flow of Granular Material

Having introduced and discussed some of the basic equations of open channel flows of granular material, it is appropriate at this point to pause in order to identify several key issues which remain unresolved. At the end of section 7.1.3 we briefly discussed the manner in which open-channel flows may become fully-developed. In the case of conventional open-channel flows of liquids, the flow becomes fully developed with the forces acting on an elemental length of the fluid reach equilibrium so that the depth and velocity no longer change with distance down the channel. For convenience we refer to this as a state of force-equilibrium. In liquids such force equilibria often occur because the shear stress, τ, increases with velocity. In granular material flow the variation of the shear stress with velocity, u, is less clear and consequnetly the circumstances under which force equilibrium may be reached are not obvious. If we assume that the wall friction, τ, is Coulombic so that f is a simple constant then equation

(6) would predict that equilibrium could only occur for one particular inclination, θ, given by $\tan^{-1} f$. Experiments (Bailard (1978), Ahn *et al.*(1989a and b)) seem to suggest that such a conclusion is too simplistic since τ depends on other factors such as the solid fraction, α.

But the evolution of a granular material flow depends on more than the momentum considerations of equation (6). Since the granular temperature affects the solid fraction and the stresses, it is also necessary that the flow reach "thermal" equilibrium as well as force equilibrium. In other words, in a fully developed flow the processes of production, conduction and dissipation of granular heat must also reach equilibrium before the stresses and the solid fraction become invariant with position, z. While the experimental evidence is not, as yet, conclusive, recent results (Johnson *et al.*(1990), Ahn *et al.*(1989a and b)) suggest that in a granular material flow the process of reaching thermal equilibrium is slower than the process of reaching force equilibrium so that the evolution is governed primarily by the former process.

It follows that one should observe different flows depending on how the granular material is supplied at the entrance to the channel for then the initial thermal state will be different. Johnson and Jackson (1990) have recently explored this feature in a valuable experimental study. They observed that for low mass flow rates the fully developed flow depends only on the mass flow rate with other conditions fixed and that it is independent of the entry conditions (how materials are fed into the channel). At higher mass flow rates it was unclear whether the flows in their channel were indeed fully developed. But unlike the low mass flow case, the flows far from the entry depended on both the flow rate and the entry conditions.

Johnson *et al.* (1990) also discuss how the flow develops as the granular material enters the channel. Because of difficulties in measuring the depth of flow, h, the nondimensionalized mass hold-up, m_T^*, defined as $m_T^* = \alpha_m h/d$ (instead of the solid fraction or velocity) was presented as a function of the nondimensionalized mass flow rate, $\dot{m}^*$. Since fully developed flow was desired, the channel was set at relatively small inclinations ($12° \sim 18°$); for larger inclinations the flow would accelerate over the length of the channel. Two methods of feeding granular material to the channel were investigated; "loose entry" and "dense entry." To achieve loose entry conditions, particles were dropped into the channel from a height, resulting in an energetic initial state at the channel entrance. Dense entry conditions were obtained by using a conventional sluice-gate which produced quite unenergetic initial conditions. For a given channel inclination and at low mass flow rates both loose and dense entry conditions yielded the same fully developed flow. With loose entry conditions, the energetic initial flow decelerated and the flow depth increased as the flow approached the fully developed state. On the other hand, with dense entry conditions, the unenergetic flow accelerated and the depth increased. However, as the mass flow rate increased, the two methods of entry yielded different results. In both cases the density and the depth of flow increased, but the increases in the case of dense entry were much larger than for loose entry. Likewise, the decrease in the velocity was much greater for dense entry. Further increase in the mass flow rate led to densities which were similar for both entry conditions, but the depth of flow for the dense entry was much larger, resulting in a lower velocity for the same mass flow rate. At the higher channel inclinations, dense entry flows were observed to be supercritical at low mass flow rate but subcritical at high mass flow rates. These results clearly suggest that channel flows of granular material can only be understood when the processes of production, dissipation and conduction of granular temperature are fully understood.

7.2 PROFILES OF CHANNEL FLOWS

7.2.1 Measurement of Profiles in Channel Flows

Few experimental studies of the detailed profiles of channel flows are reported in the literature because of the obvious difficulties involved in making point measurements of velocity, solid fraction and granular temperature within the interior of granular flows. Ridgway and Rupp (1970) used a horizontal knife-edge to split the channel flow into layers and, by measuring the mass flow rate in each layer, derived density profiles assuming uniform velocity. Augenstein and Hogg (1978) obtained vertical velocity distributions over the depth by examining the trajectories of sand particles leaving the channel exit. Bailard (1978) measured profiles in channen flows of sand with a layer of sand grains glued to the base of the channel. He used a horizontal splitter-collector assembly which was a combination of the techniques of both Ridgway and Rupp (1970) and Augenstein and Hogg (1974, 1978). By estimating the velocities from the trajectories of the particles at the end of the chute, the solid concentrations were calculated from the mass flux profiles. The concentration profiles showed a maximum at approximately mid-depth with reduced solid fractions both at the base and at the free surface. The velocity profiles were almost linear. More recently Drake and Shreve (1986) have taken high-speed motion pictures of nearly steady, uniform flows through the side walls of a channel. Their narrow channel was designed so that flows of a single particle width could be examined. Shirai *et al.* (1977) introduced a small probe made of three optical fibers of 0.2 mm in diameter to measure the particle velocity within the granular bed. Ishida and Shirai (1979) inserted fiber optic probes into the channel flow. In general, insertion of fiber optic probes causes interference with the flow, resulting in an unacceptable alteration of the flow (see Johnson and Jackson (1990)). The surface was covered with very rough sandpaper, and the velocity at the channel base was close to zero. Except for the region near the base, the velocity profile appeared to be linear, and the velocity gradient increased as the channel inclination became higher. Savage (1979) used fiber optic probes to measure the velocity of flow through glass side walls. Note, however, that the flow profile was probably affected by the presence of the side wall. Roughened rubber sheets were applied to the surface of the channel base and the velocity profile showed an inflexion point. Near the free surface, particle passages past the probes were infrequent, indicating the saltation of particles in the low density surface cloud. Johnson *et al.* (1990) also made similar efforts to obtain velocity profiles at the side walls using fiber optic probes. Both rough and smooth base surface conditions were investigated. Velocity profiles for both supercritical and subcritical flows were obtained for a smooth base. The rough base lined with a layer of sandpaper resulted in higher velocity gradients than the smooth base.

Ahn *et al.* (1989a) also employed fiber optic probes to obtain profiles of the velocity, solid fraction and one component of velocity fluctuation at the side walls. As shown in Figure 7-3(a), the velocity profiles were linear. It was also observed that the velocity gradient was higher for a rubberized base than for a smooth base. Profiles of the velocity fluctuation in the flow direction were presented for the first time. Figure 7-3(b) shows that the profile is fairly linear and that the fluctuations are larger at the free surface than at the channel base. These overall features are in contrast to the results obtained by Campbell and Brennen (1985b).

In their computer simulation, granular temperature near the solid

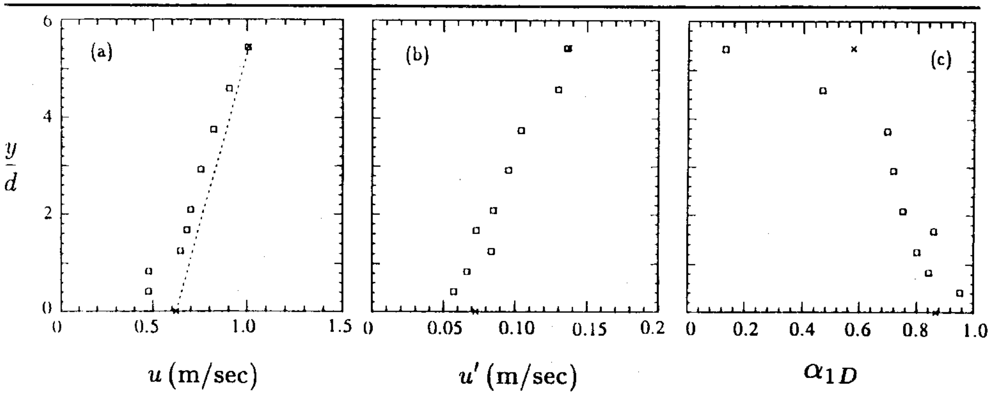

FIGURE 7-3. Vertical profiles at the side wall, □, for the (a) mean velocity, (b) velocity fluctuation, and (c) linear concentration as a function of coordinate y normalized by the particle diameter, d. Also shown is similar data at the center of the chute (×). Dotted line is the assumed velocity profile at the center of the chute. Data were taken at $\theta = 17.8°$ on the rubberized surface; $\alpha_m = 0.30$, $h_o = 25.4$ mm, and $d = 3.04$ mm.

wall was substantially higher than near the free surface and the profile was far from linear. Ahn *et al.* also observed that the gradient of velocity fluctuations for the rubberized surface was larger than for the smooth surface. The profile of linear concentration which is a measure of the solid fraction is presented in Figure 7-3(c). In general, the linear concentration decreases monotonically except for the regions affected by the "corner effect." (The "corner effect" is that particles in the corner made of the channel base and the sidewall are arranged in a distinct line which has high solid fraction and low velocity.) The gradual decrease of the linear concentration near the free surface makes it difficult to define the location of the free surface. The shape of this density profile is quite different from those of Bailard (1978) and Campbell and Brennen (1985) where the density near the base is lower than in the center of the flow.

Compared to computer simulations of simple shear flows or Couette flows, relatively little work has been done on the computer simulation of gravity flows. Campbell and Brennen (1985b) simulated channel flows with two-dimensional disks, presenting the profiles of velocity, solid fraction, and granular temperature over the depth of flow, as shown in Figure 7-4. Their simulations employed periodic boundaries, which imply steady, fully developed flow. The results

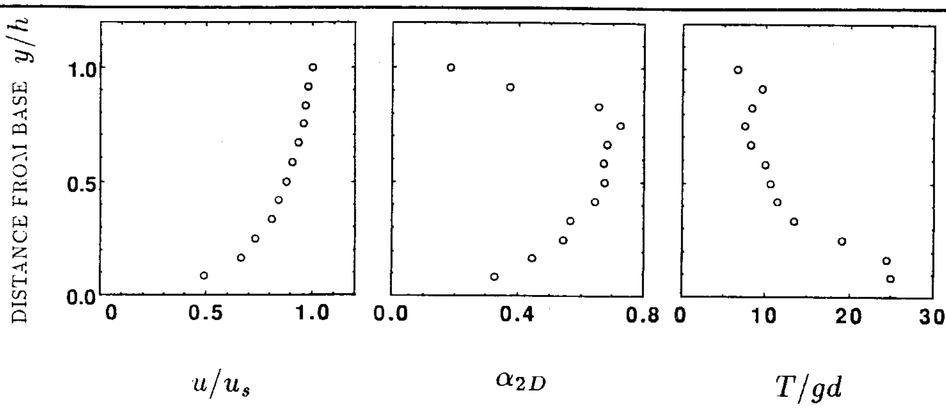

FIGURE 7-4. Profiles of the velocity nondimensionalized by the surface velocity, u_s, of the two-dimensional solid fraction α_{2D} and of the temperature T nondimensionalized by $gd/2$ from a computer simulation of two-dimensional discs flowing down a plane inclined at 30° ; $\epsilon_p = 0.6$ and $\epsilon_w = 0.8$ (Campbell and Brennen (1985b)).

showed that the granular temperature decreases substantially with the distance from the channel base, and that the solid fraction as observed in Bailard (1978) has its maximum value in the bulk of the flow and is smaller both at the free surface and at the base. Walton *et al.* (1988) used three-dimensional spheres to simulate gravity flow of particles through arrays of cylindrical horizontal rods and down inclined chutes. They employed a discrete particle model to simulate the chute flow tests conducted by Drake and Shreve (1986).

7.2.2 Types of Channel Flow Profile

As discussed in the previous section, the profiles of the granular temperature and solid fraction obtained by Ahn *et al.* (1989a) and Campbell and Brennen (1985b) are quite different. That is, in the experiments of Ahn *et al.*, the granular temperature increases and the solid fraction decreases monotonically with the distance from the channel base whereas, in the computer simulations of Campbell and Brennen, the temperature decreases with distance from the base and the solid fraction exhibits a maximum with lower values at the base and near the free surface. The analysis of Johnson and Jackson (1990) yielded both types of profile of the solid fraction but they do not present the corresponding profiles of granular temperature. For the rough surface with the inclination of 19°, the solid fraction decreased monotonically with distance from the base. A similar profile was manifest with a smooth surface at 16° inclination when the flow was dilute. However, at different type of profile closer to that of Campbell and Brennen occurred for very dense flow under the same conditions.

Ahn *et al.* (1989b) used the constitutive relations and governing equations of Lun *et al.* (1984) to generate solutions the fully developed chute flow of granular material. The solutions depend upon two parameters (the coefficient of restitution between particles, ϵ_p, and the channel inclination, θ) and three boundary values at the channel base (the values of solid fraction, granular temperature, and mean velocity). The results showed the significant role played by granular conduction in determining the profiles of granular temperature, solid fraction and mean velocity in chute flows. They also demonstrated that there exist two types of the fully developed flow depending on the choice of ϵ_p and $\tan\theta$. The two types are designated Type I and Type II. Type I flows were similar to those observed in the experiments of Ahn *et al.* (1989a); the granular temperature at the free surface is higher than at the channel base and fluctuation energy is conducted toward the base. In this type of flow the solid fraction monotonically decreases with the distance from the base and the free surface may not be well defined. On the other hand, Type II flow is characterized by higher granular temperature at the base than in the bulk and granular conduction transfers fluctuation energy from the base toward the bulk. Associated with these variations are a low solid fraction at the base and a more compact mass riding on this dilated layer. Thus the free surface of these Type II flows is well defined. The simulations of Campbell and Brennen produced Type II flows. Though Bailard (1978) did not measure granular temperature, his measured profiles of solid fraction are of Type II.

Ahn *et al.* (1989b) also discussed what determines the type of fully developed flow and Figure 7-5 presents a qualitative criterion. For a given inclination, fully developed flows of granular materials with high ϵ_p are of Type I, and particles with low ϵ_p yield fully developed flow of Type II. On the other hand, for a given granular material, fully developed flow with high inclination is of Type I, and the flow with low inclination is of Type II. (Here the borderline is a weak function of solid fraction, and near the borderline it is possible to have a flow of some of the characteristics of both Types I and II. Ahn *et al.* (1989b) referred to this flow as Type III.) It should be noted that the criterion in Figure 7-5 holds only if there exists a fully developed flow. The criterion is also consistent with physical intu-

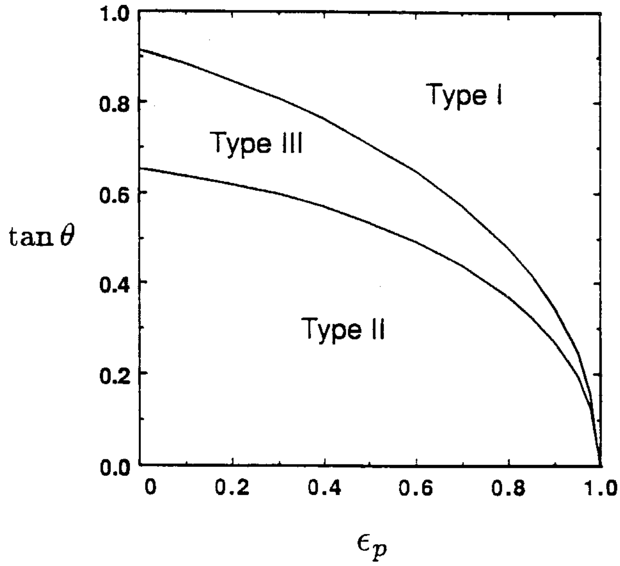

FIGURE 7-5. Flow Regimes in Channel Flow.

ition. The value $\tan\theta$ serves as a gravity force which generates the translational fluctuation energy through shear work. At larger values of $\tan\theta$, fluctuation energy production dominates. The dissipation of the fluctuation energy is a strong function of ϵ_p. For smaller ϵ_p, the fluctuation energy dissipation dominates. When the rate of generation of fluctuation energy is larger than the rate of dissipation, the channel base region has to absorb the excessive fluctuation energy. Otherwise, no fully developed flow exists for that $\tan\theta$ and ϵ_p. This implies that the coefficient of restitution between the base wall and the particle, ϵ_w, must be somewhat smaller than ϵ_p. (The role of the surface roughness is not clear.) In this case, the fluctuation energy is conducted from the bulk to the wall boundary and the granular temperature is expected to be smaller near the wall while larger temperatures occur at the free surface. On the other hand, when the rate of dissipation is larger than the rate of generation, the wall boundary will supply the fluctuation energy to the flow. Unless the rate of generation of fluctuation energy at the wall exceeds the rate of dissipation and thus supplies fluctuation energy to the bulk there can be no fully developed flow for that inclination and granular material. Therefore, ϵ_w is required to be somewhat larger than ϵ_p in order to maintain fully developed flow. In this case, the granular temperature near the wall is higher than near the free surface and the fluctuation energy is conducted from the wall to the free surface. (In the case where ϵ_w is smaller than ϵ_p for a given channel inclination so that dissipation dominates, fully developed flow could be established by external means, for example, by vibrating the channel base.) In summary, the surface boundary conditions of the channel base determine whether fully developed flow is possible or not for given $\tan\theta$ and ϵ_p. These parameters also determine whether the fully developed flow is of Type I or Type II.

This criterion is consistent with the results in the literature. Ahn *et al.* (1989a) used glass beads which have very high ϵ_p. (Lun and Savage (1986) estimated $\epsilon_p \simeq 0.95$ for glass beads.) The channel base was an aluminum with and without the rubber layer and ϵ_w was estimated to be about 0.7 and 0.5 respectively. For the case of Figure 7-3 where the channel inclination was 17.8°, Type I flow is expected and was observed in their experiments. In the computer simulations of Campbell and Brennen (1985), $\epsilon_p = 0.6$ and $\epsilon_w = 0.8$ with $\theta = 30°$. The criterion predicts the Type II flow which is consistent with the result of the simulation as shown in Figure 7-4. However, the results of Bailard (1978) can not be judged from this criterion because the channel inclination is high (34° $\sim$ 39°) and ϵ_p has an unknown but probably small value.

Richman and Marciniec (1988) sought analytical solutions for channel flows by employing the constitutive theory of Jenkins and Richman (1985) for identical, smooth, nearly elastic spheres. They were concerned with a flat channel base wall to which identical smooth hemispherical particles were attached (see Jenkins and Richman (1986)). They provide the profiles of the granular temperature, solid fraction and velocity. Their results also show that there are two types of flow and that the profiles of two types of flow are consistent with those of Ahn *et al.* (1989b) (Type I and Type II flows). For this particular boundary condition and a given granular material, the analytical results indicate when fully developed flow is possible in terms of the inclination, mass flow rate, and solid fraction. For example, for Type I flow, fully developed flow can be established at high inclination for small mass flow rate and at low inclination for large mass flow rate. On the other hand, for Type II flow, higher mass flow rate can yield fully developed flow at higher inclination.

7.2.3 Transverse Profiles in Channel Flows

Transverse velocity profiles in channel flows have been investigated by Ishida and Shirai (1979), Savage (1979), and Ahn *et al.* (1989a). In Savage's experiments, the profiles at the free surface were obtained using marker particles and a movie camera. For smooth walls, the velocity profiles were quite uniform. For rough walls, the flows were very slow and the velocity profiles were roughly parabolic with much smaller velocities at the sidewalls than in the center. Using fiber optic probes, Ahn *et al.* obtained the velocity profiles both at the free surface and at the channel base. Comparison of the profiles on the free surface and on the channel base showed that the flow at the free surface was more uniform and less affected by the side wall than the flow at the base. It was also observed that the higher the velocity (higher channel inclination), the less significant the side wall effect. Indeed nonuniformity due to the side walls was significant only at the base and only at low velocities (low inclinations).

7.3 RHEOLOGICAL IMPLICATIONS OF THE CHANNEL FLOWS

7.3.1 Rheological Behavior in Channel Flows

The rheological behavior which is manifest in open channel flows of granular material has been examined by Patton *et al.* (1987). They used the momentum equation (6) to evaluate the shear stress, τ, at the channel base. The normal stress was calculated assuming a hydrostatic variation in pressure over the depth of flow. Both stresses were normalized by $\rho_s d^2(U/h)^2$ and presented as functions of α (the shear rate was approximated as U/h). The normalized stresses showed fairly good correlations, and asymptotes at high solid fraction were very clear at high solid fraction. The friction coefficient was also plotted against the Froude number. It remained fairly constant over the wide range of the low Froude number, but as the Froude number became high the friction coefficient seemed to increase substantially though the data was quite scattered since, at these large Fr, the shear stress calculation using equation (6) is quite sensitive to $d\alpha_m/dz$ and dh/dz, quantities which are difficult to measure accurately. Since a high Froude number in the experiments of Patton *et al.* corresponds to high u and low α, the increasing friction coefficient with decreasing solid fraction is consistent with the trend observed in shear cell experiments and in the computer simulations.

Perhaps the most complete experimental investigation of the rheological behavior in the channel flow is found in Ahn *et al.* (1989a).

Two important instruments were used in their experiments; one is a gauge to measure shear stress, and the other is a set of two fiber optic probes to measure velocity, velocity fluctuations, and local solid fraction. In order to measure shear stress of flowing material at the channel base, a rectangular hole was cut into the channel base and replaced by a plate supported by strain-gauged flexures sensitive to the shearing force applied to the plate. The clearance between the plate and the rest of the channel base was adjusted to be about 0.2 mm, much smaller than the particle sizes. A system of fiber optic probes, similar to that devised by Savage (1979), was developed to measure particle velocities and their fluctuations at the channel base and at the free surface. Only one component of velocity fluctuations, u', was measured in a root-mean-square sense, and it should be some measure of the granular temperature. In addition to the mean solid fraction which was obtained by measuring the mean velocity, the depth of flow, and mass flow rate, local solid fraction was also estimated as follows. The number of particle passages per unit time detected by the probe was divided by the mean velocity to obtain the characteristic particle spacing, s, and in turn the linear concentration, α_{1D}, was calculated as $\alpha_{1D} = d/s$. An estimate of the local solid fraction near the wall, α_w, was calculated using $\alpha_w = \pi\alpha_{1D}^3/6$. Finally, the shear rate was approximated as $\Delta u/h$ where Δu is the velocity difference between at the free surface and at the wall. The normal stress was calculated in a manner similar to Patton *et al.* (1987).

Ahn *et al.* (1989a) presented the stresses with various normalizations, based on application of Lun *et al.* (1984) to the channel flow. They suggested that, for fully developed flow, the normal and shear stresses should be normalized by $\rho_s(d\Delta u/h)^2/\tan^2\theta$ and $\rho_s(d\Delta u/h)^2/\tan\theta$ respectively. For general flow (not necessarily simple shear flow or fully developed flow), it was suggested that the normal stress should be normalized by $\rho_s(u')^2$, and the shear stress by $\rho_s(d\Delta u/h)u'$. Some results are presented in Figures 7-6 through 7-8, and also are the theoretical results of Lun *et al.* (1984) included in the figures for comparison.

In Ahn *et al.* (1989a), the parameter S introduced by Savage and Jeffrey (1981) was examined with an approximation of $d(\Delta u/h)/u'$. The theoretical results of Lun *et al.* (1984) suggest that S or $S/\tan\theta$ should be a function only of α and ϵ_p for the case of simple shear flow or fully developed flow respectively. Figure 7-9 shows the results of some data which were close to fully developed flow, and the result

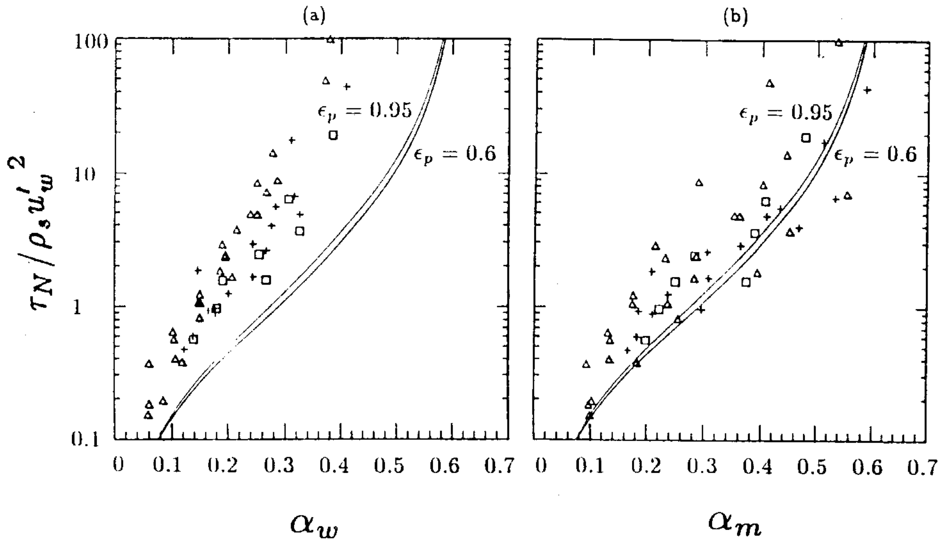

FIGURE 7-6. The normalized normal stress, $\tau_N/\rho_s u_w'^2$, presented as a function of (a) wall solid fraction, α_w and (b) mean solid fraction, α_m. $\square$, the smooth surface; $+$, the moderately smooth surface; $\triangle$, the rubberized surface. The solid lines are the results of Lun *et al.*[1984].

of Lun *et al.* (1984) is also plotted for comparison. The results show that the lower solid fraction, the lower ratio of the shear rate to the granular temperature. This is consistent with physical intuition that at low solid fraction particles move more freely, characterized by high granular temperature and thus by low shear rate as discussed in Section 7.1. This also implies that the ratio becomes zero at the free surface where no shear motion is expected.

Ahn *et al.* (1989a) also provided some data on friction coefficient for various surface conditions. First, the friction coefficient, f, was examined as a function of solid fraction, α. For the smooth aluminum surfaces, f is invariant with α. But for the rubberized surface which had higher Coulombic friction coefficient, f increased with decreasing α, and this is consistent with the results of other literature (for example, Savage and Sayed (1984) and Campbell (1989)). The friction coefficient was also plotted against u_w'/u_w where u_w and u_w' are the velocity and its fluctuation at the wall respectively. (See Figure 7-10.) For the smooth surfaces, all the data were clustered at one region. For the rubberized surface, f seemed to correlate quite well with u_w'/u_w; f increased with increasing u_w'/u_w. This phenomenon

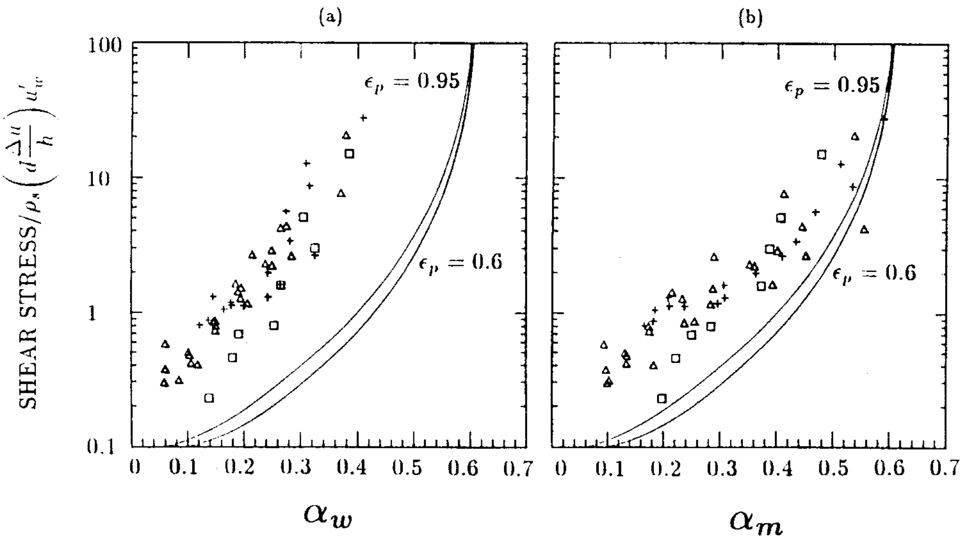

FIGURE 7-7. The normalized shear stress, $\tau_S/\rho_s d(\Delta u/h)u'_w$, presented as a function of (a) wall solid fraction, α_w, and (b) mean solid fraction, α_m. □, the smooth surface; +, the moderately smooth surface; △, the rubberized surface. The solid lines are the results of Lun *et al.*[1984].

was independent of particle size.

To explain these observations, they suggested the following. When a particle collides with a wall such that the shear stress at the contact point exceeds a shear stress limit which the surface can withstand for the given normal stress at the contact point, slip will occur. Then the ratio of the shear stress to the normal stress at the contact point is adjusted to the Coulombic friction coefficient of the surface, i.e. $f = \mu_c$. On the other hand, when the ratio at the impact does not exceed μ_c, there will be no slip between the contact surfaces of the particle and the wall. In this case, f is different from μ_c. For the smooth surface where μ_c was low, slip occurred at the contact between particles and the surface, and the slip condition resulted in the constant friction coefficient equal to μ_c. However, for the rubberized surface with high μ_c, a no-slip condition was applied, resulting in varying friction coefficient with solid fraction.

They also discussed why the friction coefficient for the rubberized surface varies with the solid fraction and u'_w/u_w. When there is no slip, the following relation results from a simple analysis of the oblique

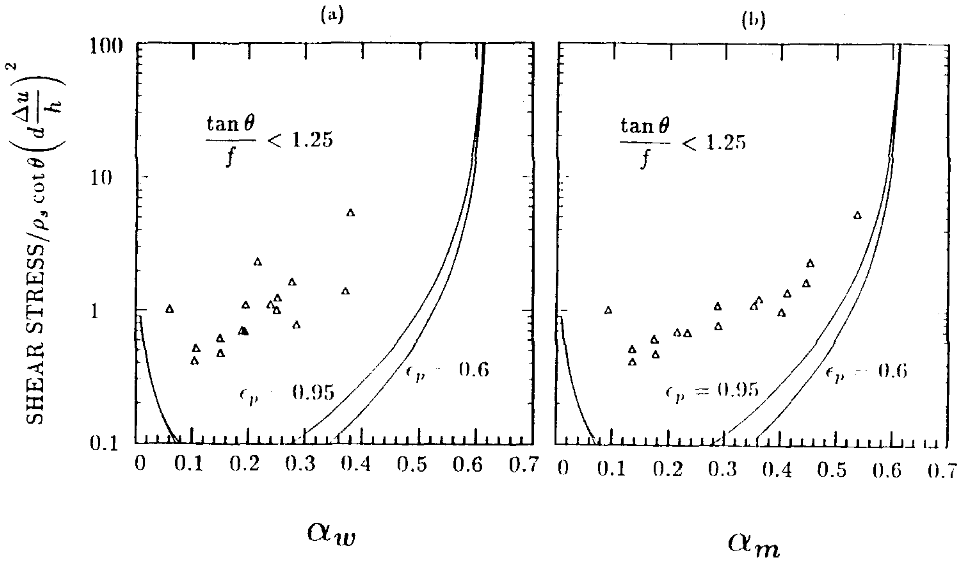

FIGURE 7-8. The normalized shear stress, $\tau_S \tan\theta/\rho_s(d\Delta u/h)^2$, presented as a function of (a) wall solid fraction, α_w, and (b) mean solid fraction, α_m. Data only for $\tan\theta/f < 1.25$. The solid lines are the results of Lun *et al.*[1984].

impact of a single sphere on the flat surface (see Ahn (1989)):

$$f = \frac{2}{7}\left(1 - \frac{\omega_1 d}{2u_1}\right)\frac{\tan\beta_1}{1+\epsilon_w}$$

where ω_1 is the rotational rate before impact, and u_1 is the velocity tangential to the wall before impact. The impact angle β_1 is defined by $\tan^{-1}(u_1/v_1)$ where v_1 is the velocity normal to the wall before impact and ϵ_w is the wall-particle coefficient of restitution. In this equation, the friction coefficient or the ratio of the shear stress to the normal stress at the surface depends on the ratio of rotational velocity to tangential velocity, $\omega_1 d/2u_1$, and on the impact angle, β_1. In their experiments, the value of β_1 could not be estimated. Another factor influencing f is $\omega_1 d/2u_1$. Campbell (1988) has shown that next to the wall ω is considerably larger than the mean value, but that with a small distance from the wall ω is slightly less than the mean value. Therefore, when a particle next to the wall with high ω hits the wall, the friction coefficient will be low, but if a particle at a distance from the wall with low ω comes down and collides with the wall, the friction coefficient will be relatively high.

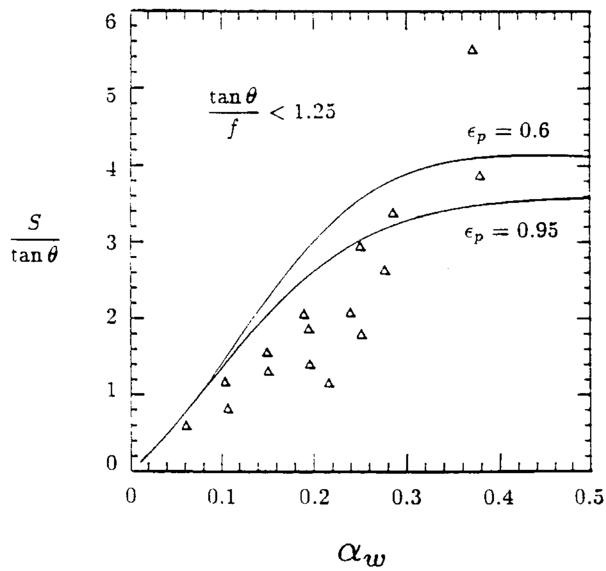

FIGURE 7-9. The parameter, $S/\tan\theta = d(\Delta u/h)/u'_w \tan\theta$, against wall solid fraction, α_w. Data only for $\tan\theta/f < 1.25$. The solid lines are the results of Lun *et al.*[1984].

These phenomena suggest a possible explanation for the increase in the friction coefficient as the solid fraction decreases. At low solid fraction, particles move more freely from one layer to another. Thus more particles in the upper layers with small values of $\omega d/2u$ move down to the boundary and collide with the wall. Because friction is measured in a statistical sense as a sum of frictions due to individual particles colliding with the wall, f is therefore high at low solid fraction. On the other hand, at high solid fraction and low granular temperature, very few particles in the upper layer with low $\omega d/2u$ penetrate to the wall. As a result, particles next to the wall with high rotational velocity will dominate collisions at the wall. Thus f would be smaller at high solid fraction. Furthermore, when higher u'_w/u_w exists, particles with low ω in the upper layer more easily move down to the boundary and collide with the wall. That is, as u'_w/u_w increases, the intrusion of particles with low ω from the upper layer into the boundary becomes more frequent, causing f to increase. As a result, the friction coefficient appears to be a fairly linear function of u'_w/u_w for the rubberized surface as shown in Figure 7-10.

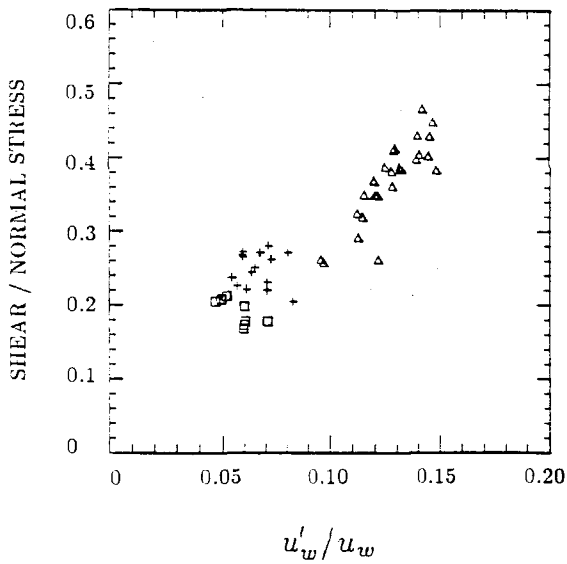

FIGURE 7-10. Friction coefficient at the wall, $f = \tau_S/\tau_N$, presented as a function of longitudinal velocity fluctuation at the wall normalized by mean velocity, u'_w/u_w. $\square$, the smooth surface; $+$, the moderately smooth surface; $\triangle$, the rubberized surface.

7.4. CONCLUDING REMARKS

One of the simplest granular material flows which can be generated in the laboratory is that down an inclined chute or channel. It therefore serves as a very convenient vehicle against which to test an existing knowledge of the rheology of granular material flows. In this review we have tried to demonstrate that much remains to be learned about the rheology from these flows which, when examined in detail, turn out to be far from simple. Perhaps the most important conclusion which should be drawn from this review is that we have reached a stage at which advancement in our understanding of the rheology of granular material flows will be led by our ability to devise methods to make detailed measurements of all of the fundamental properties of these flows, namely the mean velocity, solid fraction, fluctuating velocity and normal and shear stresses. In this review we have described some recent but incomplete attempts to experimentally document channel flows in this way. These attempts show that the kinetic theory models for granular material flow advanced by Savage (1984), Jenkins *et al.* (1985) and others have some validity but that more needs to be done before these models could be considered verified and validated. The other analytical approach, namely the computer simulation of granular material flows by Campbell and Brennen (1985a,b), Walton (1984) and others, also appears to have considerable value and yet many of the details necessary for these simulations (such as the collision mechanics) also require experimental validation.

Apart from the constitutive laws governing the mechanics of the flowing granular material there also remain some very important questions regarding the boundary conditions which should be applied to model solid walls or free surfaces which are the common boundaries of granular material flow. In this review we have described how the slip at a solid wall (channel base) is a function of the roughness of that wall and how that slip can effect the entire flow. The laws governing the slip are not at all well understood. Moreover, when one examines the situation more closely it is clear that a boundary condition on the granular temperature is also necessary and this, too, is a subject of considerable debate at the present time.

Another feature of channel flows which we have tried to highlight in this review is the lack of knowledge regarding the manner in which these flows approach equilibrium (if indeed they ever do) as they progress down the channel. Like open-channel flows of liquid they can approach a state in which the forces on an elemental length balance so that further acceleration or deceleration does not occur. But unlike open-channel flows of liquid, granular material flows will only reach equilibrium when the processes of production, dissipation and conduction of granular fluctuation energy have reached equilibrium. Though the work by Jackson has shed some very valuable light on this subject, much remains to be done before our understanding of this aspect of granular flow could be considered in any way complete.

ACKNOWLEDGEMENTS

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NOTATION

a, b, c	Profile shape parameters
d	Particle diameter
f	Friction coefficient ($\tau P/\rho_s \alpha_m gwh$ or ratio of shear stress to normal stress)
Fr	Froude number $= U/(gh)^{\frac{1}{2}}$
g	Acceleration due to gravity
h	Depth of flow
p	Pressure
P	Perimeter of flow in contact with solid walls
s	Mean spacing between particle centers
S	Parameter $d\ du/dy/T^{\frac{1}{2}}$
T	Granular temperature, $(\langle u'^2\rangle + \langle v'^2\rangle)/2$
w	Channel width
u	Velocity of flow parallel with channel walls
u', v'	Fluctuating velocity components
U	Cross-sectionally averaged velocity $= u_m$
Δu	Velocity difference $= u_s - u_w$
v	Velocity of flow normal to channel base
y	Coordinate measured perpendicular to channel base
z	Coordinate measured parallel to channel walls
α	Solid fraction
α_{ID}	Linear concentration, d/s
β	Inclination of particle trajectory to wall normal
δ	Velocity gradient
θ	Channel inclination to horizontal
ρ_s	Particle density
φ	Internal friction angle
μ_c	Coulomb friction angle
μ_F	Liquid viscosity
τ	Shear stress at wall
τ_{ij}	Stress tensor
ϵ_p	Coefficient of restitution between particles
ϵ_w	Coefficient of restitution between particle and wall
ω	Particle rotational velocity

Subscripts

m	denotes quantity averaged over cross-section of channel flow
s	denotes quantity at the free surface
w	denotes quantity at the channel base

1 denotes quantity upstream of shock or before collision
2 denotes downstream of shock or after collision

Superscript

$'$ denotes fluctuating component

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