Solution to Problem 240C

The entrance length, L, can be estimated by assuming the flow is fully developed when the thickness of the boundary layer that forms on the inside of the pipe is approximately equal to the radius of the pipe. It is reasonable to use $\delta_{0.99}$ as an appropriate estimate of the boundary layer thickness so that the entrance length is given by

$$\delta_{0.99} = R$$

For a Blasius boundary layer, the 99% thickness, $\delta_{0.99}$, is given by

$$\delta_{0.99} = 4.9 \left(\frac{\nu x}{U}\right)^{\frac{1}{2}}$$

and therefore the above equation becomes

$$4.9 \left(\frac{\nu x}{U}\right)^{\frac{1}{2}} = R$$

so that

$$L = \frac{1}{24} \frac{R^2 U}{\nu}$$

and

$$\frac{L}{R} = \frac{1}{24} \frac{UR}{\nu} = \frac{1}{24} \text{Re}$$

The volume flow rate, Q, is given by

$$Q = AU = \pi R^2 U.$$

and with $\nu = \mu/\rho$ the expression for L/R becomes

$$\frac{L}{R} = \frac{1}{24} \frac{UR}{\nu}$$

and therefore

$$L = \frac{1}{24} \frac{UR^2}{\nu} = \frac{1}{24} \frac{Q\rho}{\pi\mu}$$

Substituting $Q = 10\ cm^3/s$, $\mu = 0.001\ kg/m\ s$, and $\rho = 1000\ kg/m^3$, the result becomes

$$L = 0.13\ m$$