Solution to Problem 108B

At the contact circle, the force pulling the bubble down is S times the circumference of the circle:

$$S2\pi R\sin\theta$$

The vertical component of this force is:

$$S2\pi R \sin^2\theta$$

This must be balanced by the buoyancy force acting upward which is:

$$\rho g V$$

where V is the volume of the bubble:

$$V = \pi h^2 (R - \frac{h}{3})$$

where h is the height of the bubble:

$$h = R(1+\cos\theta)$$

So the buoyancy force is:

$$\rho g \pi R^3 (1+\cos\theta)^2 (1 - \frac{1+\cos\theta}{3})$$

Therefore equilibrium requires that:

$$S2\pi R\sin^2\theta = \rho g \pi R^3 (1+\cos\theta)^2 (1 - \frac{1+\cos\theta}{3})$$

and, after some algebra, this yields:

$$R = \left[\frac{6S(1-\cos\theta)}{\rho g (1+\cos\theta)(2-\cos\theta)} \right]^{\frac{1}{2}}$$