Solution to Problem 420A:

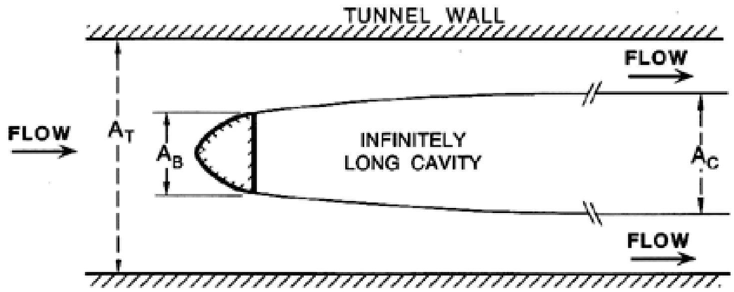

Figure 1: Body with infinitely long cavity under choked flow conditions.

In the choked flow limit of an infinitely long cavity, application of the equations of conservation of mass, momentum, and energy lead to some simple relationships for the parameters of the flow. Referring to the above figure, consider a body with a frontal projected area of A_B in a water tunnel of cross-sectional area, A_T. In the limit of an infinitely long cavity, the flow far downstream will be that of a uniform stream in a straight annulus, and therefore conservation of mass requires that the limiting cross-sectional area of the cavity, $A_C = \beta A_B = \beta A_T/\alpha$, be given by

$$\frac{A_C}{A_T} = \frac{\beta}{\alpha} = 1 - \frac{U_\infty}{q_C} = 1 - (1+\sigma_C)^{-\frac{1}{2}} \tag{1}$$

which leads to

$$\sigma_C = 1 - \left(1 - \frac{\beta}{\alpha}\right)^{-2} \tag{2}$$

This is the "choked cavitation number" under assumptions 1, 2 and 3. The corresponding choked tunnel velocity, U_C, is given by

$$U_C^2 = \frac{2(p_\infty - p_V)}{\rho_L\left[1 - (1-\beta/\alpha)^{-2}\right]} \tag{3}$$

If the vapor density, ρ_V, and the vapor flow is to be included then the continuity equation will be changed to

$$\rho_L U_T A_T = \rho_L U_C (A_T - A_C) + \rho_V U_C A_C \tag{4}$$

or

$$\frac{U_T}{U_C} = 1 - \frac{\beta(1-\gamma)}{\alpha} \tag{5}$$

and therefore the modified choked cavitation number, σ_C^* would be

$$\sigma_C^* = \frac{U_T}{U_C} = \frac{\alpha^2}{\left[\alpha - \beta(1-\gamma)\right]^2} - 1 \tag{6}$$