Solution to Problem 406A:

This problem is concerned with the growth of boiling bubbles at a surface to which heat is being supplied at a rate Q (heat/unit area/unit time). If the nucleation site density is N (sites/unit area) and if the bubbles are modeled as spherical with a radius, $R(t)$, where t is time, then we seek the bubble growth rate, dR/dt, for steady state boiling in which the mean temperature of the liquid remains unchanged. It is assumed that both the latent heat, $\mathcal{L}$, and the density of the vapor in the bubbles, ρ_V, are known and fixed.

First note that the heat supplied to each nucleation site per unit time is Q/N. Therefore the mass of liquid evaporated at each site per unit time is $Q/\mathcal{L}N$ and this must be equal to the mass of vapor generated at each site per unit time. Therefore the volume of vapor generated at each site per unit time is $Q/\rho_V\mathcal{L}N$. And since this must be equal to the volume rate of increase of the bubbles it follows that

$$\frac{Q}{\rho_V\mathcal{L}N} = \frac{d}{dt}\left(\frac{4}{3}\pi R^3\right) = 4\pi R^2\frac{dR}{dt} \tag{1}$$

or

$$\frac{dR}{dt} = \frac{Q}{4\pi R^2\rho_V\mathcal{L}N} \tag{2}$$