Solution to Problem 404B:

Given the distribution function of nuclei or microbubbles in a water tunnel:

$$N_1(R) \;=\; N^*/R^{3.5} \qquad R \;>\; R_{min} \tag{1}$$

where $N^* = 10^{-5}$ (units $m^{-0.5}$) , $R_{min} = 0.00002m$ and R is the microbubble radius in meters. This distribution is evaluated at atmospheric pressure, p_1, and at a known temperature, T.

[A] Find the microbubble population density in number/cm^3.

$$\text{Population} \;=\; \int_{R_{min}}^{\infty} N_1(R)\; dR \;=\; \frac{N^*}{2.5R_{min}^{2.5}} \;=\; 2.24\times 10^6\; m^{-3} \;=\; 2.24 \text{ per } cm^3 \tag{2}$$

[B] Find the distribution at a different pressure, p_2, if the bubbles contain a fixed mass of noncondensable gas. If the pressure is changed from p_1 to p_2 while the temperature remains the same then pV = constant and the new bubble radius, R_2, is

$$R_2 \;=\; \left(\frac{p_1}{p_2}\right)^{1/3} R_1 \quad \text{and} \quad dR_2 \;=\; \left(\frac{p_1}{p_2}\right)^{1/3} dR_1 \tag{3}$$

Therefore the number of bubbles with sizes between R_1 and $R_1 + dR_1$ per unit volume is $N_1(R_1)dR_1$ and this becomes the same as the number per unit volume at the new pressure with size between R_2 and $R_2 + dR_2$. In other words

$$N_2(R_2)dR_2 \;=\; N_1(R_1)dR_1 \tag{4}$$

$$N_2(R_2) \;=\; N_1(R_1)\left(\frac{p_2}{p_1}\right)^{1/3} \;=\; \frac{N^*}{R_2^{7/2}}\left(\frac{p_1}{p_2}\right)^{5/6} \tag{5}$$