Solution to Problem 352D

Design 1
We use the normal shock relations to get the pressure change across the shock.

$$\frac{p_e}{p_a} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right) = 7.125$$

Design 2
Finding the incoming Mach number normal to the surface of the oblique shock:

$$M_{1n} = M \sin\beta = 2.5 \sin 30^\circ = 1.25$$

Calculating the pressure and Mach number changes across the oblique shock:

$$\frac{p_2}{p_a} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{1n}^2 - 1\right) = 1.656$$

$$M_{2n}^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} = 0.6604$$

$$\Rightarrow M_{2n} = 0.8126$$

$$M_2 = \frac{M_{2n}}{\sin\left(\beta - \theta\right)} = 2.303$$

Now calculating the pressure change across the normal shock:

$$\frac{p_e}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_2^2 - 1\right) = 6.022$$

The pressure ratio between the engine and the ambient is then the combination of the pressure ratios across each of the shocks.

$$\frac{p_e}{p_a} = \frac{p_2}{p_a}\frac{p_e}{p_2} = 9.97$$