## An Internet Book on Fluid Dynamics

## Solution to Problem 320A:

A rocket engine can be modelled by a reservoir of gas at high temperature $\left(3000^{\circ} \mathrm{K}\right)$ feeding gas to a convergent/divergent nozzle:


The flow reaches critical conditions at the throat, is subsonic upstream of the throat and supersonic in the divergent section between the throat and the exit. The pressure at exit is atmospheric $\left(105 \mathrm{~kg} / \mathrm{ms}^{2}\right)$ and the supersonic flow at the exit is traveling at $M_{\text {exit }}=2.5$.
(a) From the isentropic flow table, at $M=2.5, T / T_{0}=0.444$ and therefore the temperature of the flow at the exit is $0.444 \times 3000=1332^{\circ} \mathrm{K}$.
(b) Also from the isentropic flow table, at $M=2.5, p / p_{0}=0.059$ and therefore the pressure in the reservoir, $p_{0}=100000 / 0.059=1.69 \times 10^{6} P a$.
(c) Also from the isentropic flow table, at $M=2.5, A_{\text {exit }} / A^{*}=2.637$ and therefore the area of the exit divided by the area of the throat is 2.637 .
(d) The mass flow rate per unit area of the exit cross-section, $\dot{m} / A_{\text {exit }}$, is

$$
\begin{equation*}
\frac{\dot{m}}{A_{\text {exit }}}=\rho_{\text {exit }} M_{\text {exit }} c_{\text {exit }}=p_{\text {exit }} M_{\text {exit }}\left[\frac{\gamma}{\mathcal{R} T_{\text {exit }}}\right]^{1 / 2}=484 \mathrm{~kg} / \mathrm{m}^{2} \mathrm{~s} \tag{1}
\end{equation*}
$$

(e) The thrust produced by the engine per unit area of the exit cross-section is

$$
\begin{equation*}
\rho_{\text {exit }}\left(M_{\text {exit }} C_{\text {exit }}\right)^{2}=\gamma p_{\text {exit }} M_{\text {exit }}^{2}=8.75 \times 10^{5} \mathrm{~N} \tag{2}
\end{equation*}
$$

