Solution to Problem 312A:

Consider the steady frictionless flow of a perfect gas through a pipe of constant, uniform cross-sectional area. Heat is added to this flow through the pipe walls so that the total temperature, T_0, of the gas increases by an amount dT_0, over a small length of the pipe. Find a relation for the correspondingly small change in the Mach number (denoted by dM) in terms of dT_0, the Mach number, M, and the temperature, T, of the flow (the expression also contains the ratio of the specific heats, γ).

The continuity equation yields:

$$\frac{du}{u} + \frac{d\rho}{\rho} \ = \ 0 \tag{1}$$

and the momentum equation yields:

$$\frac{dp}{\rho} + u \ du \ = \ 0 \tag{2}$$

The definition of the stagnation temperature yields

$$dT_0 \ = \ dT + \frac{u}{c_p} du \tag{3}$$

where $c_p = \gamma \mathcal{R}/(\gamma - 1)$. The definition of the Mach number, $M = u/\sqrt{\gamma \mathcal{R} T}$, yields

$$\frac{dM}{M} \ = \ \frac{du}{u} - \frac{dT}{2T} \tag{4}$$

The perfect gas law yields

$$dp \ = \ \rho \mathcal{R} dT + \mathcal{R} T d\rho \tag{5}$$

By elimination

$$\frac{dM}{M} \ = \ \frac{(1 + \gamma M^2)}{2(1 - M^2)} \frac{dT_0}{T} \tag{6}$$