Solution to Problem 276A

Since

$$\bar{u} = C u_\tau \left(\frac{y}{\epsilon}\right)^{\frac{1}{7}}$$

and $\bar{u} = U$ at $y = \delta$ it follows that

$$u_\tau = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}} = \frac{U}{C}\left(\frac{\epsilon}{\delta}\right)^{\frac{1}{7}}$$

But the Karman Momentum Integral Equation for a case in which U is independent of x is

$$\frac{\tau_w}{\rho} = \alpha U^2 \frac{d\delta}{dx}$$

and elimnating τ_w/ρ from the last two equations gives

$$\frac{d\delta}{dx} = \frac{\epsilon^{\frac{2}{7}}}{\alpha C^2}\delta^{-\frac{2}{7}}$$

which can be integrated to yield (after applying $\delta = 0$ at $x = 0$)

$$\delta = \left[\frac{9\epsilon^{\frac{2}{7}}}{7\alpha C^2}x\right]^{\frac{7}{9}}$$