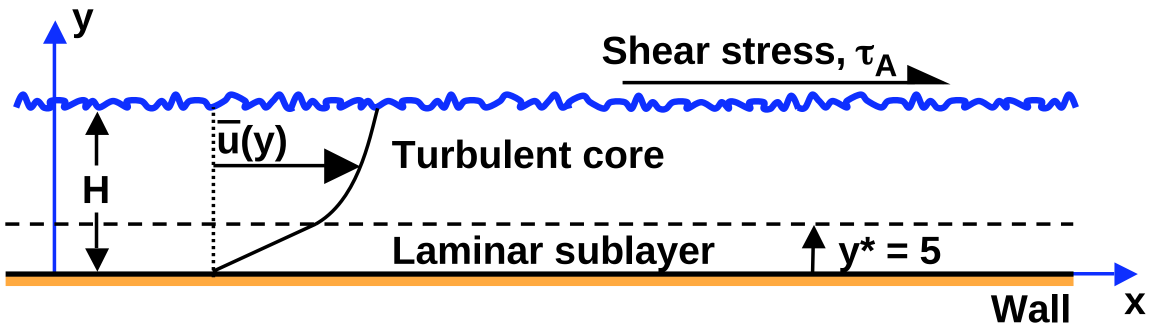
Solution to Problem 272A

Either by considering the momentum theorem applied to the control volume or by observing the similarity to Couette flow,



one can conclude that the shear stress, σ_{xy}, is a constant throughout the flow. Hence $\sigma_{xy} = \tau_A$.

- In the *turbulent core*, since the viscous stresses are negligible

$$\sigma_{xy} = \tau_A = -\rho \overline{u'v'} = \rho \kappa^2 y^2 \left(\frac{d\overline{u}}{dy}\right)^2$$

  by Prandtle's mixing length theory. Hence

$$\kappa y \frac{d\overline{u}}{dy} = \sqrt{\frac{\tau_A}{\rho}} = u_\tau = \text{constant}$$

$$\frac{\overline{u}}{u_\tau} = \frac{1}{\kappa} \ln y + C$$

  where C is an integration constant.

- Within the *laminar sublayer*, we now know $u_\tau = \sqrt{\frac{\tau_A}{\rho}}$

$$u^* = \frac{\overline{u}}{u_\tau} = y^* = \frac{u_\tau y}{\nu}$$

  Therefore at the edge of the laminar sublayer

$$y^* = 5$$

$$\frac{\overline{u}}{u_\tau} = 5$$

$$y = \frac{5\nu}{u_\tau}$$

Hence to find the constant, C

$$C = 5 - \frac{1}{\kappa} \ln \left(\frac{5\nu}{u_\tau}\right)$$

Therefore in the turbulent core

$$\frac{\overline{u}}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{u_\tau y}{5\nu}\right) + 5$$

$$(\overline{u})_{y=H} = \sqrt{\frac{\tau_A}{\rho}} \left[\frac{1}{\kappa} \ln \left(\frac{H}{5\nu}\sqrt{\frac{\tau_A}{\rho}}\right) + 5\right]$$