Solution to Problem 270D

To employ the stated hypothesis that the laminar sub-layer thicknesses could be defined by the distance from the wall at which the Reynolds' shear stress is equal to the laminar viscous stress we first note that the Reynolds' shear stress is given by

$$-\rho\overline{u'v'} = \rho\kappa^2\delta_{LSL}^2\left(\frac{d\overline{u}}{dy}\right)^2$$

using Prantdl's mixing length model with a universal constant, κ. Moreover the laminar viscous stress is given by

$$\mu\left(\frac{d\overline{u}}{dy}\right)_{y=\delta_{LSL}}$$

Equating the two expressions yields the following expression for δ_{LSL}:

$$\delta_{LSL}^2 = \frac{\nu}{\kappa^2}\left[\left(\frac{d\overline{u}}{dy}\right)_{y=\delta_{LSL}}\right]^{-1}$$

Then making use of the assumption that the shear, $(d\overline{u}/dy)$, is constant within the laminar shear layer and therefore equal to τ_w/μ and using the definition for the friction factor, $f = 8\tau_w/\rho V^2$, the expression for δ_{LSL} can be written as

$$\delta_{LSL}^2 = \frac{\rho\nu^2}{\kappa^2}\left(\frac{8}{\rho V^2 f}\right)$$

where V is the volumetric mean velocity. Introducing the pipe radius, R, and the Reynolds number, $Re = 2RV/\nu$ we can write the above expression for δ_{LSL} as

$$\frac{\delta_{LSL}}{R} = \frac{4\times 2^{\frac{1}{2}}}{\kappa}\,\frac{1}{Re\;f^{\frac{1}{2}}}$$

which for $\kappa = 0.4$ yields

$$\frac{\delta_{LSL}}{R} = \frac{14.1}{Re\;f^{\frac{1}{2}}}$$

In comparison the criterion that the laminar sublayer thickness is defined by $y* = 5$ yields

$$\frac{\delta_{LSL}}{R} = \frac{20\times 2^{\frac{1}{2}}}{Re\;f^{\frac{1}{2}}} = \frac{28.3}{Ref^{\frac{1}{2}}}$$