Solution to Problem 268B

Consider Prandtl's Mixing Length Model:

$$-\rho\overline{u'v'} = \rho l^2 (\frac{\partial \bar{u}}{\partial y})^2 \text{ where } l = \kappa y \tag{1}$$

This implies that:

$$\kappa^2 = \frac{-\overline{u'v'}}{y^2(\frac{\partial \bar{u}}{\partial y})^2} = \frac{-\frac{\overline{u'v'}}{U_\infty^2}}{(\frac{y}{\delta})^2(\frac{\partial \bar{u}/U_\infty}{\partial y/\delta})^2} \tag{2}$$

and, therefore, in terms of the quantities in the graph provided:

$$\kappa = \frac{[\frac{1}{20}(-20\frac{\overline{u'v'}}{U_\infty^2})]^{1/2}}{(\frac{y}{\delta})\frac{\partial \bar{u}/U_\infty}{\partial y/\delta}} \tag{3}$$

For various y/δ the quantity $-20\overline{u'v'}/U_\infty^2$ can be read from the graph and the quantity $\frac{\partial \bar{u}/U_\infty}{\partial y/\delta}$ can be found by measuring the slope of the graph for $\bar{u}/U_\infty$ against y/δ.

Approximate results are given in the following table. Note that κ is only crudely constant. However, the assumption of a constant value yields velocity distributions and wall shear stresses that are reasonable engineering approximations.

Table 1: **Tabulate values of the Karman constant, κ:**

y/δ	=	0.1	0.2	0.4	0.6	0.8
$-20\ \overline{u'v'}/U_\infty^2$	=	0.027	0.026	0.020	0.0114	0.0040
$\partial(\bar{u}/U_\infty)\ /\ \partial(y/\delta)$	=	≈ 1	0.62	0.41	0.32	0.167
κ	=	0.37	0.29	0.19	0.12	0.11
$\ell/\delta\ =\ \kappa y/\delta$	=	0.037	0.058	0.077	0.075	0.085