Solution to Problem 240D

The Blasius laminar boundary layer solution for a flat plate yields a boundary layer momentum thickness, $\delta_m(x)$, given by

$$\delta_m(x) = 0.664 \left(\frac{\nu x}{U}\right)^{1/2}$$

where U is the velocity of the boat relative to the water. Then the skin friction drag on the hull (plate) with length $x = L$ is:

$$D = \delta_m(L)\rho U^2 W = 0.664\rho U^{3/2} W \left(\nu L\right)^{1/2}$$

where ρ and ν are the water density and kinematic viscosity and W is the effective width of the plate. By substituting the given values we obtain:

$$D = 2.09 U^{3/2} \ kg \ m/s^2$$

If the wind speed is U_0, the force on the sail is:

$$F = \frac{1}{2}\rho_a \left(U_0 - U\right)^2 C_D A_s$$

where ρ_a is the air density and C_D and A_s are the drag coefficient and frontal projected area of the spinnaker. For a steady forward speed, without acceleration or deceleration, this force must equal the drag. Assuming $C_D \approx 1$ it follows that

$$2.09 U^{3/2} = \frac{1}{2}\rho_a \left(U_0 - U\right)^2 A_s$$

which represents a non-linear algebraic equation for U given U_0. Solving this numerically (or by trial and error) we obtain:

$$U \approx 3 \ m/s$$

The power generated by the sail is:

$$P_s = UD = 33 \ kg \ m^2/s^3$$

To generate the same power would require n rowers (each of whom can generate P_r power) where n is:

$$n = \frac{P_s}{\eta P_r} = \frac{33}{0.2 \times 74.9} = 2.2$$

Rowers cannot usefully be divided into pieces, hence we would need at least 3 rowers!