Solution to Problem 220G

Because gravity is negligible and the pressure in all jets far from the impact point is atmospheric, Bernoulli's equation requires that the velocity in all three jets is the same (and equal to U). A fraction of the incident jet, αA, is deflected forward, and the remaining fluid, $(1-\alpha)A$, is deflected backward, where A is the area of the incident jet.

Define a control volume above the plate with a boundary along the surface of the plate and other boundaries far from the impact point and far above the plate. Consider the momentum theorem applied to this control volume; specifically consider the component of the momentum and the forces tangential to the plate. Since the fluid is inviscid, there are no shear stresses between the fluid and the plate. Furthermore since the pressures far from the impact point are all atmospheric, there is no net pressure force on this control volume in a direction tangential to the plate. Therefore, only the contributions from the momentum fluxes remain and they lead to

$$\underbrace{(\rho\alpha AU)\,U}_{\text{forward deflected jet}} + \underbrace{[\rho\,(1-\alpha)\,AU]\,(-U)}_{\text{backward deflected jet}} + \underbrace{(\rho AU)\,(-U\cos\theta)}_{\text{incident jet}} = 0$$

Solving for α yields

$$\alpha = \frac{1+\cos\theta}{2}$$