Solution to Problem 220B:

First we must make a Galilean transformation into a frame of reference in which the plate is fixed and the flow is steady. To do this we subtract U^* from all the velocities:



We now apply the momentum theorem to the control volume shown by the red dashed line in the figure. There are no net forces since the pressure is the same everywhere. There is no net force on the plate or applied to the fluid by the plate since it is free to move. Therefore the x-momentum flux out must equal the x-momentum flux in to the control volume. Since the deflected jets have no x-momentum it follows that

$$-\rho A_1 (U_1 - U^*)^2 + \rho A_2 (U_2 + U^*)^2 = 0$$
(1)

or

$$U^* = \frac{A_1^{1/2}U_1 - A_2^{1/2}U_2}{A_1^{1/2} + A_2^{1/2}}$$
(2)