## An Internet Book on Fluid Dynamics

## Solution to Problem 137A:

The surface of the cylinder in the $z$-plane is given by $z=R e^{i \theta}$ where $0 \leq \theta \leq 2 \pi, \theta$ being an angle. In the $\zeta$-plane this maps to

$$
\begin{equation*}
\zeta=R e^{i \theta}-R e^{-i \theta}=2 R i \sin \theta=\xi+i \eta \tag{1}
\end{equation*}
$$

Consequently the surface is at $\xi=0$ (flat plate) and $\eta=2 R \sin \theta$ so the plate extends to $\eta= \pm 2 R$. Therefore the plate width is $4 R$.

The velocity components, $u$ and $v$, in the $\zeta$-plane are given by

$$
\begin{equation*}
\frac{d f}{d \zeta}=u-i v=\frac{d f}{d z}\left[\frac{d \zeta}{d z}\right]^{-1}=U\left[1-\frac{R^{2}}{z^{2}}\right]\left[1+\frac{R^{2}}{z^{2}}\right]^{-1} \tag{2}
\end{equation*}
$$

and on the plate surface

$$
\begin{equation*}
(u-i v)_{O n z=R e^{i \theta}}=i U \tan \theta \tag{3}
\end{equation*}
$$

Therefore on the surface of the flat plate

$$
\begin{equation*}
u=0 \quad \text { and } \quad v=-U \tan \theta \tag{4}
\end{equation*}
$$

Hence $\left|u^{2}+v^{2}\right|=U^{2}$ when $\tan \theta=1$ or $\theta= \pm \pi / 4$, that is to say at the points $\eta= \pm \sqrt{2} R$.

