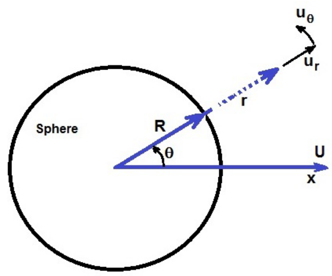
Solution to Problem 133A:

The potential flow around a cylinder of radius, R, traveling in the x direction with velocity, U, in a fluid



at rest far from the cylinder, has a velocity potential, $\phi = -(UR^2/r)\cos\theta$ and velocity components:

$$u_r = \frac{\partial \phi}{\partial r} = \frac{UR^2}{r^2}\cos\theta \quad ; \quad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{UR^2}{r^2}\sin\theta \tag{1}$$

[Note that on the surface of the cylinder $u_r = U \ \cos\theta$, $u_\theta = U\sin\theta$.]

(a) The total kinetic energy in the flow is

$$= \frac{\rho}{2}\int_{\text{Volume of fluid}} (u_r^2 + u_\theta^2) dV \tag{2}$$

$$= \frac{\rho}{2}\int_R^\infty \int_0^{2\pi} \frac{U^2R^4}{r^4}(\cos\theta)^2 + \frac{U^2R^4}{r^4}(\sin\theta)^2 \quad d\theta \ dr \tag{3}$$

$$= \frac{\rho}{2}\int_R^\infty \frac{2\pi U^2R^4}{r^4} \ dr \tag{4}$$

$$= \frac{\pi}{2}\rho U^2 R^2 \tag{5}$$

(b) Therefore when U increases from U to $U+\delta U$, the kinetic energy increases by $\pi\rho r^2 U\delta U$. This additional energy must come from the work done by the cylinder on the fluid due to the drag, F, acting in the negative x direction.

(c) If this change occurs in a time δt, the distance over which the force F acts is $U\delta t$ and therefore

$$F \ u\delta t = \pi\rho R^2 u \ \delta U \quad \text{and} \quad F = \pi\rho R^2 \frac{\delta U}{\delta t} \tag{6}$$

and the quantity $\rho\pi R^2$ is known as the "added mass".