## An Internet Book on Fluid Dynamics

## Solution of Problem 122B:

The potential flow around a sphere is generated by the superposition of a point doublet whose potential flow has the form $B \cos \theta / r^{2}$ and a uniform stream, $U x$ : Therefore


$$
\begin{equation*}
\phi=U x+\frac{B \cos \theta}{r^{2}}=\cos \theta\left\{U r+\frac{B}{r^{2}}\right\} \tag{1}
\end{equation*}
$$

It follows that the velocities in the $r$ and $\theta$ directions, $u_{r}$ and $u_{\theta}$, are

$$
\begin{align*}
& u_{r}=\frac{\partial \phi}{\partial r}=\cos \theta\left\{U-\frac{2 B}{r^{3}}\right\}  \tag{2}\\
& u_{\theta}=\frac{1}{r} \frac{\partial \phi}{\partial \theta}=-\frac{\sin \theta}{r}\left\{U r+\frac{B}{r^{2}}\right\} \tag{3}
\end{align*}
$$

But on the surface of the sphere, $r=R$, we must have $u_{r}=0$. Therefore

$$
\begin{equation*}
U-\frac{2 B}{R^{3}}=0 \text { and therefore } B=\frac{U R^{3}}{2} \tag{4}
\end{equation*}
$$

Hence

$$
\begin{equation*}
u_{\theta}=\frac{\sin \theta}{r}\left\{U r+\frac{U R^{3}}{2 r}\right\} \tag{5}
\end{equation*}
$$

and on the surface

$$
\begin{equation*}
\left\{u_{\theta}\right\}_{r=R}=-\frac{\sin \theta}{R}\left\{U R+\frac{U R}{2}\right\}=-\frac{3 U}{2} \sin \theta \tag{6}
\end{equation*}
$$

and therefore the ratio of the maximum velocity on the surface to the uniform stream velocity is $3 / 2$.

