Problem 120J

The velocity potential for the total flow is given by

$$\phi = Ux + \frac{B(x+a)}{(x+a)^2+y^2} + \frac{B(x-a)}{(x-a)^2+y^2}$$

$$\psi = Uy - \frac{By}{(x+a)^2+y^2} - \frac{By}{(x-a)^2+y^2}$$

Uniform stream | Doublet @ x = -a | Doublet @ x = a

The velocities are given by

$$u = \frac{\partial\phi}{\partial x} = U + \frac{B\left\{y^2-(x+a)^2\right\}}{\left\{(x+a)^2+y^2\right\}^2} + \frac{B\left\{y^2-(x-a)^2\right\}}{\left\{(x-a)^2+y^2\right\}^2}$$

$$v = \frac{\partial\phi}{\partial y} = -\frac{2B(x+a)y}{\left\{(x+a)^2+y^2\right\}^2} - \frac{2B(x-a)y}{\left\{(x-a)^2+y^2\right\}^2}$$

At the critical value for B, the bodies touch at the origin, which means that the origin should be a stagnation point, with $u=0$.

$$u\Big|_{x=0,y=0} = U - \frac{2B}{a^2}$$

Therefore the condition for flow around a single body is

$$B > \frac{Ua^2}{2}$$