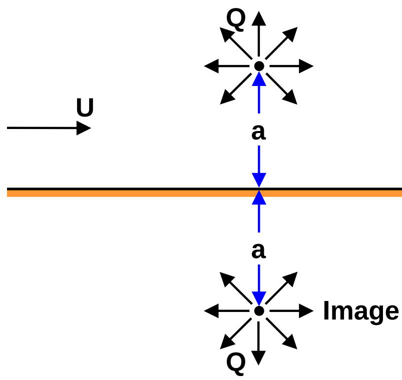
Solution to Problem 120D

The source requires an image source in order to satisfy the boundary condition on the wall:



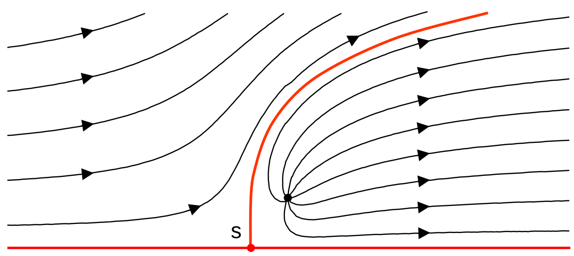
and therefore the velocity potential, ϕ, for the flow is

$$\phi = \underbrace{Ux}_{\text{Uniform Stream}} + \underbrace{\frac{Q}{4\pi}\ln\left\{x^2 + (y-a)^2\right\}}_{\text{Source}} + \underbrace{\frac{Q}{4\pi}\ln\left\{x^2 + (y+a)^2\right\}}_{\text{Image Source}}$$

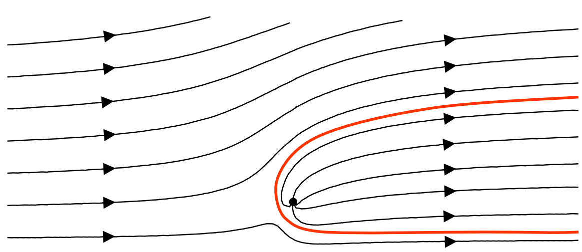
It follows that

$$u|_{y=0} = \left.\frac{\partial\phi}{\partial x}\right|_{y=0} = U + \frac{Qx}{\pi(x^2+a^2)}$$

Now the flow from the source will only come in contact with the wall if Q is strong enough so that the flow pattern looks like:



If it is weaker then:



Therefore, if there is a stagnation point, s, the source fluid will contact the wall. If there is no stagnation point it will not. Now a stagnation point where $u|_{y=0} = 0$ occurs when

$$Qx = -\pi(x^2+a^2)U$$

$$x = \frac{1}{2}\left[\pm\left(\frac{Q^2}{\pi^2 U^2} - 4a^2\right)^{\frac{1}{2}} - \frac{Q}{\pi U}\right]$$

Consequently, for a stagnation point to exist requires that

$$\begin{aligned} & \frac{Q}{\pi U} > 2a \\ \rightarrow \quad & Q > 2\pi a U \end{aligned}$$

and this is the condition that the source fluid reach the wall.