## Solution to Problem 120C

This problem is to find the angular rate of rotation of a hurricane and the velocity of the wind at the edge of the core. The properties of the hurricane are:

$$R = 40\ m \qquad Q = 5000\ m^2/s \qquad p_\infty - p_R = \Delta p = 1500\ kg/m^2 s \qquad \rho = 1.2\ kg/m^3$$

The flow outside the core of the hurricane can be modelled as the combination of a sink and a vortex.

$$\phi = \underbrace{-\frac{Q}{2\pi}\ln r}_{\text{Sink}} + \underbrace{\frac{\Gamma}{2\pi}\theta}_{\text{Vortex}}$$

The velocity components will then be:

$$u_r = \frac{\partial \phi}{\partial r} = -\frac{Q}{2\pi r} \quad \text{and} \quad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{\Gamma}{2\pi r}$$

Evaluating the velocity at the edge of the core ($r = R$):

$$u_r = -\frac{Q}{2\pi R} \quad \text{and} \quad u_\theta = \frac{\Gamma}{2\pi R}$$

$$\Rightarrow |u|^2_{r=R} = {u_r}^2 + {u_\theta}^2 = \frac{Q^2}{4\pi^2 R^2} + \frac{\Gamma^2}{4\pi^2 R^2}$$

Since the flow is irrotational outside of the core, we can apply Bernoulli's equation. Applying the equation at the edge of the core and evaluating the constant far from the core ($r \to \infty$):

$$\begin{aligned} \frac{1}{2}\rho|u|^2_R + p_R &= \frac{1}{2}\rho|u_\infty|^2 + p_\infty \\ \frac{1}{2}\rho|u|^2_R &= p_\infty - p_R = \Delta p \\ |u|^2_{r=R} &= \frac{2\Delta p}{\rho} = 50\ m/s \end{aligned}$$

Solving for $\Gamma$:

$$\begin{aligned} \frac{2\Delta p}{\rho} &= \frac{Q^2}{4\pi^2 R^2} + \frac{\Gamma^2}{4\pi^2 R^2} \\ \Gamma &= \sqrt{\frac{8\pi^2 R^2 \Delta p}{\rho} - Q^2} \end{aligned}$$

Consequently the angular rate of rotation of the hurricane is:

$$\omega = \frac{u_\theta}{R} = \frac{\Gamma}{2\pi R^2} = 1.147\ rad/s$$