Solution to Problem 120B

The fluid injection at the point B can be modelled as a source of strength 2Q. In order to satisfy the boundary condition on the vertical axis, an image source of the same strength needs to be added at $x = -a$. This gives a total velocity potential of :

$$\phi = \frac{1}{2}C\left(x^2 - y^2\right) + \frac{2Q}{2\pi}\ln\left[\sqrt{(x-a)^2 + y^2}\right] + \frac{2Q}{2\pi}\ln\left[\sqrt{(x+a)^2 + y^2}\right]$$

Consequently the velocity u is

$$u = \frac{\partial\phi}{\partial x} = Cx + \frac{Q}{\pi}\frac{(x-a)}{\{(x-a)^2 + y^2\}} + \frac{Q}{\pi}\frac{(x+a)}{\{(x+a)^2 + y^2\}}$$

and therefore on the x-axis

$$u|_{y=0} = Cx + \frac{Q}{\pi(x-a)} + \frac{Q}{\pi(x+a)}$$

Therefore the point $x = x^*$ where $(u)_{y=0} = 0$ is given by

$$Cx^* + \frac{Q}{\pi(x^* - a)} + \frac{Q}{\pi(x^* + a)} = 0$$

and therefore

$$x^* = \left[a^2 - \frac{2Q}{\pi C}\right]^{\frac{1}{2}}$$