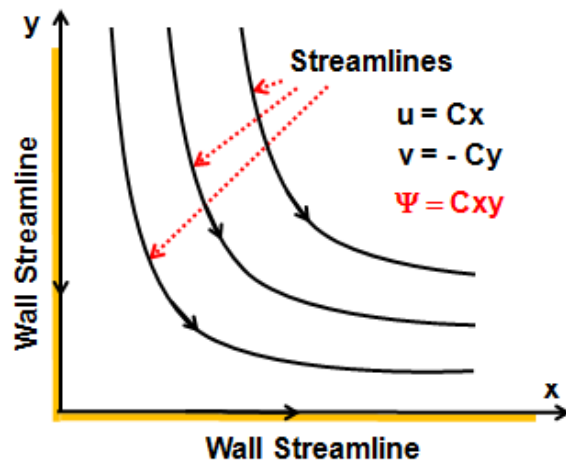## Solution to Problem 115I:
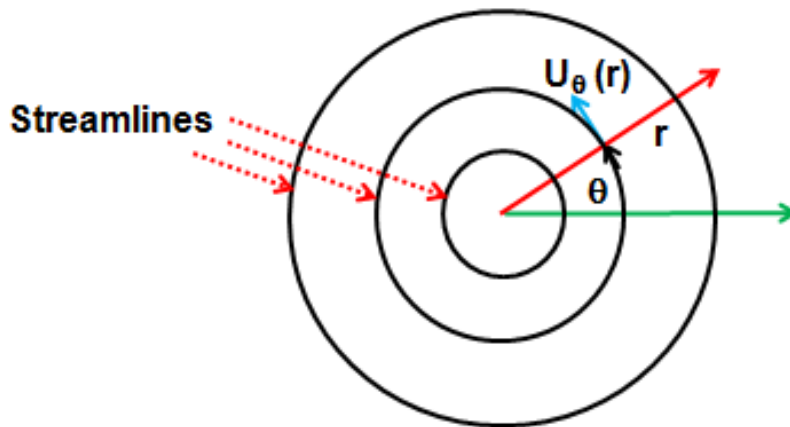
The following simple flows:

[a] Stagnation Point Flow, $(u, v, w) = (kx, -ky, 0)$: The streamlines are rectangular hyperbolae in which

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \tag{1}$$

so this flow is irrotational.

[b] Free Vortex Flow, $(u_r, u_\theta, u_z) = (0, -\Gamma/2\pi r, 0)$: The streamlines are circles with

$$u = -u_\theta \sin\theta = -\frac{\Gamma}{2\pi} \sin\theta = \frac{\Gamma}{2\pi r} \frac{y}{(x^2 + y^2)} \tag{2}$$

$$v = u_\theta \cos\theta = -\frac{\Gamma}{2\pi} \cos\theta = -\frac{\Gamma}{2\pi} \frac{x}{(x^2 + y^2)} \tag{3}$$

$$w = 0 \tag{4}$$

and

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \tag{5}$$

so this flow is irrotational.

[c] Solid Body Rotation: $(u_r, u_\theta, u_z) = (0, \Omega r, 0)$:

$$u = -\Omega r \sin\theta = -\Omega y \tag{6}$$

$$v = \Omega r \cos\theta = \Omega x \tag{7}$$

$$w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \Omega - (-\Omega) = 2\Omega \tag{8}$$

Note the vorticity is uniform everywhere and equal to twice the rate of rotation. So this flow is not irrotational.