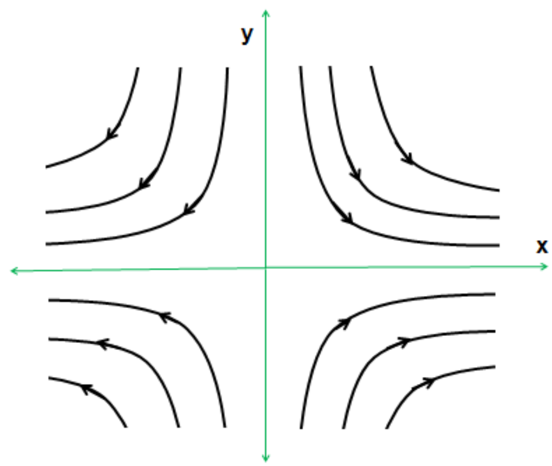
Solution to Problem 115E:

On the planar, incompressible flow given by $\psi = Axyt$:

(a) Streamlines are lines of constant ψ at some specific time, t, and the equation for those lines is therefore

$$xy \;=\; \psi/At \;=\; \text{constant} \tag{1}$$

so the flow is as follows:



(b) The velocities of the flow are:

$$u(x,y,t) \;=\; \frac{\partial\psi}{\partial y} \;=\; Axt \tag{2}$$

$$v(x,y,t) \;=\; -\frac{\partial\psi}{\partial x} \;=\; -Ayt \tag{3}$$

(c) The pathlines of the flow: Since $u = Axt = dx/dt$ along a pathline it follows by integration that

$$\ln x \;=\; At^2/2 \;+\; \text{constant} \tag{4}$$

and the constant is $\ln x_0$ so that

$$x \;=\; x_0 \exp\left(At^2/2\right) \tag{5}$$

Similarly since $v = -Ayt = dy/dt$ along the pathline

$$y \;=\; y_0 \exp\left(-At^2/2\right) \tag{6}$$