Problem 272C

Consider a steady turbulent planar Poiseuille flow of an incompressible fluid of density, ρ, between two smooth flat plate distance, h, apart and driven by a pressure gradient, $-dp/dx$, in a direction, x, parallel with those plates. If the friction factor, f, of this duct is defined as

$$f \;=\; \frac{2h}{\rho V^2}\left(-\frac{dp}{dx}\right) \tag{1}$$

where V is the average volumetric velocity of the flow, find the relation between wall shear stress, τ_w, and ρ, f and V. If the friction velocity, u_τ, is defined as $\sqrt{\tau_w/\rho}$ what is the relation between u_τ/V and f?

If the mean fluid velocity profile, $\overline{u}(y)$, in the duct (y is the distance from the mid-line of the duct) is given by

$$\overline{u}(y) \;=\; 5.75\log_{10}\left(\frac{yu_\tau}{\nu}\right) + 5.5 \tag{2}$$

what is the relation between f and the Reynolds number, $Re = hV/\nu$, of the flow (ν is the kinematic viscosity of the fluid)?