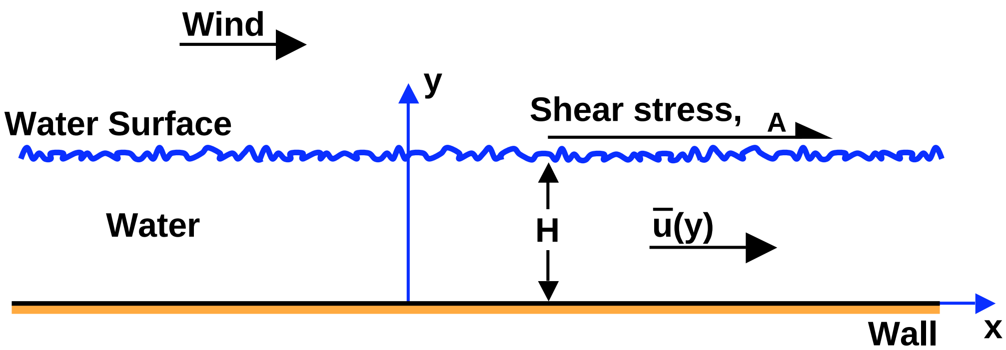
Problem 272A

A high wind drives a film of water over a solid surface at such a speed that the flow in the film becomes turbulent. This occurs because the wind applies a shear stress, τ_A, to the surface of the water:



The thickness of the film, H, and the mean water velocity, $\bar{u}(y)$, are constant in time and with position, x. Using the assumptions listed below, find an expression for the mean velocity on the water surface, $\bar{u}(H)$, in terms of τ_A, H, ρ (the water density), ν (the kinematic viscosity of the water), and the Karman universal constant, κ. The assumptions:

- The laminar sublayer next to the solid surface (in which $u^* = y^*$) extends to $y^* = 5$ where the mean velocity is to be matched with that of the turbulent flow in the rest of the water film.
- Outside the laminar sublayer, the Reynolds stresses dominate and the viscous component of the shear stress can be neglected.
- Prandtl's mixing length theory is to be used with a Karman universal constant denoted by κ.