Problem 269A

This problem involves the development of a "law of the wall" for turbulent flow near a rough wall (rather than a smooth wall as was assumed in class). The typical height of the roughness on the wall is represented by a length, ϵ.

1. What is the condition for which the roughness will extend to the same height as the hypothetical laminar sublayer that would be present if the wall were smooth? [The answer involves the wall shear stress, τ_w, the fluid density, ρ and the fluid kinematic viscosity, ν as well as ϵ. Assume the laminar sublayer for a smooth wall extends to $y^* = 5$.]

2. If the roughness is larger than the thickness of the hypothetical laminar sublayer, then we imagine that there is no laminar sublayer and the turbulent flow, dominated by the Reynolds shear stress, extends all the way to the wall. Consequently there is no region in which the laminar shear stress is significant and therefore the viscosity should not appear in the modified law of the wall. Repeat the dimensional arguments used to find the "law of the wall" in the case of a smooth wall but now without viscosity and with the quantity ϵ. What is the modified "law of the wall" under these circumstances?

3. Using the above result and using Prandtl's mixing length model with a universal constant, κ, determine the velocity profile in the turbulent flow near a rough wall when the roughness size, ϵ, is greater than the thickness of the hypothetical laminar sublayer.