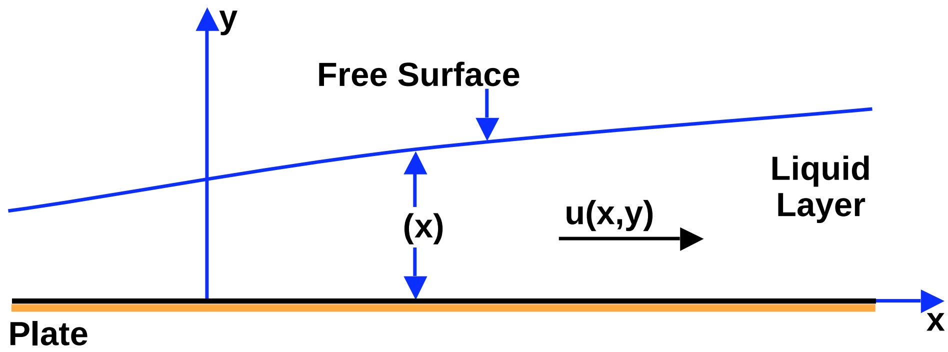
Problem 250D

A thin layer of viscous fluid (kinematic viscosity, ν) is flowing over a flat horizontal plate. The flow is steady, planar and laminar. The upper *free* surface experiences a uniform pressure and surface tension effects are negligible. The thickness of the layer, $\delta(x)$, which is to be determined is assumed to vary slowly with x so that methods similar to those used for approximate boundary analysis are applicable :



Assume self-similar velocity profiles within the layer so that the following profile parameters can be regarded as known constants :

$$\alpha = \int_0^1 \left(\frac{u}{U}\right) d\left(\frac{y}{\delta}\right)$$

$$\beta = \left(\frac{d(u/U)}{d(y/\delta)}\right)_{y=0}$$

$$\gamma = \int_0^1 \left(\frac{u}{U}\right)^2 d\left(\frac{y}{\delta}\right)$$

where $U(x)$ is the velocity in the x direction on the surface of the liquid. Note that U will be a function of x. Also assume that the pressure is constant everywhere in the liquid; in other words neglect the effect of gravity on the pressure.

Find an expression for the thickness of the layer, $\delta(x)$, in terms of the known constants ν, α, β, γ, the mass flow rate per unit distance perpendicular to the figure, m, and the density, ρ, of the liquid. Assume that the thickness at $x = 0$ has a known value, δ_0.