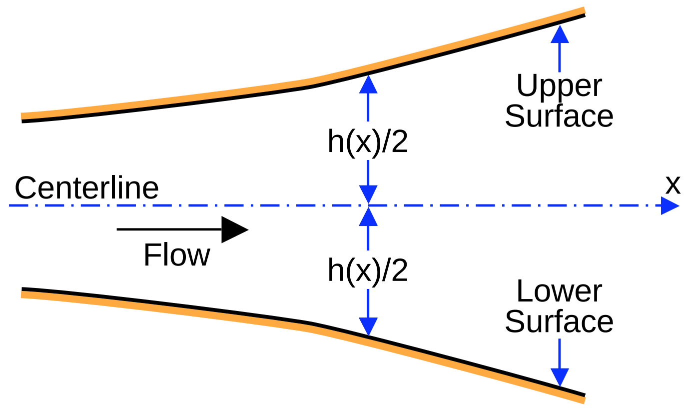
Problem 241C

The working section of a water tunnel consists of a duct with a rectangular cross-section. The width of the cross-section, b (perpendicular to sketch), is constant but the height, $h(x)$, may vary with longitudinal distance, x, measured along the centerline of the duct:



Laminar boundary layers form on the upper and lower surfaces of the working section and would cause an acceleration of the flow if the height h were constant with x. [A similar effect would be caused by the front and back surfaces but we ignore this for the purposes of this problem and assume that there are no boundary layers on the front and back surfaces.] A water tunnel designer wishes to select the function $h(x)$ in order to ensure that the pressure and velocity outside the boundary layer (say, on the centerline) vary with x in a specified way. The designer decides to use the Falkner-Skan solutions and functions of the form

$$h(x) = h_0 + Hx^k \tag{1}$$

where h_0, H and k are constants and the boundary layers begin at $x = 0$. Find:

1. The value of k which produces zero longitudinal pressure gradient in the tunnel. Also find the expression for H in terms of b, the kinematic viscosity, ν, and the velocity, U, of the flow on the centerline.

2. The value of k which produces a uniform acceleration, $C = dU/dx$, on the centerline of the tunnel. Also find the relation between C and h_0 .

[Note: The answer to the second question contains a dimensionless integral, $I(m)$, which is a function of the Falkner-Skan parameter, m, and is defined by

$$I(m) = \int_0^\infty \left[1 - \frac{dF}{d\eta}\right] d\eta \tag{2}$$

where $F(\eta)$ and η are respectively the Falkner-Skan solution and similarity variable. You may express your answer to the second question using one of these functions.