Problem 210A

A horizontal oil pipeline is 30 km long, has a circular cross-section with an internal diameter of 0.4 m and a friction factor, f, of 0.02. A pump at inlet generates flow through the pipeline. The total pressures at inlet to the pump and at discharge from the end of the pipeline are equal to atmospheric pressure at all times. When running, the pump generates a total head rise, ΔH, which may be a function of the instantaneous flow rate, Q (in m/s), through the pipeline. With everything initially at rest, the pump is switched on at time, $t = 0$, and the flow rate, $Q(t)$, increases up to some asymptotic value, $Q(\infty)$, at large times. Assuming that the oil is incompressible and that the pipe is rigid find

(i) the rate of increase of the flow rate with time, dQ/dt, at $t = 0$,

(ii) the asymptotic flow rate, $Q(\infty)$,

(iii) the integral which would need to be evaluated to find $Q(t)$ if

(a) the pump generates a constant ΔH of 200 m independent of Q

(b) the pump generates a ΔH which decreases with Q according to

$$\Delta H = 200 - 1000Q \quad for \quad Q > 0$$

where ΔH is in m and Q is in m^3/s.