Problem 150N

A spherical gas bubble of radius, R, is expanding at a radial velocity, V, in a Newtonian liquid of viscosity, μ, and density, ρ. The expansion is spherically symmetric so that the only non-zero velocity in the liquid is u_r, the radial velocity. The surface tension of the interface is S. What is the relation between the gas pressure, p_G, and the pressure, p, in the liquid immediately surrounding the bubble? The answer includes R, V, μ, ρ and S.

Note: The constitutive laws for a Newtonian liquid when written in spherical coordinates, (r, θ, ϕ), with velocities u_r, u_θ, u_ϕ in the r, θ, ϕ directions become:

$$\sigma_{rr} = -p + 2\mu \frac{\partial u_r}{\partial r} \quad ; \quad \sigma_{\theta\theta} = -p + 2\mu \left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right)$$

$$\sigma_{\phi\phi} = -p + 2\mu \left(\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} + \frac{u_\theta \cot\theta}{r} \right)$$

$$\sigma_{r\theta} = \mu \left(r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta} \right) \quad ; \quad \sigma_{r\phi} = \mu \left(\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{u_\phi}{r}\right) \right)$$

$$\sigma_{\theta\phi} = \mu \left(\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi} + \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{u_\phi}{\sin\theta}\right) \right)$$