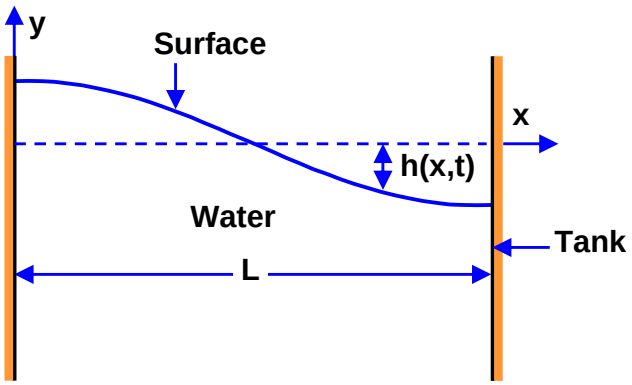
Problem 130C

Water is sloshing back and forth between two infinite vertical walls separated by a distance, L:



The flow is assumed to be planar, incompressible, inviscid potential flow. The free surface is devoid of surface tension and is at constant atmospheric pressure. Its position is described by $h(x,t)$ as indicated in the sketch. The wave height, $h(x,t)$, is small so that the assumptions of linear water wave theory may be used. An appropriate velocity potential for this flow is

$$\phi = Ae^{ky} \cos kx \sin \omega t$$

where A, k and ω are undetermined constants.

(a) What are the four boundary conditions which a solution to this flow must satisfy ?

(b) Find the series of values which are possible for the wavelength, λ ($\lambda = 2\pi/k$), of the free surface waves. Each of these wavelengths corresponds to a particular mode of sloshing.

(c) Use the kinematic condition on the free surface to determine the shape of the free surface, $h(x,t)$, as a function of A, k, ω, x and t.

(d) Use the dynamic condition on the free surface to determine the frequency, f ($f = \omega/2\pi$), for each of the modes of sloshing. Denote the acceleration due to gravity by g.