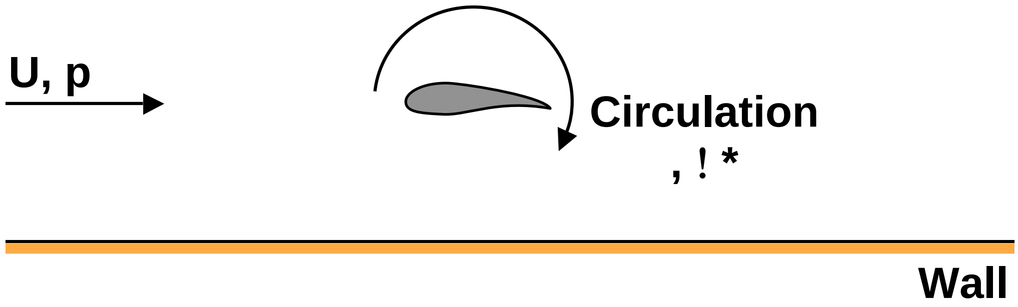
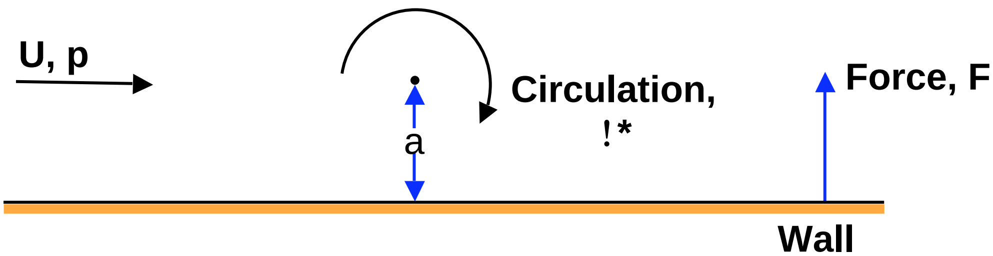
Problem 120H

To a first approximation, the potential flow around an airfoil may be simulated by a simple potential vortex. In this problem we consider some of the consequences of the planar potential flow around an airfoil close to a plane wall by simulating the following flow,



by the following:



Note the convention used for the direction of rotation of the circulation, Γ^* which is opposite to that we used in class and therefore a sign change is necessary. Calculate the force, F (per unit depth normal to the sketch), acting on the wall in the upward direction assuming that the pressure underneath the wall is p_∞, the same as in the flow far from the airfoil. (Thus each element of length, dx, of the wall contributes $(p_\infty - p)dx$ to the force, F.)

Since no net force can act on the body of the fluid, it follows that a downward force of magnitude F acts on the airfoil. Use this result to derive an expression for the lift, L (per unit depth), acting on the foil in terms of ρ (the fluid density), Γ^* (the circulation of the vortex as defined in the sketch), U (the free stream velocity), and a (the distance from the vortex to the wall). Note that the lift is the force on the foil in a direction perpendicular to the oncoming stream. What can you conclude regarding the effect of the neighbouring wall on the lift?

Note:

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+a^2)} = \frac{\pi}{a} \quad ; \quad \int_{-\infty}^{+\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{2a^3}$$