Problem 102D

A vertical cylinder containing a liquid of density, ρ_L , has air of density, ρ_A , injected into it so that the cylinder is filled with a bubbly mixture. Any unit volume of this mixture contains a volume α of air and a volume $(1 - \alpha)$ of liquid; the quantity α is called the void fraction. What is the effective density of the mixture, ρ , in terms of α , ρ_L , and ρ_A ?

Since $\rho_A \ll \rho_L$ you may set $\rho_A = 0$ for the purposes of the second part of this problem.



The legs of an inverted air-liquid manometer are connected to the sides of the cylinder at two points which are a height, H, apart as shown in the figure. The levels in this manometer register an elevation difference which we will denote by h. Assume that the bubbles in the cylinder do not enter the manometer tubes which are therefore filled with pure liquid. Show that the manometer can be used to measure the void fraction by finding the relation between α and the lengths h and H.